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Integer Programming for Optimal Control of Geostationary Station Keeping of Low-Thrust Satellites

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Abstract: A control scheme is elaborated to perform the station keeping of a geostationary satellite equipped with electric propulsion. The use of electric thrusters imposes to take into account some additional mutually exclusive constraints on the control function that can be reformulated as logical constraints. The resulting fuel optimal station keeping problem is thus not solved with classical methods, either direct or indirect, but is transformed into a linear integer programming problem. The linearised relative velocity of the satellite is computed and some linear constraints on this velocity are added to the original station keeping problem in order to perform the station kepping over one week after another. Simulation results validate the efficiency of the optimal control thrusts obtained by the solution of the overall linear integer programming problem.

Keywords: Satellite control, GEO satellite station keeping, electric propulsion, integer programming.

1. INTRODUCTION

Satellites operating in Geostationary Earth Orbit (GEO) naturally drift outside their station keeping (SK) window (a rectangular box of a given geographical longitude and latitude range) due to orbital disturbing forces. Performing an accurate SK strategy is therefore necessary by using electric and/or chemical thrusters in order to counteract the effect of orbital perturbations.

Chemical propulsion systems have been and are still widely used. For these propulsion systems with high thrust capabilities, SK control laws are usually designed assuming an impulsive idealisation of the thrust, as described for example in Soop (1994). Electric propulsion for station keeping has been used since the late sixties (see, e.g. the work of Barrett (1967) and Hunziker (1970)) and some theoretical developments have been presented in the eighties by Anzel (1988) and Eckstein (1980). Nowadays, the SK of GEO satellites can be performed reliably by electric as well as by the chemical propulsion. The bigger specific impulsion of electric thrusters leads to a consequent savings in fuel consumption, what enables a reduction of the satellite mass and increased payload capacity and/or improved satellite lifetime.

Considering these technological and operational features, optimal control strategies for electrical SK, taking various constraints into account (minimum elapsed time between two consecutive firings, on-off profile of the thrusters, thrust allocation) have to be carefully designed. Some references as for example Sidi (1997), Campan et al. (1995) or Soop (1994) have established some rule of thumb based on the physical analysis of the effect of the external forces in order to perform the SK. In general, the problem of station keeping can be expressed as an optimal control problem even if the above mentioned constraints, inherent to the use of electric thrusters, prevent us to solve it with classical methods.

Optimal SK control problems may be solved according to several approaches. When simplifying assumptions are used, analytical control laws may be obtained, as in Sukhanov and Prado (2012). Otherwise, it is in general necessary to resort to numerical methods, such as direct collocation based methods as described in Hull (1997) and Betts (1998). For this kind of approaches integrating the satellite dynamic, the state and the control variables are discretised to produce a non linear programming problem and get an optimal open loop control. To deal with on-off models of the thrusts, rectangular profiles may be generated from a continuous one with the Pulse Width Modulation technique (see Vazquez et al. (2015) and the references therein). Losa et al. (2005) has formulated a method based on differential inclusion and Losa et al.

(2006) and Gazzino et al. (2016) have implemented a decomposition technique. In this last reference, an equivalent problem has been tackled by means of a two-step methodology combining the application of the maximum principle on a simplified version of the SK problem and a transcription method initialized with the solution of the first step. Even if the proposed two-step decomposition succeeds in finding a feasible plan of manoeuvers, the difficulty of tuning the numerous parameters due to their high sensitivity clearly leaves room for improvement.

The approach developed in this paper is a more straightforward process with few tuning parameters and only one step. If the operational constraints cited above are difficult to account for, the present work take advantage to transform the linearised SK optimal control problem in a linear integer programming problem, using a discretisation of the control variables. Unlike the direct collocation method used in Gazzino et al. (2016), the dynamic equation of the satellite evolution is integrated beforehand to provide its state transition matrix. Using the computed transition matrix, the dynamic constraint is removed and the optimal variable set is only composed of the control vector. Instead of removing the operational constraints, that are difficult to deal with in the Pontryagin Maximum Principle framework, and finding afterwards an equivalent control strategy respecting these constraints, their particular disjuntive structure is used in order to express them as logical constraints such that the control variables are now binary variables in this formulation. A simple way of transforming the logical constraints in binary variables constraints is investigated. A realistic numerical example illustrates the efficiency of the proposed approach and some comparison are made between a rule of thumb for SK strategies found in the literature and the systematic optimisation based approach presented in this paper.

2. STATION KEEPING PROBLEM

2.1 Dynamic Modeling

Let us consider a satellite equipped with 4 electric thrusters mounted on the anti-nadir face. The position of the satellite on its orbit is described with the equinoctial orbit elements as defined in Battin (1999):

$$\boldsymbol{x}_{eoe} = \begin{bmatrix} a & e_x & e_y & i_x & i_y & \ell_{M\Theta} \end{bmatrix}^t \in \mathbb{R}^6, \tag{1}$$

where a is the semi-major axis, (e_x, e_y) are the eccentricity vector components, (i_x, i_y) are the inclination vector components, $\ell_{M\Theta} = \omega + \Omega + M - \Theta$ is the mean longitude where Ω is the right ascension of the ascending node, ω is the argument of perigee, M is the mean anomaly and $\Theta(t)$ is the right ascension of the Greenwich meridian. The dynamics of the satellite may be represented by the following non linear state-space model:

$$\frac{d\boldsymbol{x}_{eoe}}{dt} = \boldsymbol{f_L}(\boldsymbol{x}_{eoe}, t) + \boldsymbol{f_G}(\boldsymbol{x}_{eoe}, t)\boldsymbol{u}, \tag{2}$$

where $\boldsymbol{f_L} \in \mathbb{R}^6$ is the Lagrange contribution part of the external force model described by the CNES ORANGE model (cf. Campan and Brousse (1994)) and $\boldsymbol{f_G} \in \mathbb{R}^{6\times 3}$ is the Gauss contribution part.

In order to deal with the station keeping problem, the relative state of the satellite with respect to the station keeping state:

$$\boldsymbol{x}_{sk} = \begin{bmatrix} a_{sk} & 0 & 0 & 0 & 0 & \ell_{M\Theta_{sk}} \end{bmatrix}^t, \tag{3}$$

is defined, where a_{sk} is the synchronous semi-major axis and $\ell_{M\Theta_{sk}}$ is the station mean longitude.

The relative dynamics are developed by linearisation of Equation (2) about the station keeping point (3). By denoting $\boldsymbol{x} = \boldsymbol{x}_{eoe} - \boldsymbol{x}_{sk}$ the relative state model reads:

$$\dot{\boldsymbol{x}} = A(t)\boldsymbol{x} + \boldsymbol{D}(t) + B(t)\boldsymbol{u}, \tag{4}$$

where the matrices:

$$A(t) = \frac{\partial}{\partial \boldsymbol{x}_{eoe}} \left(\overrightarrow{f_L}(\boldsymbol{x}_{eoe}(t), t) \right) \Big|_{\boldsymbol{x}_{eoe} = \boldsymbol{x}_{sk}} \in \mathbb{R}^{6\times 6}, \tag{5}$$

$$B(t) = f_G(\boldsymbol{x}_{sk}, t) \in \mathbb{R}^{6\times 3}, \tag{6}$$

$$\boldsymbol{D}(t) = \boldsymbol{f_L}(\boldsymbol{x}_{sk}, t) \in \mathbb{R}^6, \tag{7}$$

are obtained from the linearisation of functions $\boldsymbol{f_L}$ and $\boldsymbol{u} \mapsto f_G\boldsymbol{u}$.

As opposed to what is done in the literature, the linearisation is not performed with respect to an equilibrium point, but around a fictitious geostationary point. In other words, the equinoctial orbital elements $\boldsymbol{x}_{eoe}$ undergo all disturbing external forces whereas the geostationary equinoctial elements $\boldsymbol{x}_{sk}$ evolve following a keplerian motion. These two different dynamics for the state vector and the station keeping state vector explains the term $\boldsymbol{D}(t)$ in the relative state model (4).

Recalling that 4 thrusters are available to realize the control, the transcription of the station keeping problem expressed in terms of the control vector $\boldsymbol{u}$ would require to solve an allocation problem to find a right combination of thrusts. An alternative consists in considering directly the 4 thrusts provided by the 4 engines in the satellite dynamic. The control $\boldsymbol{u}(t)$ can be written as a linear combination of the 4 thrusts such that $\boldsymbol{u} = \Gamma\boldsymbol{F}$, where $\boldsymbol{F} \in \mathbb{R}^4$, $\Gamma \in \mathbb{R}^{3\times 4}$. In order to model the on-off nature of the control profile, the thrust vector $\boldsymbol{F} = F_{max}\begin{bmatrix} F_1 & F_2 & F_3 & F_4 \end{bmatrix}^t$ is composed of four normalized variables scaled by F_{max}, the mximum thrust delivered by each propeller. The thrust direction matrix $\Gamma = \begin{bmatrix} \boldsymbol{\Gamma}_1 & \boldsymbol{\Gamma}_2 & \boldsymbol{\Gamma}_3 & \boldsymbol{\Gamma}_4 \end{bmatrix}^t \in \mathbb{R}^{3\times 4}$ is defined such that each thrust direction $\boldsymbol{\Gamma}_j \in \mathbb{R}^3$ are given by :

$$\boldsymbol{\Gamma_j} = \frac{1}{m}\begin{bmatrix} -\sin\theta_j\cos\alpha_j & -\sin\theta_j\sin\alpha_j & -\cos\theta_j \end{bmatrix} \tag{8}$$

where angles θ_j and α_j are defined exactly as in Anzel (1988). Finally, the relative state model for the addressed SK problem is given by:

$$\dot{\boldsymbol{x}} = A(t)\boldsymbol{x} + \boldsymbol{D}(t) + B(t)\Gamma\boldsymbol{F}, \tag{9}$$

The geographical coordinates of the satellite:

$$\boldsymbol{y}_{eoe} = T(\boldsymbol{x}_{eoe}, t)\boldsymbol{x}_{eoe}, \tag{10}$$

are the variables of interest since the station keeping problem consists in constraining them in the vicinity of

the station position $\boldsymbol{y}_{sk} = \begin{bmatrix} r_{sk} & 0 & \lambda_{sk} \end{bmatrix}^t$ where r_{sk} is the synchronous radius and λ_{sk} is the station keeping geographical longitude. The relative geographical position with respect to the station-keeping position is denoted by:

$$\boldsymbol{y} = \boldsymbol{y}_{eoe} - \boldsymbol{y}_{sk} = T(\boldsymbol{x}_{sk}, t)\boldsymbol{x} = C(t)\boldsymbol{x}, \tag{11}$$

by linearising Equation (10).

The relative geographical velocity of the satellite is computed by linearising the derivative of Equation (10). As done with the linearisation of the derivative of the state vector, the station-keeping position is a fictitious point supposed to evolve following a keplerian motion, whereas $\boldsymbol{y}_{eoe}$ undergoes all the external forces. The dynamics of the relative geographical position reads thus:

$$\dot{\boldsymbol{y}}(t) = E(t)\boldsymbol{x}(t) + C(t)\boldsymbol{D}(t) + F_{max}C(t)B(t)\Gamma\tilde{\boldsymbol{F}}(t), \tag{12}$$

$$\text{with } E(t) = 2H(t) + C(t)A(t) + \dot{C}(t) \in \mathbb{R}^{3\times 6} \tag{13}$$

$$\text{and } H(t) = \left.\frac{\partial[T(\boldsymbol{x}_{eoe}, t)]}{\partial \boldsymbol{x}_{eoe}}\right|_{\boldsymbol{x}=\boldsymbol{x}_{sk}} \boldsymbol{D}(t), \tag{14}$$

2.2 Station Keeping as a Constrained Optimal Control Problem

The objective of the system is to maintain the longitude and the latitude of the satellite in a box defined by its size δ on a fixed time horizon by acting on the orbital parameters via the 4 thrusters. An open loop control law can thus be obtained by solving an associated optimal control problem. In this context, optimality means that a minimum fuel-solution is looked for to extend the operational life time of the satellite. Therefore, performing minimum-fuel station keeping amounts to minimizing the following performance index under the dynamic constraint (9):

$$\hat{J} = \int_0^T \sum_{\text{thruster } i=1}^{4} \left(|u_{R_i}(t)| + |u_{T_i}(t)| + |u_{N_i}(t)|\right) dt$$

$$\Leftrightarrow J = \sum_{\text{thruster } i=1}^{4} ||\boldsymbol{\Gamma}_i||_1 \int_0^T F_i(t)dt. \tag{15}$$

2.2.1 Station Keeping Requirements (Orbital Constraints)

Station keeping constraints are imposed to ensure that the satellite stays in the geographical box. This box is defined in the plane (latitude,longitude) of width $2\delta \times 2\delta$ centered on the station keeping geographical position. By denoting $\boldsymbol{y} = \boldsymbol{y}_{eoe} - \boldsymbol{y}_{sk}$, the station-keeping constraints over a finite time interval can be written as:

$$|[0\ 1\ 0]\boldsymbol{y}(t)| \leqslant \delta \text{ and } |[0\ 0\ 1]\boldsymbol{y}(t)| \leqslant \delta\ \forall t \in [0, T]. \tag{16}$$

The initial condition is given by $\boldsymbol{x}(0) = \boldsymbol{x}_0$ whereas the final state $\boldsymbol{x}(t_f)$ is free. The constraints on the geographical position may not suffice to perform the station keeping over several time intervals. For instance, the final position at $t = T$ may be close to the boundary of the SK window with the satellite velocity pointing outward. In such a case, it may not be possible for a limited-propulsion satellite to remain in the SK window for the ensuing time interval. For this reason, a constraint on the final velocity of the geographical variables is added:

$$|[0\ 1\ 0]\dot{\boldsymbol{y}}(T)| \leqslant \nu \text{ and } |[0\ 0\ 1]\dot{\boldsymbol{y}}(T)| \leqslant \nu, \tag{17}$$

where the parameter ν is chosen small.

2.2.2 Operational Constraints on Actuation

Beside the station keeping geographical constraints and the usual bounds on the maximum thrust, technological operational constraints on the actuation have to be taken into account:

(a) actuators can only provide on-off thrusts;
(b) thrusters cannot be active simultaneously;
(c) each thrust must last at least T_l (s);
(d) two successive thrusts of a given thruster must be separated of an interval of latency equal to $T_s = mT_l$ (s) with $m \in \mathbb{N}^\star$;
(e) two thrusts of two different thrusters must be separated by an interval of latency equal to $T_d = T_l$ (s).

Finally, the SK optimal control problem reads:

$$\min_{\boldsymbol{F}(t)} J = \sum_{i=1}^{4} ||\boldsymbol{\Gamma}_i||_1 \int_0^T F_i(t)dt, \text{ such that:}$$

$$\begin{cases} \dot{\boldsymbol{x}} = A(t)\boldsymbol{x} + \boldsymbol{D}(t) + B(t)\Gamma\boldsymbol{F}, & \\ \boldsymbol{x}(0) = \boldsymbol{x}_0, & \text{initial condition,} \\ |\dot{y}_2(T)| \leqslant \nu, |\dot{y}_3(T)| \leqslant \nu, & \text{final conditions} \\ |y_2(t)| \leqslant \delta \text{ and } |y_3(t)| \leqslant \delta\ \forall t \in [0, T] & \text{SK. const.} \\ \text{conditions } (a) - (e) & \text{control const.} \end{cases} \tag{18}$$

3. INTEGER FORMULATION OF THE SK PROBLEM

As proposed earlier, the Optimal Control Problem (OCP) (18) will be addressed in the sequel by means of a direct methodology. Note that solving (18) with indirect methods by applications of the classical optimality conditions (Pontryagin principle) is difficult due to the complexity of the operational constraints involved and the presence of state constraints. In the proposed methodology, the particluliar disjunctive structure of the operational constraints is investigated in order to transform them into linear integer constraints. The whole station keeping problem is then recast as a linear integer programming problem and solved with dedicated solvers.

3.1 Control Parametrization and Operational Constraints

We first parameterize the control profiles and then discretise the constraints in order to obtain an integer linear programming problem.

As mentioned before in condition (a), the electrical engines only produce on/off thrusts so that $\boldsymbol{F}_i$ at time τ_i can be considered to be a 4-dimensional binary vector expressed by:

$$\boldsymbol{F}(t) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \end{bmatrix}^t, \tag{19}$$

where $s_l \in \{0,1\},\ l = 1,\dots,4$.

The time interval $[0,\ T]$ is divided in N equal intervals $[\tau_i, \tau_{i+1}]$ with:

$$0 = \tau_0 < \tau_1 < \cdots < \tau_{N-1} < \tau_N = T. \tag{20}$$

The thrust vector $\boldsymbol{F}(t)$ will be considered constant over each intervals $[\tau_i,\ \tau_{i+1}]$, so that $\boldsymbol{F}(t)$ is completely parameterized by the vectors $\{\boldsymbol{F}_1,\dots,\boldsymbol{F}_N\}$:

$$\boldsymbol{F}(t) = \boldsymbol{F}_i, \quad \text{if } t \in [\tau_{i-1},\ \tau_i],\ i = 1,\dots,N. \tag{21}$$

In order to cope with condition (c), the length of each interval is chosen such that $\tau_{i+1} - \tau_i = T_l$.

Beside this, two different thrusters cannot thrust simultaneously according to conditions (b). Thus, the control profile of the satellite can be considered at each time to be a binary word that has to be chosen among the five possibilities:

$$\forall i,\ \boldsymbol{F}_i = \begin{bmatrix}(s_1)_i\\(s_2)_i\\(s_3)_i\\(s_4)_i\end{bmatrix} \in \left\{ \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \right\}. \tag{22}$$

With such a parametrisation of the control, it is possible to write the operational constraints (b)-(e) as logical constraints. Let us consider a discretisation $\{\tau_i\}$ of the time interval $[0,T]$ in sub-intervals of length T_l. Defining $F_i = S(\tau_i)$, the constraints (c) is automatically satisfied. The constraint (d) can be transformed to:

$$\forall i,\ \Big[(\boldsymbol{F}_i \neq 0)\ \&\ (\boldsymbol{F}_{i+1} = 0)\Big] \Rightarrow \Big[(\boldsymbol{F}_{i+2} \neq \boldsymbol{F}_i) \\ \&\ (\boldsymbol{F}_{i+3} \neq F_i)\ \&\dots\&\ (\boldsymbol{F}_{i+m} \neq F_i)\Big]. \tag{23}$$

The constraint (e) can be expressed as:

$$\forall i,\ \boldsymbol{F}_i \neq 0 \Rightarrow \boldsymbol{F}_{i+1} = \{0, \boldsymbol{F}_i\}. \tag{24}$$

3.2 State Transition and States Constraints Transcription

Using the transition matrix of the differential equations of the satellite state vector evolution (9) as defined in Antsaklis (2003), the state vector trajectory, the geographical positions and velocities can be evaluated at each discretisation point: $\forall j = 0,\dots,N$,

$$\boldsymbol{x}_j = \Phi_{j,0}\boldsymbol{x}_0 + \sum_{i=1}^{j} \Phi_{j,i} B_i^{\Phi} \boldsymbol{F}_i + \boldsymbol{D}^{\Phi}(\tau_j), \tag{25a}$$

$$\boldsymbol{y}_j = C_j \Phi_{j,\tau_0}\boldsymbol{x}_0 + \sum_{i=1}^{j} C_j \Phi_{j,i} B_i^{\Phi} \boldsymbol{F}_i + C_j \boldsymbol{D}_j^{\Phi}, \tag{25b}$$

$$\dot{\boldsymbol{y}}_j = \sum_{i=1}^{j} E_j \Phi_{ji} B_i^{\Phi} \boldsymbol{F}_i + C_j \tilde{B}_j \boldsymbol{F}_j + E_j \Phi_{j,0}\boldsymbol{x}_0 \\ + E_j \boldsymbol{D}_j^{\Phi} + C_j \boldsymbol{D}_j, \tag{25c}$$

with $\boldsymbol{x}_j = \boldsymbol{x}(\tau_j)$, $\boldsymbol{y}_j = \boldsymbol{y}(\tau_j)$, $\dot{\boldsymbol{y}}_j = \dot{\boldsymbol{y}}(\tau_j)$, $C_j = C(\tau_j)$, $\boldsymbol{D}_j = \boldsymbol{D}(\tau_j)$, $E_j = E(\tau_j)$, $\Phi_{j,0} = \Phi(\tau_j, 0)$, $\Phi_{j,i} = \Phi(\tau_j, \tau_i)$, $B_i^{\Phi} = \left(\int_{\tau_{i-1}}^{\tau_i} \Phi(\tau_i, s) B(s) ds\right) F_{max}\Gamma$, $\tilde{B} = F_{max} B \Gamma$ and $\boldsymbol{D}_i^{\Phi} = \int_0^t \Phi(\tau_i, s)\boldsymbol{D}(s) ds$.

This leads to the following discretisation of the SK constraints (16) and of the velocity constraints (17):

$$\begin{cases} \sum_{i=1}^{j} \beta C_j \Phi_{ji} B_i^{\Phi} \boldsymbol{F}_i \leqslant \delta - C_j \Phi_{j,0}\boldsymbol{x}(0) - C_j \boldsymbol{D}_j^{\Phi}, \\ -\sum_{i=1}^{j} \beta C_j \Phi_{ji} B_i^{\Phi} \boldsymbol{F}_i \leqslant \delta + C_j \Phi_{j,0}\boldsymbol{x}(0) + C_j \boldsymbol{D}_j^{\Phi}, \\ \sum_{i=1}^{N} \beta E_N \Phi_{Ni} B_i^{\Phi} \boldsymbol{F}_i \leqslant \nu - E_N \Phi_{N,0}\boldsymbol{x}_0 - E_N \boldsymbol{D}_j^{\Phi} - C_N \boldsymbol{D}_N, \\ -\sum_{i=1}^{N} \beta E_N \Phi_{Ni} B_i^{\Phi} \boldsymbol{F}_i \leqslant \nu + E_N \Phi_{N,0}\boldsymbol{x}_0 + E_N \boldsymbol{D}_j^{\Phi} + C_N \boldsymbol{D}_N, \end{cases} \tag{26}$$

with β being either [0 1 0] or [0 0 1]. The discretisation of the SK requirements are rewritten in matrix format as:

$$\begin{cases} \mathcal{B}_s \bar{\boldsymbol{F}} \leqslant d - \mathcal{B}_x x(0) - \boldsymbol{\mathcal{B}_c}, \\ -\mathcal{B}_s \bar{\boldsymbol{F}} \leqslant d + \mathcal{B}_x x(0) + \boldsymbol{\mathcal{B}_c}, \\ \mathcal{G}_s \bar{\boldsymbol{F}} \leqslant \upsilon - \mathcal{G}_x x(0) - \boldsymbol{\mathcal{G}_c}, \\ -\mathcal{G}_s \bar{\boldsymbol{F}} \leqslant \upsilon + \mathcal{G}_x x(0) + \boldsymbol{\mathcal{G}_c}, \end{cases} \tag{27}$$

where $\bar{\boldsymbol{F}} = [\boldsymbol{F}_1^t, \dots, \boldsymbol{F}_N^t]^t$. With the transformation of the thrusters operational constraints of Section 2.2.2 and the previous discretisation of the dynamics equation, it is possible to use an integer programming formulation to solve the minimum fuel SK problem.

3.3 Actuators Constraints and Integer Programming

Considering that the parameter vectors F_i are composed of binary variables s_i (cf. (19)), the constraint (b) is expressed as:

$$\forall i,\ (s_1)_i + (s_2)_i + (s_3)_i + (s_4)_i \leqslant 1. \tag{28}$$

Constraint (c) is satisfied by definition of the control parametrization. In order to express constraints (d) and (e), an auxiliary binary variable $\gamma \in \{0,1\}^N$ is used and is defined as:

$$\forall i,\ (s_1)_i + (s_2)_i + (s_3)_i + (s_4)_i = \gamma. \tag{29}$$

The variable γ_i expresses the fact that one of the four thrusters is firing on interval $[\tau_i, \tau_{i+1}]$. If $\gamma_i = 0$, all thrusters are off. Thanks to γ, the constraint (d) reads:

$$\forall k = 1,\dots,m,\ \forall i = 1,\dots,N-k-1,\ \forall l = 1,\dots,4, \\ (s_l)_{i+k} + (s_l)_i + \gamma_i - \gamma_{i+1} \leqslant 0, \tag{30}$$

and the constraint (e) is given by:

$$\forall i = 1,\dots,N-1,\ \forall l = 1,\dots,4,\ (s_l)_{i+1} - (s_l)_i + \gamma_i \leqslant 1. \tag{31}$$

The fuel-consumption is expressed with the control profile S by:

$$J_s = \sum_{i=1}^{N} [1\ 1\ 1\ 1]\boldsymbol{F}_i, \tag{32}$$

and the final SK linear integer programming problem is defined by:

$$
\min_{(s_l)_i,\gamma_i} \; J_s = \sum_{i=1}^{N} [1\ 1\ 1\ 1]\boldsymbol{F}_i
$$

$$
\text{s.t.} \begin{cases}
\mathcal{B}_s\bar{\boldsymbol{F}} \leqslant d - \mathcal{B}_x\boldsymbol{x}(0) - \mathcal{B}_c, \\
-\mathcal{B}_s\bar{\boldsymbol{F}} \leqslant d + \mathcal{B}_x\boldsymbol{x}(0) + \mathcal{B}_c, \\
\mathcal{G}_s\bar{\boldsymbol{F}} \leqslant \upsilon - \mathcal{G}_x\boldsymbol{x}(0) - \mathcal{G}_c \\
-\mathcal{G}_s\bar{\boldsymbol{F}} \leqslant \upsilon + \mathcal{G}_x\boldsymbol{x}(0) + \mathcal{G}_c, \\
(s_1)_i + (s_2)_i + (s_3)_i + (s_4)_i \leqslant 1, \\
(s_1)_i + (s_2)_i + (s_3)_i + (s_4)_i = \gamma_i, \\
(s_l)_{i+k} + (s_l)_i + \gamma_i - \gamma_{i+1} \leqslant 0, \\
(s_l)_{i+1} - (s_l)_i + \gamma_i \leqslant 1.
\end{cases} \tag{33}
$$

4. NUMERICAL RESULTS

In this section, simulation results obtained with the proposed methodology are presented. Let us consider a satellite of mass 4850 kg equipped with 4 electric thrusters oriented in the directions North-East, North-West, South-East and South-West. This satellite has to be controlled in order to remain close to its geostationary position at a fixed longitude $\bar{\lambda}$ and a fixed latitude $\bar{\varphi} = 0$. The SK problem has been solved with Gurobi (see Gu et al. (2010)) and the Matlab parser Yalmip (see Löfberg (2004)) for a time horizon $T = 1$ week. The initial relative geographical position is $\boldsymbol{y}(0) = [0\ 0.03^\circ\ \ -0.04^\circ]^t$ and the relative initial geographical velocity is $\dot{\boldsymbol{y}}(0) = [0\ 0\ 0]^t$.

The benefits of the constraints on the geographical variables derivatives in the integer formulation (33) are checked by solving the SK problem on one hand with the final constraint on the geographical variables derivatives and on the second hand without the the final velocity constraints. On Figure 1 and 2, the geographical positions for two cases defined above are drawn. It is clear that these differences in the design have a high impact on the final trajectory, as well as on the control profile (see Figure 3).

In order to perform the station keeping one week after another, the position in the SK window at the end of a week is used as the initial position for the following one. If the constraints on the geographical variables derivatives at that time are not included in the optimisation stage, the satellite trajectory may end near the boundary of the SK window with an outward pointing velocity, so that the optimisation problem can be infeasible the week after. For the above initial conditions, this phenomenon occurs at week 3. If the constraints on the derivatives of the geographical parameters at the final time are added, the station keeping problem can be solved week after week, but the price to pay is a fuel consumption increase by three times. With the integer formulation of the SK problem, final velocity constraints are much easily handled than with the Pontryagin Maximum Principle involved in Gazzino et al. (2016). Therefore, it is much simpler to optimise over several weeks with the proposed linear integer programming formulation.

It is possible to find in the literature some rules of thumb for geostationary SK strategies, in particular in Campan et al. (1995), Soop (1994) and Sidi (1997). In these references, the effect of the perturbing forces on the orbit is studied, and some general SK laws are derived. The North-South effect of the Sun and the Moon attractions are the most pregnant forces and these forces must be corrected each half orbit, once in the North direction and half an orbit later in the South direction. The East-West drift is meanwhile corrected by setting different thrust durations for each thrusters. This SK strategy was usefully used in an industrial context by Anzel (1995). The control profile of Figure 3 shows that the two South thrusters have a thrust at the beginning of each day, and the two North thrusters have a thrust half a day later. On the control profile, the thrusts have always different durations in order to compensate for the East-West drift. The physical rule of thumb can thus be recovered from a systematic optimisation process.

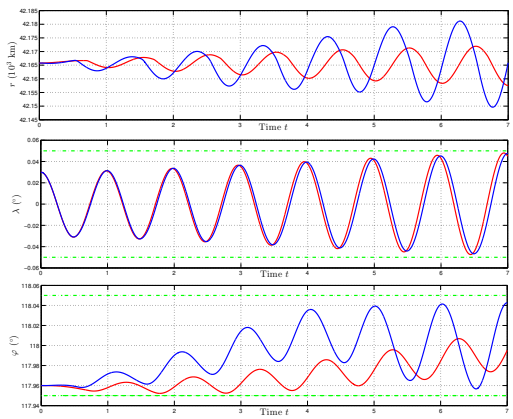

Fig. 1. Relative geographical position.
– : with final constraint on the geographical variables derivatives.
– : without final constraint on the geographical variables derivatives.

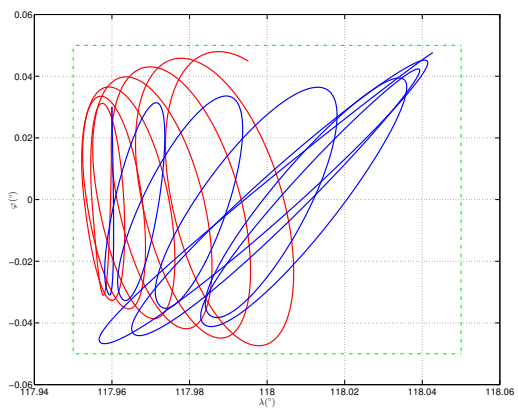

Fig. 2. Station Keeping window.
– : with final constraint on the geographical variables derivatives.
– : without final constraint on the geographical variables derivatives.

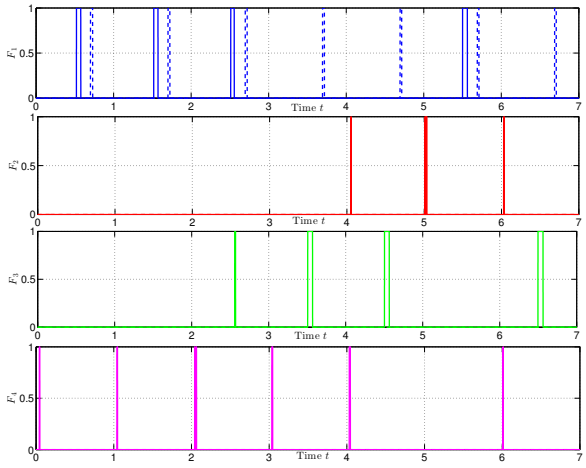

Fig. 3. Control profiles.
– : with final constraint on the geographical variables derivatives.
- - : without final constraint on the geographical variables derivatives.

5. CONCLUSION

In this paper, the optimal control formulation of the minimum fuel station keeping problem has been transformed into a linear integer programming problem to cope with the on-off characteristics of the electrical thrusters and various operational constraints. The methodology has been illustrated with an example that highlights the interest of constraining the derivative of the geographical variable at the final moment of the optimisation horizon in order to prevent the satellite to fly out the station keeping window. The optimal SK strategy based on an optimisation method that could be derived in this paper matches the general rules of the literature for station keeping that have been derived by analysing the physical effect of the disturbing forces on the satellite orbit. The transition matrices used to build up the linear integer programming problem have been obtained under zero-holder assumption. However, in order to improve the accuracy of our results, further investigations have to be made to provide either closed-form expression or numerically certified approximations for the linearised dynamic.

REFERENCES

Antsaklis, P.J. (2003). *Linear Systems*. Cambridge Aerospace Series. Birkhäuser, Boston, {M}assachusetts, {USA}.

Anzel, B.M. (1988). Controlling a stationary orbit using electric propulsion. In *DGLR/AIAA/JSASS 20th International Electric propulsion Conference*, 306–314. Garmisch-Partenkirchen, Germany.

Anzel, B.M. (1995). Method and apparatus for a Satellite Station Keeping.

Barrett, C.C. (1967). On the Application of electric Propulsion to Satellite Orbit Adjustement and Station Keeping. In *American Institute of Aeronautics and Astronautics, Electric propulsion and Plasmadynamics Conference*. Colorado Springs, Colorado. doi: 10.1017/CBO9781107415324.004.

Battin, R.H. (1999). *An Introduction to the Mathematics and Methods of Astrodynamics*. Education. AIAA. doi: 10.2514/4.861543.

Betts, J.T. (1998). Survey of Numerical Methods for Trajectory Optimization. *Journal of Guidance, Control, and Dynamics*, 21(2), 193–207. doi:10.2514/2.4231.

Campan, G., Alby, F., and Gautier, H. (1995). Les techniques de maintien à poste de satellites géostationaires. In *Mécanique Spatiale*, chapter 15, 983–1085. Cépaduès-Editions, Toulouse, France, cnes edition.

Campan, G. and Brousse, P. (1994). ORANGE: Orbital analytical model for geosynchronous satellite. *Revista Brasileira de Ciencias Mecanicas*, 16(16), 561–572.

Eckstein, M.C. (1980). Optimal Station Keeping by Electric Propulsion With Thrust Operation Constraints. *Celestial Mechanics*, 21, 129–147.

Gazzino, C., Arzelier, D., Losa, D., Louembet, C., Pittet, C., and Cerri, L. (2016). Optimal Control for Minimum-Fuel Geostationary Station Keeping of Satellites Equipped with Electric Propulsion. In *20th IFAC Symposium on Automatic Control in Aerospace - ACA 2016*. IFAC, Sherbrooke, Canada. URL `https://hal.archives-ouvertes.fr/hal-01275305`.

Gu, Z., Rothberg, E., and Bixby, R. (2010). Gurobi 4.0.2. software.

Hull, D.G. (1997). Conversion of optimal control problems into parameter optimization problems. *Journal of Guidance, Control, and Dynamics*, 20(1), 57–60.

Hunziker, R.R. (1970). Low-Thrust Station Keeping Guidance for a 24-Hour Satellite. *AIAA Journal*, 8(7), 1186–1192.

Löfberg, J. (2004). Yalmip : A toolbox for modeling and optimization in matlab. In *In Proceedings of the CACSD Conference*. Taipei, Taiwan.

Losa, D., Lovera, M., Drai, R., Dargent, T., and Amalric, J. (2005). Electric Station Keeping of Geostationary Satellites: a Differential Inclusion Approach. *Proceedings of the 44th IEEE Conference on Decision and Control*, 7484–7489. doi:10.1109/CDC.2005.1583369.

Losa, D., Lovera, M., Marmorat, J.p., Dargent, T., and Amalric, J. (2006). Station Keeping of Geostationary Satellites with On-Off Electric Thrusters. *2006 IEEE International Conference on Control Applications*, 2890–2895. doi:10.1109/CCA.2006.286049.

Sidi, M.J. (1997). *Spacecraft Dynamics and Control*. Cambridge University Press.

Soop, E.M. (1994). *Handbook of Geostationary Orbits*. Kluwer Academic Publishers Group.

Sukhanov, A. and Prado, A. (2012). On one approach to the optimization of low-thrust station keeping manoeuvres. *Advances in Space Research*, 50(11), 1478–1488. doi:10.1016/j.asr.2012.07.028.

Vazquez, R., Gavilan, F., and Camacho, E.F. (2015). Pulse-Width Predictive Control for LTV Systems with Application to Spacecraft Rendezvous. *arXiv preprint arXiv:1511.00869*.