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# Target Tracking via a Circular Formation of Unicycles<sup>\*</sup>

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**Abstract:** This paper deals with the problem of encircling a moving target with a fleet of unicycle-like vehicles. A new control law is developed to steer the vehicles to a circular formation whose center tracks the target. The novelty of this paper lies in the fact that the control law only uses the velocity of the target and the relative positions of the agents with respect to it, expressed in the local frame of each vehicle. Communication between agents is used to maintain the vehicles equally spaced along the circular formation. Simulation results show the effectiveness of the proposed strategy.

*Keywords:* Target tracking, autonomous mobile robots, motion control, cooperative control

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## 1. INTRODUCTION

Formation control is a relevant problem in the wide area of motion coordination of multi-agent systems that has been the focus of research documented in the literature (see Martínez et al. (2007), Beard et al. (2001), Egerstedt and Hu (2001) and the references therein). In particular, circular formation control of multi-agent systems has been the subject of considerable research effort over the last years because of its multiple applications, such as target tracking and source-seeking missions of the type described in Leonard et al. (2007) and Ögren et al. (2004). Circular formation for nonholonomic vehicles was studied by Marshall et al. (2004) and Lan et al. (2010) using cyclic pursuit strategies. Circumnavigation around a fixed target using only bearing measurements was proposed by Zheng et al. (2015). Circular collective motion of a network of unicycle agents was studied by Sepulchre et al. (2008). In all of the above, the center of the desired circular formation is fixed.

A relevant and challenging problem is to consider time-varying formations in which the center of the formation is time-varying. Indeed, time-varying configurations are appropriate for some applications where the agents perform collaborative tasks that require the group to move towards an *a priori* unknown direction and to adapt to a desired formation geometry. For instance, in source seeking applications, the formation is driven to follow the gradient of the signal emitted by a source of interest, see Ögren et al. (2004) and Briñón-Arranz et al. (2016). Target tracking problems also require the consideration of time-varying formations. Cooperative approaches to meet this challenge using a fleet of vehicles have been studied in the literature (Klein and Morgansen (2006); Lan et al. (2010)). Based on cyclic pursuit strategies, the work of Kim and Sugie (2007) describe a cooperative control law for moving target-capturing with a fleet of vehicles modeled as simple

integrators. In Goncalves et al. (2010) and Frew et al. (2008) a vector field control approach is presented that enforces a simple integrator vehicle to converge to time-varying target trajectories. Circular formation control with a moving center for vehicles modeled by simple or double integrator dynamics was addressed in Swartling et al. (2014). In a recent paper by Yu and Liu (2016), the circular formation control problem of multiple unicycle vehicles was studied. Using cyclic pursuit techniques, the control law proposed guarantees that the vehicles describe a circular trajectory around a common fixed center using only local information. Mallik et al. (2016) proposed a deviated cyclic pursuit strategy to stabilize a group of unicycle vehicles to a circular formation around a target, using only bearing angle information of the target and the neighbours. In the two references above, the theoretical results hold for a fixed target and the potential application of both proposed strategies in the case of time-varying target is only shown through simulations.

Time-varying circular formation control for nonholonomic agents is a far more challenging problem. Based on previous circular formation control results described in Sepulchre et al. (2008), a new controller design to make nonholonomic vehicles converge to a circular motion following a time-varying reference for its center is provided by Briñón-Arranz et al. (2014). The main idea is to apply affine transformations to the circular trajectories of a stable autonomous exosystem and to enforce the multi-agent system to track these transformed trajectories. The absolute positions of the agents in a global reference frame as well as the velocity and acceleration of the time-varying center are needed to compute the designed tracking controller to ensure global asymptotic stabilization of the circular formation. A distributed reconfigurable control law is proposed in Lan et al. (2010) to enforce a group of vehicles to follow and encircle a moving target while adopting an evenly spaced formation. Only the target velocity and local information in addition to communication between neigh-

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bouring agents are needed. However, the proposed strategy ensures only local stability. Furthermore, the tracking position errors with respect to the target are locally bounded but do not converge to zero.

The main objective of this paper is to devise a method to track the position of a moving target with a circular formation of unicycle vehicles. Each agent is able to measure its relative position with respect to the target in its own reference frame. Based on the ideas from Briñón-Arranz et al. (2014), an autonomous stable exosystem is used to generate the circular trajectories and a new tracking control design stabilizes the agents to a circular motion around the moving target. Only the velocity of the target expressed in the local frame is known, the acceleration is not required. In addition, the vehicles have communication capabilities and thus, they are able to communicate with their neighbors in order to achieve a uniform radial distribution of the circular formation.

The paper is organized as follows. Section 2 starts by introducing the model adopted for the vehicles and the communication network topology, after which the problem of target tracking via a circular formation of unicycles is formulated. The proposed control strategy is presented in Section 3. Section 4 describes the main contribution, a nonlinear control law to stabilize the agents to a circular formation tracking a time-varying center. The performance of the proposed strategy is analyzed through numerical simulations in Section 5. Finally, the paper contains the main conclusions and discusses issues that warrant future research.

## 2. PROBLEM FORMULATION

### 2.1 Model of the agents

In what follows, a group of  $N$  vehicles are modeled as unicycles, subject to simple nonholonomic constraints, such that the kinematic model of agent  $i = 1, \dots, N$  is defined by

$$\begin{aligned} \dot{\mathbf{r}}_i &= \mathbf{R}(\theta_i) [v_i, 0]^T \\ \dot{\theta}_i &= u_i \end{aligned} \quad (1)$$

where  $\mathbf{r}_i \in \mathbb{R}^2$  is the position vector of vehicle  $i$  in a given inertial frame,  $\theta_i$  its heading angle,  $\mathbf{R}(\theta)$  denotes the rotation matrix from body to inertial reference frame defined by  $\mathbf{R}(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ , and  $(v_i, u_i)$  are the control inputs consisting of linear and rotation speed.

### 2.2 Communication topology

In the set-up adopted, each vehicle can communicate with a set of neighboring vehicles. We assume that the communication network of the multi-agent system is represented by an undirected graph. The set of vertices of the graph is denoted by  $V = \{1, \dots, N\}$ ,  $E$  represents the set of edges such that  $(i, j) \in E$  if agents  $i$  and  $j$  communicate with each other. Thus,  $\mathcal{G} = (V, E)$  is the corresponding undirected graph and  $\mathbf{L}$  its Laplacian matrix. Let  $\mathcal{N}_i = \{j \in V : (i, j) \in E\}$  be the set of neighbors of agent  $i$  and  $|\mathcal{N}_i|$  its number of neighbors. If the graph  $\mathcal{G}$  is connected then  $\mathbf{L}\mathbf{x} = 0$  if and only if  $\mathbf{x} = \mathbf{1}x_0$ .

### 2.3 Key objective

The key objective is to derive a cooperative control law such that a group of agents reaches a formation encircling and tracking a moving target describing a trajectory  $\mathbf{c}(t) \in \mathcal{C}_1(\mathbb{R} \rightarrow \mathbb{R}^2)$ , i.e.  $\mathbf{c}(t)$  is a continuously differentiable function of time. To accomplish this task, the agents will be deployed in a uniform distributed pattern along a circular formation whose center is the moving target.

In the present approach we consider that the only information available for each vehicle is its relative position with respect to the target expressed in its body frame, i.e.,  $\mathbf{R}(\theta_i)^T(\mathbf{r}_i - \mathbf{c})$ , rather than in the global inertial frame. It is also assumed that the velocity of the target  $\dot{\mathbf{c}}(t)$  is known for all the vehicles, also expressed in the local coordinate frame of each agent, i.e.,  $\mathbf{R}(\theta_i)^T \dot{\mathbf{c}}$ .

## 3. CONTROL DESIGN

In previous work by Briñón-Arranz et al. (2014), we extended a circular formation control law presented in Sepulchre et al. (2008) to time-varying formations, not only circular, using an exosystem and a tracking control design. A translation control was proposed in which the agents are stabilized to a circular formation whose center tracks a time-varying reference  $\mathbf{c}(t)$ . It was assumed that the agents can compute their own absolute position  $\mathbf{r}_i$  and moreover, in order to track the reference  $\mathbf{c}(t)$ , both the first and second order derivatives,  $\dot{\mathbf{c}}(t)$  and  $\ddot{\mathbf{c}}(t)$ , were known. In other words,  $\mathbf{c}(t) \in \mathcal{C}_2(\mathbb{R} \rightarrow \mathbb{R}^2)$ , i.e.  $\mathbf{c}(t)$  was a continuous and twice differentiable function of time.

In the present paper, the goal is to design a new formation control law which relaxes the assumptions of the problem. We improve substantially the above result by developing a target tracking control law that only requires information on the relative positions between the agents and the target. Additionally, instead of both the first and second order derivatives, in this approach only the velocity of the target  $\dot{\mathbf{c}}(t)$  expressed in the local frame of each agent is known. Thus,  $\ddot{\mathbf{c}}(t)$  is not required and no global information such as the absolute positions of the agents and target are needed. Consequently, the amount of required information decreases considerably. We will see that this is made possible at the price of introducing a position error which can be made arbitrarily small.

### 3.1 Control strategy

Consider that each vehicle is able to compute the relative position vector  $\mathbf{r}_i - \mathbf{c}$ , where  $\mathbf{c}$  represents the position of the target. The objective for each agent is to encircle and track the target, i.e., to describe a circular motion around  $\mathbf{c}(t)$ . This objective can be also expressed as follows: design control laws  $(v_i, u_i)$  for the multi-agent system (1) such that the relative vector  $\mathbf{r}_i - \mathbf{c}$  describes a circular motion about the origin.

In order to exploit the circular control law described in Sepulchre et al. (2008), we introduce an exosystem represented by the multi-agent dynamics (3). The main idea is thus to make the exosystem converge to a circular motion centered at the origin and to use the exosystem trajectories  $\hat{\mathbf{r}}_i$  as references for each relative vector  $\mathbf{r}_i - \mathbf{c}$ ,

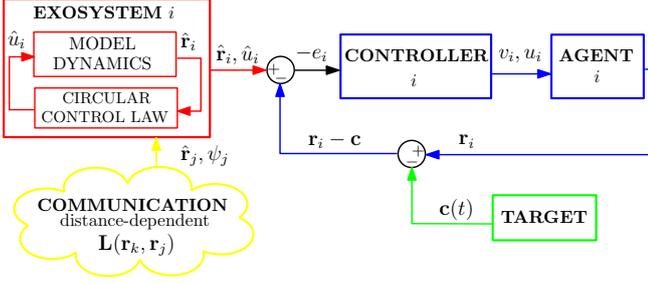


Fig. 1. Structure for feedback design including a decentralized exosystem. Each agent  $i$  computes its own exosystem that communicates with other exosystems by means of a communication protocol depending on the distance between the agents (the Laplacian matrix  $\mathbf{L}$  depends on the agents' states  $\mathbf{r}_i$ ).

see Fig 1. Each agent is able to measure its relative position expressed in its body frame, i.e.,  $\mathbf{R}(\theta_i)^T(\mathbf{r}_i - \mathbf{c})$ . The error between the reference and the relative position vector expressed in the body coordinates of agent  $i$  is defined by

$$\mathbf{e}_i = \mathbf{R}(\theta_i)^T((\mathbf{r}_i - \mathbf{c}) - \hat{\mathbf{r}}_i) = \mathbf{R}(\theta_i)^T(\mathbf{r}_i - (\hat{\mathbf{r}}_i + \mathbf{c})). \quad (2)$$

The problem is then to design control laws  $(v_i, u_i)$  for the multi-agent system (1) such that the position errors  $\mathbf{e}_i$  converge to zero for all  $i = 1, \dots, N$ .

### 3.2 Autonomous stable exosystem

The collective motion of a group of nonholonomic agents has been extensively studied in the literature (see Leonard et al. (2007) and Sepulchre et al. (2008)). In these references, the authors proposed a circular formation control law for a group of nonlinear agents modeled as unicycle dynamics. In all of the above references, the vehicles are represented by (1) with unit linear velocity, i.e.  $v_i = 1, \forall i$ . In order to exploit these previous results, we consider an autonomous exosystem represented by the unicycle model

$$\begin{aligned} \dot{\hat{\mathbf{r}}}_i &= \mathbf{R}(\psi_i) [v_0, 0]^T \\ \dot{\psi}_i &= \hat{u}_i \end{aligned} \quad (3)$$

where  $v_0 = |\omega_0|R_0$  is the constant linear velocity,  $R_0$  denotes the desired radius,  $\omega_0 \neq 0$  is the angular velocity,  $\psi_i$  represents the angular orientation of the velocity vector  $\hat{\mathbf{r}}_i$ , and  $\hat{u}_i$  is the control input. In order to be self-consistent, we state the following lemma which summarizes the results presented by Sepulchre et al. (2008):

*Lemma 1.* Let  $\omega_0 \neq 0$  and  $\kappa > 0$  be two control parameters and  $R_0 > 0$  be the radius of the desired circular motion. Then, the control law

$$\hat{u}_i = \omega_0(1 + \kappa \hat{\mathbf{r}}_i^T \hat{\mathbf{r}}_i) - \frac{\partial U}{\partial \psi_i} \quad (4)$$

ensures that the multi-agent system (3) converges to a circular motion centered at the origin of the coordinates frame with radius  $R_0$  and the direction of rotation is determined by the sign of  $\omega_0$ . Moreover, the agents are distributed along the circle following a pattern defined by an equilibrium point of the potential function  $U(\psi)$ .

The proof of Lemma 1 can be found in Sepulchre et al. (2008). In order to clarify this result, we present here a sketch of the proof. Define  $\hat{\mathbf{r}} = (\hat{\mathbf{r}}_1^T, \dots, \hat{\mathbf{r}}_N^T)^T$ ,  $\psi = (\psi_1, \dots, \psi_N)^T$ , and consider the Lyapunov function

$$S(\hat{\mathbf{r}}, \psi) = \frac{1}{2} \sum_{i=1}^N \left\| \dot{\hat{\mathbf{r}}}_i - \omega_0 \mathbf{R}_{\frac{\pi}{2}} \hat{\mathbf{r}}_i \right\|^2 \geq 0, \quad (5)$$

where  $\mathbf{R}_{\frac{\pi}{2}} = \mathbf{R}(\frac{\pi}{2})$ . At the equilibrium points of the previous Lyapunov function, i.e.,  $S(\hat{\mathbf{r}}, \psi) = 0$ , the dynamics of the exosystem (3) satisfy  $\dot{\hat{\mathbf{r}}}_i - \omega_0 \mathbf{R}_{\frac{\pi}{2}} \hat{\mathbf{r}}_i = 0, \forall i$ . Thus, the position vector and the velocity vector are perpendicular, i.e.,  $\hat{\mathbf{r}}_i^T \dot{\hat{\mathbf{r}}}_i = 0$ . This condition leads to the kinematic relation for the rotation of a rigid body. In other words, at equilibrium the vectors  $\hat{\mathbf{r}}_i$  turn around the frame origin. Considering the proposed control law (4) with  $U(\psi) = 0$  and evaluating the derivative of  $S(\hat{\mathbf{r}}, \psi)$  along the solutions of the resulting closed-loop system (3) yields

$$\dot{S}(\hat{\mathbf{r}}, \psi) = \sum_{i=1}^N \omega_0 \hat{\mathbf{r}}_i^T \dot{\hat{\mathbf{r}}}_i (\omega_0 - \hat{u}_i) = -\kappa \sum_{i=1}^N (\omega_0 \hat{\mathbf{r}}_i^T \dot{\hat{\mathbf{r}}}_i)^2 \leq 0.$$

In conclusion,  $S(\hat{\mathbf{r}}, \psi)$  is a suitable Lyapunov function for the exosystem (3), and by the LaSalle Invariance Principle the solutions converge to the largest invariant set for which  $\dot{S} = 0$ . Consequently, the dynamics of the exosystem satisfies  $\dot{\hat{\mathbf{r}}}_i = \omega_0 \mathbf{R}_{\frac{\pi}{2}} \hat{\mathbf{r}}_i$  which corresponds to a circular motion around de origin.

### 3.3 Uniformly distributed circular formation

Inspired by the synchronization problem of coupled oscillators, Sepulchre et al. (2008) introduced a potential function  $U(\psi)$  which depends on the relative headings and induces a repulsion force to enlarge the angular distance between two connected agents in the circle. The agents then compute the control law (4) with

$$\frac{\partial U}{\partial \psi_i} = \frac{\kappa_u}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \sum_{m=1}^{[\mathcal{N}_i/2]} \frac{\sin(m(\psi_i - \psi_j))}{m}, \quad (6)$$

where  $\kappa_u$  is a positive constant and  $[\mathcal{N}_i/2]$  is the largest integer less than or equal to  $\mathcal{N}_i/2$ . The evenly spaced state corresponding to the uniform distribution of the agents along the circle is a critical point of  $U(\psi)$ . In this situation, the stability of the autonomous exosystem can be proved using, as in Lemma 1, the Lyapunov function  $\tilde{S}(\hat{\mathbf{r}}, \psi) = \kappa S(\hat{\mathbf{r}}, \psi) + U(\psi)$ , where  $S(\hat{\mathbf{r}}, \psi)$  is defined in (5). Differentiating  $\tilde{S}(\hat{\mathbf{r}}, \psi)$  along the solutions of (4) yields  $\dot{\tilde{S}}(\hat{\mathbf{r}}, \psi) \leq 0$  and thus, applying LaSalle Invariance Principle we conclude that the uniform distribution state is locally asymptotically stable for fixed connected graphs.

In our previous work, Briñón-Arranz et al. (2014), we extended this result to distance-dependent communication graphs. This means that each agent can only receive information from its nearest neighbors. In this situation, a communication area for each vehicle is introduced, defined by the critical communication distance  $\rho$  which depends on the characteristics of the communication devices and the environment of the agents. The distance-dependent graph is now time-varying because the position of vehicles is changing in time. Based on graph theory, the distance-dependent Laplacian matrix  $\mathbf{L}(\mathbf{r}(t))$  is defined as:

$$\mathbf{L}_{i,j} = \begin{cases} |\mathcal{N}_i|, & \text{if } i = j, \\ -1, & \text{if } \|\mathbf{r}_i - \mathbf{r}_j\| \leq \rho, \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

In this situation, due to the geometrical properties of the circle, if the critical communication distance  $\rho$  satisfies the condition

$$\rho > 2R_0 \sin \frac{\pi}{N}, \quad (8)$$

then the state in which the agents are evenly spaced is the only critical point of  $U(\psi)$  and thus uniform distribution of the agents along the circle is achieved. The details of the proof can be found in Briñón-Arranz et al. (2014). Note that each vehicle computes its own virtual exosystem (3) and communicates the virtual quantity  $\psi_i$  to its neighbors.

#### 4. TARGET TRACKING WITH A CIRCULAR FORMATION OF VEHICLES

In this section, we present a new control strategy to solve the target tracking problem with a group of unicycle vehicles. The main idea is to consider the circular trajectories of the autonomous exosystem  $\hat{\mathbf{r}}_i$  as references to be tracked for the relative vectors  $\mathbf{r}_i - \mathbf{c}$ . In other words, the aim is to enforce convergence of each  $\mathbf{r}_i(t)$  to the desired position  $\hat{\mathbf{r}}_i(t) + \mathbf{c}(t)$  composed by the circular trajectory generated by the exosystem and the position of the target which is moving with velocity  $\dot{\mathbf{c}}(t)$ . Consider the tracking error defined by (2). Then, the error dynamics satisfy

$$\begin{aligned} \dot{\mathbf{e}}_i &= \mathbf{R}(\theta_i)^T (\dot{\mathbf{r}}_i - (\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}})) - u_i \mathbf{R}_{\frac{\pi}{2}} \mathbf{R}(\theta_i)^T (\mathbf{r}_i - (\hat{\mathbf{r}}_i + \mathbf{c})) \\ &= [v_i, 0]^T - \mathbf{R}(\theta_i)^T (\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}}) - u_i \mathbf{R}_{\frac{\pi}{2}} \mathbf{e}_i. \end{aligned}$$

As described in Vanni (2007), we introduce the vector  $\boldsymbol{\delta} = [-\delta, 0]^T$ , where  $\delta$  is an arbitrarily small positive constant, such that:

$$\begin{aligned} \dot{\mathbf{e}}_i &= [v_i, 0]^T - \mathbf{R}(\theta_i)^T (\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}}) - u_i \mathbf{R}_{\frac{\pi}{2}} (\mathbf{e}_i - \boldsymbol{\delta}) - u_i \mathbf{R}_{\frac{\pi}{2}} \boldsymbol{\delta} \\ &= \Delta [v_i, u_i]^T - \mathbf{R}(\theta_i)^T (\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}}) - u_i \mathbf{R}_{\frac{\pi}{2}} (\mathbf{e}_i - \boldsymbol{\delta}), \end{aligned} \quad (9)$$

where  $\Delta = \begin{pmatrix} 1 & 0 \\ 0 & \delta \end{pmatrix}$ . Note that the new error vector ( $\mathbf{e}_i - \boldsymbol{\delta}$ ) is the distance between a point located at distance  $\delta$  from the center of mass of the agent  $i$  in the  $x$ -axis of its local reference frame and the desired position  $\hat{\mathbf{r}}_i + \mathbf{c}$ . The addition of  $\boldsymbol{\delta}$  is similar to a feedback linearization technique which makes the control variable  $u_i$  appear directly in the position error dynamics. Consequently, in order to design simultaneously both control inputs of the unicycle-like agents, the error is made to converge to a neighborhood of the origin instead of to the origin itself.

*Theorem 2.* Consider a differentiable function  $\mathbf{c}(t) : \mathbb{R} \rightarrow \mathbb{R}^2$ . Let  $R_0 > 0$  be the radius of the desired circular motion,  $\omega_0 \neq 0$ ,  $\kappa > 0$ ,  $\kappa_u > 0$  be three control parameters and  $\mathbf{K} \in \mathbb{R}^{2 \times 2}$  be a symmetric positive definite matrix. Then, the control law

$$[v_i, u_i]^T = \Delta^{-1} \left( \mathbf{R}(\theta_i)^T (\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}}) - \mathbf{K}(\mathbf{e}_i - \boldsymbol{\delta}) \right) \quad (10)$$

where  $\boldsymbol{\delta} = [-\delta, 0]^T$  with  $\delta$  being an arbitrary small positive constant,  $\Delta = \begin{bmatrix} 1 & 0 \\ 0 & \delta \end{bmatrix}$ , and the exosystem  $\dot{\hat{\mathbf{r}}}_i$  is defined by (3) with the control law  $\hat{u}_i$  defined by (4) with (6), makes all the agents defined by (1) converge to an evenly spaced circular formation whose center tracks the time-varying target position  $\mathbf{c}(t)$ . The direction of rotation is determined by the sign of  $\omega_0$ . Moreover, the relative distance between each agent  $i$  and the moving target, i.e.,  $\|\mathbf{r}_i(t) - \mathbf{c}(t)\|$ , converges to the set  $\mathcal{R} = [R_0 - \delta, R_0 + \delta]$ .

**Proof.** The proof is divided in two steps. First, the autonomous exosystem is stabilized to a fixed circular

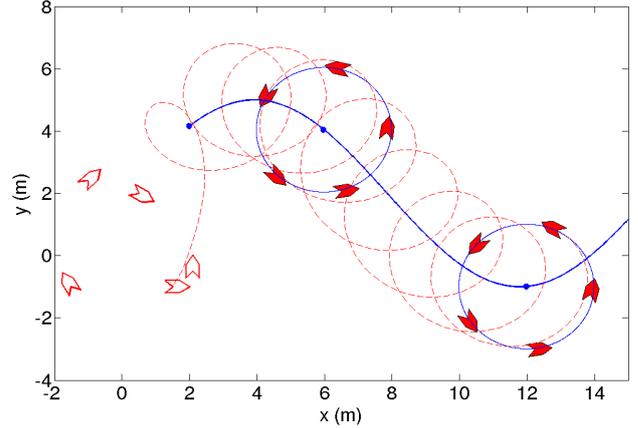


Fig. 2. Target encircling with a group of 5 agents governed by control law (10). The figure shows three snapshots: the initial condition represented by the empty red agents and two states at  $t = 20s$  and  $t = 50s$  respectively. The target is represented by a blue diamond and the blue line displays the target trajectory. The dashed red line denotes one agent's trajectory.

motion, as presented in Section 3. Therefore, thanks to the circular control law (4) with (6), when  $t \rightarrow \infty$  the trajectories of the autonomous exosystem satisfy  $\hat{\mathbf{r}}_i(t) = \omega_0 \mathbf{R}_{\frac{\pi}{2}} \hat{\mathbf{r}}_i(t)$ ,  $\forall i$ , as proved in Lemma 1. The exosystem is used as an autonomous reference generator and the circular trajectories become references to be tracked by system (1).

The second step consists of a tracking control strategy to make each unicycle vehicle follow the desired time-varying trajectory  $\hat{\mathbf{r}}_i(t) + \mathbf{c}(t)$ . Consider the error dynamics defined in (9). According to the proposed control law (10), the closed-loop error dynamics become

$$\dot{\mathbf{e}}_i = -u_i \mathbf{R}_{\frac{\pi}{2}} (\mathbf{e}_i - \boldsymbol{\delta}) - \mathbf{K}(\mathbf{e}_i - \boldsymbol{\delta}). \quad (11)$$

The stability of the error system (11) is studied using the Lyapunov function given by

$$V(\mathbf{e}_i) = \frac{1}{2} \|\mathbf{e}_i - \boldsymbol{\delta}\|^2. \quad (12)$$

Differentiating  $V(\mathbf{e}_i)$  along the solutions of the closed-loop dynamics (11) yields

$$\begin{aligned} \dot{V}(\mathbf{e}_i) &= (\mathbf{e}_i - \boldsymbol{\delta})^T (-u_i \mathbf{R}_{\frac{\pi}{2}} (\mathbf{e}_i - \boldsymbol{\delta}) - \mathbf{K}(\mathbf{e}_i - \boldsymbol{\delta})) \\ &= -(\mathbf{e}_i - \boldsymbol{\delta})^T \mathbf{K}(\mathbf{e}_i - \boldsymbol{\delta}) \leq 0. \end{aligned}$$

Therefore, it is proven that  $\mathbf{e}_i = \boldsymbol{\delta}$ ,  $\forall i$  is a globally exponentially stable equilibrium point for the error system.

Finally, we analyze the resulting closed-loop dynamics of multi-agent system (1). According to the stability analysis of the error system, when  $t$  goes to  $\infty$ ,  $\mathbf{e}_i$  tends to  $\boldsymbol{\delta}$  and  $\dot{\mathbf{e}}_i$  converges to 0. Therefore, at steady state, in view of the previous definition of the error, it follows that

$$\begin{aligned} \mathbf{r}_i - (\hat{\mathbf{r}}_i + \mathbf{c}) &= \mathbf{R}(\theta_i) \boldsymbol{\delta} \\ \dot{\mathbf{r}}_i - (\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}}) &= u_i \mathbf{R}_{\frac{\pi}{2}} (\mathbf{r}_i - \hat{\mathbf{r}}_i - \mathbf{c}). \end{aligned}$$

Consequently, the following equality holds:

$$\dot{\mathbf{r}}_i - u_i \mathbf{R}_{\frac{\pi}{2}} \mathbf{R}(\theta_i) \boldsymbol{\delta} = \dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}}.$$

Let  $\mathbf{z}_i = \mathbf{r}_i - \mathbf{R}(\theta_i) \boldsymbol{\delta}$  define the point located at distance  $\delta$  from the center of mass in the  $x$ -axis of the local coordinates frame of agent  $i$ . The key idea is to control

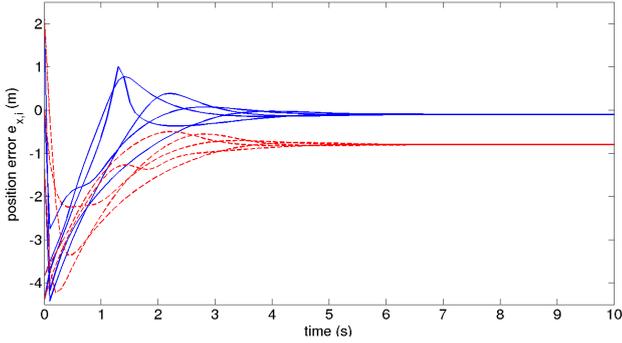


Fig. 3. Evolution of the  $x$ -component of the position error  $e_{x,i}$  for a simulation of 5 agents. Two simulations with different values of parameter  $\delta$  are displayed:  $\delta = 0.1$  (blue lines) and  $\delta = 0.8$  (red dashed lines).

$\mathbf{z}_i$  and make it converge to a circular motion with a time-varying center. Because the exosystem converges to a fixed circular motion, at steady state the following equation holds for all  $i = 1, \dots, N$ :

$$\dot{\mathbf{z}}_i = \dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}} = \omega_0 \mathbf{R}_{\frac{\pi}{2}} \hat{\mathbf{r}}_i + \dot{\mathbf{c}} = \omega_0 \mathbf{R}_{\frac{\pi}{2}} (\mathbf{r}_i - \mathbf{R}(\theta_i) \boldsymbol{\delta} - \mathbf{c}) + \dot{\mathbf{c}}.$$

We can conclude that the closed-loop dynamics describe a time-varying circular motion around the target given by

$$\dot{\mathbf{z}}_i = \underbrace{\omega_0 \mathbf{R}_{\frac{\pi}{2}} (\mathbf{z}_i - \mathbf{c})}_{\text{circular motion}} + \underbrace{\dot{\mathbf{c}}}_{\text{target velocity}}.$$

To conclude, the artificial point  $\mathbf{z}_i$  at a distance  $\delta$  from the center of mass of the agent converges to a moving circular motion around the time-varying center  $\mathbf{c}(t)$ . Therefore, the agents  $\mathbf{r}_i$  converge also to a circular motion tracking the target. In addition, the agents exchange information with their neighbors, the virtual angular quantity  $\psi_i$ , in order to maintain an evenly spaced distribution along the circle.

The previous Lyapunov analysis proved global exponential convergence of the error which implies that for all positive constant  $\varepsilon$ , there exists a time instant  $t_\varepsilon > 0$  such that,

$$\|\mathbf{e}_i(t) - \boldsymbol{\delta}\| \leq \varepsilon, \quad \forall t \geq t_\varepsilon.$$

Considering the definition of the error in (2) and developing the previous inequality, we have that for all  $t \geq t_\varepsilon$  the following expression holds:

$$\left| \|\mathbf{R}(\theta_i)^T (\mathbf{r}_i(t) - \mathbf{c}(t))\| - \|\mathbf{R}(\theta_i)^T \hat{\mathbf{r}}_i(t) + \boldsymbol{\delta}\| \right| \leq \varepsilon.$$

We know that the trajectories of the exosystem converge to circular motions centered at the origin and with radius  $R_0$ , i.e.,  $\|\hat{\mathbf{r}}_i\| = R_0$ , for all  $i$ . Then, the distance between the position of each agent  $i$  and the target satisfies

$$R_0 - \delta - \varepsilon \leq \|\mathbf{r}_i(t) - \mathbf{c}(t)\| \leq R_0 + \delta - \varepsilon, \quad \forall t \geq t_\varepsilon.$$

The previous inequality holds for any arbitrarily small value of  $\varepsilon$  and consequently, at steady state, the radius of the final circular formation  $R$  converges to the set  $\mathcal{R} = [R_0 - \delta, R_0 + \delta]$ , thus concluding the proof.

*Remark 1.* In the definition of the control law (10), one can see that the input variables  $u_i$  are multiplied by  $1/\delta$ , which corresponds to the inverse of the norm of the artificial error vector  $\boldsymbol{\delta}$ . Therefore, if one wishes to achieve an almost perfect tracking, i.e.  $\delta \ll 0$ , then the control inputs  $u_i$  may reach large values. Conversely, if one wants to limit the amplitude of the control input  $u_i$ , one will have to enlarge the tracking error, showing a tradeoff between

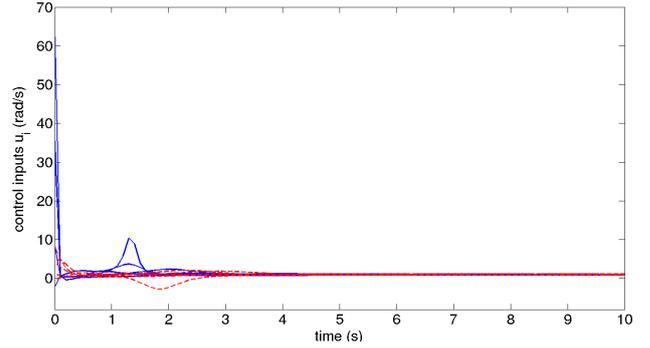


Fig. 4. Evolution of the control inputs  $u_i$  for a simulation of 5 agents. Two simulations with different values of parameter  $\delta$  are displayed:  $\delta = 0.1$  (blue lines) and  $\delta = 0.8$  (red dashed lines).

the reduction of the position error and the limitation of the control inputs  $u_i$ .

The control law (10) presents several advantages with respect to the time-varying circular control law proposed in Briñón-Arranz et al. (2014). First of all, the new control strategy allows designing directly the control inputs ( $v_i, u_i$ ) for each agent  $i$ , instead of deriving a dynamic controller where  $\dot{v}_i$  is a control input in order to deal with time-varying references  $\mathbf{c}(t)$ . In addition, the control law singularities that appear in Briñón-Arranz et al. (2014), such that  $v_i = 0$ , are avoided with the new approach. Secondly, the proposed control law (10) only requires the relative position vector ( $\mathbf{r}_i - \mathbf{c}$ ) expressed in the body frame and the target velocity  $\dot{\mathbf{c}}(t)$  to be computed, while, in Briñón-Arranz et al. (2014), the dynamic controller depends on absolute measurements of the center position  $\mathbf{c}$ , velocity  $\dot{\mathbf{c}}$  and acceleration  $\ddot{\mathbf{c}}$ , as well as the absolute position of each vehicle  $\mathbf{r}_i$ . Hence, despite the issue exposed in Remark 1, the control law (10) presents several advantages for tracking and encircling a time-varying target.

## 5. SIMULATION RESULTS

In this section, simulation results are presented. For all the simulations, the trajectory of the time-varying target is defined by  $\mathbf{c}(t) = [2 + 0.2t, 2 + 3 \sin(0.08t + 2)]^T$  in meters where  $t$  is in seconds, and the parameters of the exosystem control law (4) are chosen as  $\omega_0 = 1 \text{ rad/s}$ ,  $R_0 = 2 \text{ m}$ ,  $\kappa = 1$  and  $\kappa_u = 1$ . The critical communication radius is  $\rho = 1.2$  which satisfies condition (8).

In Fig. 2, a simulation with 5 agents modeled by (1) following and encircling the time-varying target  $\mathbf{c}(t)$  is shown. The only information available for each agent  $i$  is its relative position with respect to the target, i.e.,  $\mathbf{r}_i - \mathbf{c}$ , and the velocity of the target  $\dot{\mathbf{c}}$  both expressed in its local reference frame. The control parameters in (10) are  $\mathbf{K} = \mathbf{I}$  and  $\delta = 0.1$ , where  $\mathbf{I}$  denotes the identity matrix.

The influence of parameter  $\delta$  is analyzed in both Fig. 3 and 4. Firstly, the evolution of the position errors  $\mathbf{e}_i$  for a 5 agents simulation governed by (10) with  $\mathbf{K} = \mathbf{I}$  and two different values of  $\delta$  is shown in Fig. 3. The blue solid lines display the case in which  $\delta = 0.1$  and the red dashed ones the case  $\delta = 0.8$ . As shown in the figure, the position error converges to  $[-\delta, 0]^T$ , such that  $\mathbf{e}_{x,i}(\infty) = -0.1$  for the

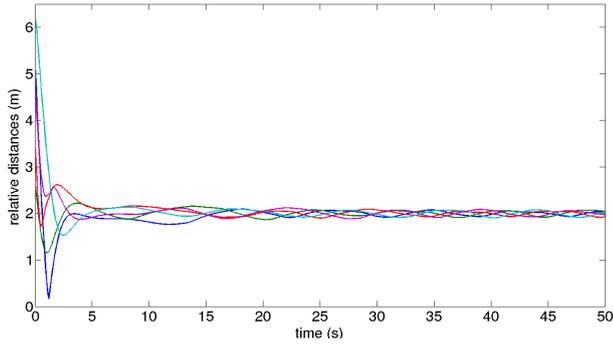


Fig. 5. Evolution of the relative distance of each agent with respect to the target  $\|\mathbf{r}_i - \mathbf{c}\|$  for a simulation of 5 agents following and encircling the time-varying target  $\mathbf{c}(t)$ .

first case and  $\mathbf{e}_{x,i}(\infty) = -0.8$  for the second one. Fig. 4 illustrates the evolution of the control inputs  $u_i$  for the same two simulations with  $\delta = 0.1$  (blue solid lines) and  $\delta = 0.8$  (red dashed lines). The bound on the control input depends strongly on the control parameter  $\delta$ :  $\|u_i\| \leq 65.02$  when  $\delta = 0.1$  and  $\|u_i\| \leq 2.768$  when  $\delta = 0.8$ . Note that the control inputs  $u_i$  converge to the value of the angular velocity  $\omega_0$ . These figures illustrate that the smaller the value of  $\delta$  the smaller the position error and the greater the control inputs are, as stated in Remark 1.

Fig. 5 displays the evolution of the relative distances with respect to the target  $\|\mathbf{r}_i - \mathbf{c}\|$  for the previous simulation with 5 agents shown in Fig. 2. The control parameters in (10) are again  $\mathbf{K} = \mathbf{I}$  and  $\delta = 0.1$ . The vehicles converge to a circular motion centered at the target position and whose radius  $R$  converges to the set  $\mathcal{R} = [R_0 - \delta, R_0 + \delta]$  defined in Theorem 2.

## 6. CONCLUSIONS

In this paper, a new control strategy to track the position of a moving target with a circular formation of nonholonomic vehicles is presented. The proposed control law only requires the relative positions of the vehicles with respect to the target and the velocity of the time-varying target both expressed in their local frame. The control law is provided with a tuning parameter,  $\delta$ , that allows for a tradeoff between tracking error and bounded control inputs  $u_i$ . In addition, the vehicles send and receive information from their neighbors in order to achieve a uniform configuration along the circular formation.

Future work will focus on applying cooperative strategies based on consensus in order to estimate the target position. Another future direction is to study the performance of the proposed control law when the velocity of the target is unknown as well as to analyze its robustness with respect to external disturbances and model uncertainty.

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