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Performance of a fixed reward incentive scheme for two-hop DTNs with competing relays

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Abstract

We analyse the performance of an incentive scheme for two-hop DTNs in which a backlogged source proposes a fixed reward to the relays to deliver a message. Only one message at a time is proposed by source. For a given message, only the first relay to deliver this message gets the reward corresponding to this message thereby inducing a competition between the relays. The relays seek to maximize the expected reward for each message whereas the objective of the source is to satisfy a given constraint on the probability of message delivery. We show that the optimal policy of a relay is of threshold type: it accepts a message until a first threshold and then keeps the message until it either meets the destination or reaches the second threshold. Formulas for computing the thresholds as well as probability of message delivery are derived for a backlogged source.

1 Introduction

The technology of Delay-Tolerant Networks (DTN) has been designed to support communications in environments where end-to-end paths between a source and a destination may not be available at all time. This technology is in particular used to enable the communication between mobile nodes scattered in outermost and sparsely populated regions. In these challenging environments, when a source node wants to transmit a message to a destination node, it can only rarely transmit directly its message to that destination, and therefore a different approach from the one used in traditional communication networks has to be used. The approach used in DTN is based on the so-called *store-carry-and-forward* paradigm. In this approach, the source node transmits its message to each and every mobile node that it meets. The latter nodes play the role of relays. They store the message and carry it, in the hope that they will eventually reach the destination and be able to deliver the message. A source can also play the role of a relay and vice versa.

Due to random node mobility and uncertainty in connectivity, DTN routing schemes usually replicate many copies of the message. In particular, *Epidemic routing* is flooding-based in nature, as relays continuously replicate and transmit messages to newly discovered contacts that do not already possess a copy of the message [25]. The advantage is that it can be guaranteed with high probability that some copy will reach the destination, and with a minimum delivery delay; but the downside is that it floods the network with message copies, leading to a significant energy consumption. Other routing schemes have been proposed as well (see Chapter 2 of Ph.D. thesis [9] for a detailed description of the different schemes). For instance, in the *two-hop routing* scheme, which is known to provide a good tradeoff between message delivery time and energy consumption [1], a relay cannot forward the message to another relay, so it stores and carries the message until the relay is in radio range of the destination.

The delivery of a message incurs a certain number of costs for a relay, in particular in terms of energy – a scarce resource in mobile networks. It can therefore be expected that, even though some nodes will cooperate out of altruism, many mobile nodes will behave selfishly, acting as "free riders" that profit from the resources of others for transmitting their own messages without offering their own resources in exchange. Clearly, if there are too many selfish mobiles nodes, the network

collapses and it is no more possible to communicate between nodes. It is therefore crucial to design incentive mechanisms to persuade mobile nodes to serve as relays.

Related work: There has been a large body of literature on incentive mechanisms for DTNs. These mechanisms can be broadly classified into three categories: reputation-based schemes [16, 27], barter-based schemes [4] and credit-based schemes [31, 8, 28, 20].

Reputation-based schemes, such as SORI [12], MobiGame [26], CONFIDANT [3] and RELICS [24], are based on a simple principle: a node's message is forwarded only if it has forwarded messages originated from others. This however requires each node to monitor the traffic information of all encountered nodes and keep track of their reputation values. In addition, these reputation values should be updated and propagated to all other nodes efficiently and effectively, which is clearly impractical due to the intermittent connectivity between nodes.

Barter-based incentive mechanisms have also been considered to enforce fair cooperation of all nodes. For example, the authors of [23] propose an incentive-aware routing protocol which is based on the Tit-for-Tat (TFT) strategy, in which each node forwards as much traffic for an encountered node as the latter forwards for it. In [4] and [5], Buttyan *et al.* propose a mechanism which is based on the principle of barter: a node relays the message of a neighbor if the latter relays a message of the former in return. One of the issues with this scheme is that a message might be not delivered to its destination if the destination has no message to forward in return.

Finally, in credit-based schemes, the credits earned by nodes from forwarding the messages of other nodes can be used to pay for the delivery of their own ones. As compared to reputation-based schemes, these schemes do not require global information sharing, but they assume the existence of a Trusted Third Party to manage the rewarding procedure. Credit-based incentive schemes are often designed using concepts from Game Theory, such as Vickrey-Clarke-Groves (VCG) auctions [29, 14] or Minority Games [7]. Other examples of credit-based schemes are Mobicent [8], SMART [31], PIS [15], INPAC [10] and FRAME [13], among others.

For most of these schemes, it has been difficult to obtain performance measures such as probability of message delivery and mean time to deliver a message with the exception of [2, 20]. In [2], a simple reward-based mechanism was proposed in which the first relay to deliver gets the reward. The author provided the expression of the success delivery probability of a packet within a fixed time τ with the assumption that a relay i will participate in delivering the packet until a certain time. It was shown that the equilibrium policy is of threshold type: relays participate until a certain time after which they are deactivated. All the computations and results are for a single message. In [20], the source offers rewards that depend upon the meeting time with the condition that only the first one to deliver the message receives its reward. Since the mobility model is random, a relay that meets the source later has lower probability of being the first to deliver the message and hence receiving the reward. The reward proposed to a relay is inversely proportional to its success probability, and is such that a relay always accepts the message. The analysis relies heavily on the assumption that the relays do not discard a message once they accept it from the source. This assumption may be realistic in participative networks in which nodes are altruistic. On the other hand, when nodes are selfish, they could decide to throw away a message once it is not longer profitable to keep it (because the probability of success is too small) and reduce their costs.

Contributions: In the present work, we analyze the incentive mechanism for message delivery in two-hop DTNs assuming a *backlogged* source (a source with infinite number of messages to send) and a *fixed reward* (may depend upon the message). When the source wants to send a message, it proposes a fixed reward to each relay it meets. The reward may vary from message to message but for a given message, the same reward is proposed to each relay. The first relay to deliver gets the reward. The relay can decide to accept or to reject the message depending on the time at which it meets the source. The cost of delivering the message for a relay consists of a linear term which depends upon the duration the relay carries this message as well as constant terms for receiving and transmitting the message.

Our setting is different from [2] in the following ways. In that work, the relays decided how long they participate in the network and during this time they accepted the message with certain probability and did not drop it. In our work, the relays can decide how long they accept they message and then how long they keep it. This gives more freedom to the relays to make their choice. Our cost structure is also different from that in [2]. The linear term in our work depends only upon the duration the relay stores a message whereas in [2] this term depends upon the time

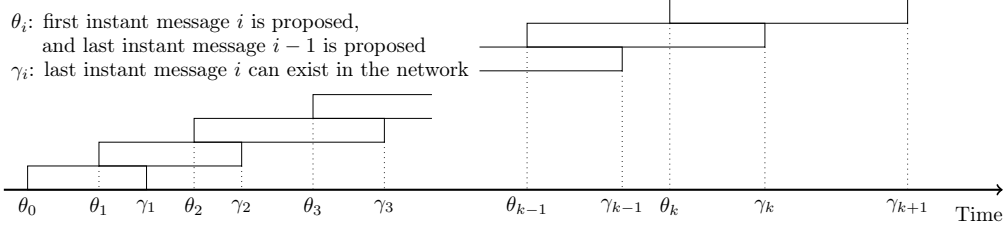


Figure 1: Message injection policy of the source at the symmetric Nash equilibrium.

the relay is participating. These two are different because in the latter case, relays accrue a cost even if they do not have a message. Furthermore, there is no cost of receiving the message from the source in [2]. The inclusion of this cost leads to a policy with two thresholds in our case as opposed to a single threshold in [2]. We also show how the performance measures depend upon the reward offered by the source thereby giving the source an explicit way to compute the reward so as to achieve its targeted performance. Finally, we consider a backlogged source as opposed to a single message in [2]. This induces dependence between the policies of messages which was not there in that work. We also remark that in the mechanism in [20], there was no possibility to drop or accept the message, and thus there was no strategic interaction between the relays.

A model similar to the one studied in this work was first considered in [19] in which the competition was modelled as a stochastic game. That model was in discrete-time, restricted to two players and a single message, and had partial results on the optimal policy. The model studied in this work is in continuous time, for an arbitrary number of players, and a backlogged (with possibly infinite number of messages) source which sequentially proposes messages. Preliminary results of this work have appeared in [17]. The results in that paper were limited to a game with just one message for the source. Here, we generalize the results to a backlogged source. Also, we give conditions for the existence and uniqueness of symmetric equilibrium, which were not given in that paper.

Main results: One of the main questions in incentive mechanisms is to determine the value of reward that an agent should propose. The main aim of this paper is to give the precise relationship between the performance measures and the reward when multiple relays are competing for message delivery. This, in turn, will help the source providing an adequate reward in order to achieve a target delivery probability. Towards this end:

- For the strategic message delivery based game described above, we show that any Nash equilibria (NE) policy of a relay is of threshold type: a relay accepts a message until a first threshold and then keeps it until it either meets the destination or reaches the second threshold. Once a message is no longer accepted by the relays, the source starts giving out the following message. The thresholds of a message depend upon its index and the reward proposed.
- The NE may not be unique. A NE could be symmetric, that is, each relay has the same two thresholds (depends upon the message) for a given message, or asymmetric. We give examples of scenarios with multiple NEs. However, we shall show that any symmetric NE is unique.
- For symmetric NE, for each message, formulas for the thresholds as well as for performance measures such as the probability of delivery and expected delay are derived as a function of the reward proposed for this message. This analysis will be called *transient* analysis, that is, for message k the quantities will depend upon k .

Figure 1 illustrates how, at the symmetric NE, messages will be injected by the source into the network. At time θ_0 , the source will start proposing message 1, which the relays will accept if they meet the source after θ_0 and before θ_1 . At θ_1 , the source will stop proposing message 1 because it knows that none of the relays will accept it, and start proposing message 2. A relay which accepted message 1 will keep it until γ_1 or until it meets the destination, whichever occurs earlier, after which it will drop this message.

Organization: The rest of this paper is organised as follows. Section 2 is devoted to model description. In Section 3, we show that the structure of the best response policy of a relay is of

threshold type. Section 4 gives the conditions for the existence and uniqueness of the symmetric Nash equilibrium. In Section 5, we present a method for recursively computing the thresholds of the symmetric equilibrium as well as the probabilities of message delivery for a backlogged source. Some conclusions are drawn in Section 6.

2 Model Description

2.1 Assumptions

Consider a network of one source, one destination, and N relays. The source and the destination are assumed to be fixed, whereas the relays move according to a given mobility model. It is assumed that the mobility pattern of any two relays are independent, and that the inter-contact times between a relay and the source (resp. the destination) are independent and identically distributed according to an exponential distribution of rate λ (resp. μ). The inter-contact processes of different relays with the source as well as with the destination are assumed to be statistically identical. We note that the assumption of exponentially distributed inter-contact times is satisfied under the Random Waypoint Mobility model [21, 11, 22, 6] and has been observed to hold in real motion traces [30].

When it meets the source, a relay is offered a fixed reward, R_k , to deliver message k . The reward is fixed in the sense that, for a given message, each relay is offered the same reward irrespective of their meeting times. The relay has a choice to either accept the message or not. There is no cost associated with rejecting the message. If it accepts the message, the relay can decide to drop the message at any time in the future at no additional cost. If during this time the relay meets the destination, then it can transmit the message to the destination and claim the reward only if it is the first one to do so for this message.

The various costs incurred for accepting and storing a message are assumed to be as follows: (i) C_r is the cost of receiving the message from the source; (ii) C_d is the cost of transmitting the message to the destination; and (iii) C_s is the cost per unit time for storing the message.

We illustrate the cost structure with an example. Suppose that relay i meets the source at time instant a , accepts the message k and decides to keep it until time b , then the expected total cost of keeping the message in the interval (a, b) will be

$$C_r + \int_a^b \mu e^{-\mu(t-a)} (C_s(t-a) + (C_d - R_k) p_k^i(t)) dt + e^{-\mu(b-a)} C_s(b-a) =: C_r + G_k^i(a, b), \quad (1)$$

where $p_k^i(t)$ is the probability that the relay is the first one to deliver this message when it meets the destination at time t . A one-time cost of C_r is incurred for accepting the message at a . From then on, a storage cost of C_s is incurred per unit of time either until b (that is, for a duration $b - a$) or until it meets the destination. If the relay meets the destination at time $t < b$ and it is the first one to meet the destination with this message then it will transmit the message to the destination and get the reward thereby incurring a net cost of $C_s(t-a) + (C_d - R_k) p_k^i(t)$. So, the second term is the expected cost incurred if the relay meets the destination before b . Finally, the last term is the storage cost incurred if the relay does not meet the destination before b . The sum of the last two terms will be denoted by $G_k(a, b)$ which is the expected cost of keeping the message in the interval (a, b) . Note that this cost depends on the strategies of the other relays through the success probability $p_k^i(t)$.

The analysis in the paper will be done for $R_k = R, \forall k$. This is done to make the notation less cumbersome. The results carry over to the case when the reward depends upon the index of the message. We shall sometimes use the notation $\bar{R} = R - C_d$. In addition, we shall assume that $R \geq R_{min} = C_r + \frac{C_s}{\mu} + C_d$. Note that R_{min} is the average cost of a relay if it were to be the only one to be competing for the message. It is therefore natural that the reward should be larger than this average cost for any relay to participate in forwarding.

It shall be assumed that a relay can store only one message at a time. Further, if a relay already has a message in its buffer, then it does not seek a new (or the same) message until it either meets the destination or drops the message. A message can be dropped only because it is no longer profitable to store this message due to a small probability of success but not because the

relay meets the source. Once it has delivered or dropped the message, the relay can seek a new one from the source.

The source has an unlimited number of messages to send to the destination, each of which it proposes sequentially. That is, to each message the source associates an interval of time during which it proposes this message to any relay it meets. We shall denote this interval for message k by $[\theta_{k-1}, \theta_k)$, where θ_{k-1} is the last time message $k-1$ was proposed.

For a given relay (called tagged relay) when it meets the source, we shall assume that the decision to accept and the duration can depend upon its history of contacts and previous decisions but not upon the history of the other relays. While the exact history of the other relays is not available to the tagged relay, we shall assume that it can compute a belief (or a probability distribution) on when the other relays will enter into competition with it for this message. This belief will be computed based upon the statistics of the mobility model, and will be denoted by $\Phi_k^i(t)$ which is the probability that relay i will enter into competition for message k on or before time t . By enter into competition on or before time t , we mean that there was time instant before t at which relay i did not have any message with index smaller than k . We shall denote by $\phi_k^i(t)$ the probability density function corresponding to $\Phi_k^i(t)$.

Let us take an example to illustrate the notion of competing for a message. Suppose there are two relays. Relay 1 meets the source for the first time at some instant between θ_1 and θ_2 when the source is proposing message 2. If relay 2 had the message 1 at time θ_1 then we say that it is not in competition for message 2 until it has this message because even if it meets the source it cannot accept message 2. Now suppose that relay 2 meets the destination at some time $t_2 \in [\theta_1, \theta_2)$. At t_2 , we say that relay 2 enters into competition with relay 1 for message 2. Of course, relay 1 does not have exact knowledge of the contact history of relay 2 but, using the statistics of the mobility model, it can compute the probability that relay 2 entered into competition at time $t \in [\theta_1, \theta_2)$. This probability distribution will be denoted by $\Phi_2^2(t)$.

A further assumption we shall make is that the relays do not know whether there will be any more messages in the future¹. Hence, they treat each message as though it were the last one. The policy for a message thus does not depend upon the future messages but could depend upon the policy for the previous messages.

3 Structure of the best-response policy

In this section, we shall show that the optimal strategy for relay i for the delivery of message k , given the strategies of the other relays is of threshold type: it accepts a message until a first threshold θ_k^i and then keeps the message until it either meets the destination or reaches the second threshold γ_k^i .

We shall refer to this optimal strategy as the best-response of player i to the strategies of the other relays. Without loss of generality, we assume that the relay comes into play for the delivery of message k at time 0. Note that all relays are immediately available when the first message is released, which is not necessarily the case for the subsequent messages.

Let $\mathcal{S} = \{0, m_s, 1, m_d, 2\}$ be the set of possible states for relay i . If, at some decision epoch, relay i is in state $x_i \in \mathcal{S}$, it may choose action a from the set of allowable actions in that state, $\mathcal{A}(x_i)$. The interpretation of states as well as the actions available in each state are summarized in Table 1. When message k is proposed for the first time by the source, it may happen that relay i still has a previous message. In this case, the relay i is not competing for message k until it either drops the previous message or meets the destination. When this happens, relay i enters state 0 and now has to calculate its optimal policy.

In the following, we shall denote by $x_i(t)$ the state of relay i at time t , and by $\mathbf{x}_{-i}(t)$ the state of the other relays. We shall refer to $\mathbf{x}(t) = (x_i(t), \mathbf{x}_{-i}(t))$ as the state of the system at time t . We emphasize that relay i does not know the state of the other relays at time 0.

The main difficulty in modelling the decision problem faced by relay i is that some actions (namely, rejecting or dropping the message) lead to an immediate change of state, or, in other words, correspond to an infinite transition rate. To circumvent this difficulty, we shall temporarily assume that when the relay makes such a decision, it stays an exponentially distributed amount of time of mean $\frac{1}{M}$ in the same state, where M is some large constant. Under this assumption, it

¹The source is assumed to be backlogged but this information is not known to the relays.

Table 1: State, action sets and costs for a relay for message k .

State	Significance	Action set	Cost
0	relay is competing for message k	\emptyset	0
m_s	relay is in contact with the source	$\{accept, reject\}$	$C_r \mathbb{1}_{\{a=accept\}}$
1	relay has the packet	$\{drop, keep\}$	$C_s \mathbb{1}_{\{a=keep\}}$
m_d	relay is in contact with the destination	\emptyset	$(C_d - R) \mathbb{1}_{\{x_j \neq 2, \forall j \neq i\}}$
2	relay quits the game	\emptyset	0

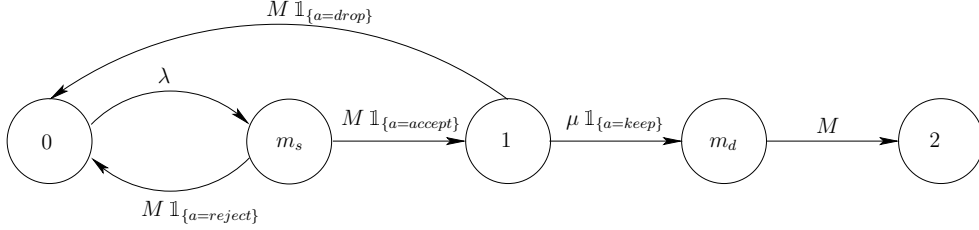


Figure 2: Controlled Markov Chain.

turns out that the optimal decision-making problem of relay i can be cast as an MDP, as we now explain.

It is clear that the stochastic process $x_i(t)$ corresponds to a controlled continuous-time Markov chain, as shown in Figure 2. The cost incurred by the relay depends on its current state, on the action it takes, as well as on the state of the other relays. In the following, $g(x_i, a, \mathbf{x}_{-i})$ denotes the cost incurred by relay i if it takes action a when the system is in state \mathbf{x} . The possible values of the costs are shown in the last column of Table 1.

We define a control law (or policy) as a function $\pi : \mathbb{R} \times \mathcal{S} \rightarrow \mathcal{A}$ such that $\pi(t, x) \in \mathcal{A}(x)$ for all $x \in \mathcal{S}$. Given the policies $\pi_{-i} = (\pi_1, \dots, \pi_{i-1}, \pi_{i+1}, \dots, \pi_N)$ of the other relays, the goal of relay i is to minimize the expected cost

$$J_i(\tau, 0; \pi, \pi_{-i}) = \mathbb{E} \left\{ \int_{\tau}^{\infty} g(x_i(t), \pi(s, x_i(t), \mathbf{x}_{-i}(t))) dt \right\}, \quad (2)$$

over all policies π . In the above equation, $J_i(s, x; \pi, \pi_{-i})$ represents the expected cost-to-go for relay i under policy π if it is in state x at time s , and τ is the first time relay i enters state 0. The cost for relay i depends upon the states and the policies of the other relays only through p_k^i (see (1)).

Let

$$J_i^*(t, x; \pi_{-i}) = \lim_{M \rightarrow \infty} \inf_{\pi} J_i(t, x; \pi, \pi_{-i}),$$

be the optimal cost-to-go for the tagged relay if it is in state $x \in \mathcal{S}$ at time t when $M \rightarrow \infty$.

Proposition 3.1. *When $M \rightarrow \infty$, the optimality equations read as follows*

$$J_i^*(t, 1; \pi_{-i}) = \min \left(0, \inf_{s \geq t} G_k^i(t, s) \right), \quad (3)$$

$$J_i^*(t, m_s; \pi_{-i}) = \min \left(0, C_r + \inf_{s \geq t} G_k^i(t, s) \right), \quad (4)$$

where $G_k^i(t, s)$ is defined in (1).

Proof. See Appendix A.1. □

From now on, we shall only consider the limiting regime $M \rightarrow \infty$. In words, Proposition 3.1 says that if relay i has the message at time t , its best-response is to keep it if and only if there

exists $s \geq t$ such that the expected cost $G_k^i(t, s)$ of keeping the message in the interval (t, s) is negative. Similarly, if relay i meets the source at time t , its best-response is to accept the message if and only if there exists $s \geq t$ such that the expected reward $-G_k^i(t, s)$ offsets the cost of receiving the message from the source. Using the optimality equations stated in Proposition 3.1, Theorem 3.1 establishes the structure of the best-response policy of a relay.

Theorem 3.1. *Given the strategies of the other relays, the best-response policy $\pi_i^*(t; x)$ of relay i is a threshold-type policy: there exists a θ_k^i and $\gamma_k^i > \theta_k^i$ such that $\pi_i^*(t; m_s) = \text{accept}$ if and only if $t \leq \theta_k^i$, and $\pi_k^*(t; 1) = \text{drop}$ if and only if $t > \gamma_k^i$. Moreover, (θ_k^i, γ_k^i) is the solution of:*

$$\gamma_k^i = \sup\{t : p_k^i(t) > \frac{C_s}{\mu(R - C_d)}\}, \quad (5)$$

$$\theta_k^i = \sup\{t : C_r + G_k^i(t, \gamma_k^i) < 0\}, \quad (6)$$

where by convention the supremum of the empty set is 0.

Proof. See Appendix A.2. □

Note that in Theorem 3.1, nothing precludes that $\gamma_k^i = \infty$, or even that $\theta_k^i = \gamma_k^i = \infty$, as shown in the following example. Consider the case where the strategy of the other relays is to never accept the message. In that case, the success probability of relay i is $p_k^i(t) = 1$ for all $t \geq 0$, and it follows from the assumption $R \geq R_{min}$ that $\gamma_k^i = \infty$. Moreover, since in that case $G_k^i(t, \infty) = \frac{C_s}{\mu} - \bar{R} < 0$ for all $t \geq 0$, we also have $\theta_k^i = \infty$. Hence, the best-response policy of player i is to always accept the message and to keep it forever. This shows in particular that, under the assumption $R \geq R_{min}$, the vector of policies in which all relays always reject the message cannot be a Nash equilibrium of our game.

4 Nash Equilibrium

The Nash equilibrium of the forwarding game is defined as a vector $\boldsymbol{\pi}^* = (\pi_1^*, \dots, \pi_N^*)$ of policies from which no relay finds it beneficial to unilaterally deviate. Formally, $\boldsymbol{\pi}^*$ is a Nash equilibrium of the game if and only if

$$J_i(\tau, 0; \pi_i^*, \boldsymbol{\pi}_{-i}^*) \leq J_i(\tau, 0; \pi, \boldsymbol{\pi}_{-i}^*), \quad \forall \pi, \forall i \quad (7)$$

A direct consequence of Theorem 3.1 is the following structural result of any Nash equilibrium.

Corollary 4.1. *At a Nash equilibrium, if any, all players use a threshold-type policy, that is, there exist vectors $\boldsymbol{\theta}_k$ and $\boldsymbol{\gamma}_k$ such that relay i uses a threshold-type strategy with parameters (θ_k^i, γ_k^i) .*

A Nash equilibrium can be asymmetric or symmetric. An asymmetric equilibrium can be of the form: relay 1 always accepts and keeps the message until it meets the destination and relay 2 never accepts. For an example of this type of asymmetric equilibrium, assume that $\lambda = \mu = 1$, and let $k = 1$. Under the given policy of relay 1, the probability of success of relay 2 at time t will be

$$p_1^2(t) = e^{-t}(1+t). \quad (8)$$

From the above equation and (5), it follows that γ^2 will be finite. Suppose that relay 2 meets the source at time 0. This is the most favorable scenario for relay 2. If it is not profitable to accept the message at time 0, then it will never be so later on. From (1), if the relay meets the source at time 0, then its total cost to go if it accepts the message and keeps it until time γ^2 will be

$$\begin{aligned} C_r + G_1^2(0, \gamma^2) &> C_r - \bar{R} \int_0^{\gamma^2} (1+t)e^{-2t} dt \\ &> C_r - \bar{R} \int_0^{\infty} (1+t)e^{-2t} dt = C_r - \frac{3\bar{R}}{4} \end{aligned}$$

That is, if $\bar{R} < \frac{4}{3}C_r$, then relay 2 will always have a positive cost of accepting and its best response will be never to accept. Of course, if $\bar{R} > C_r + C_s$ then relay 1 will always accept if it knows that relay 2 will never accept because this reward is greater than the average cost incurred by one relay. Thus, we have the claimed asymmetric equilibrium.

In the sequel, we shall study the existence and uniqueness of only the symmetric Nash equilibria, that is, equilibria in which all relays use the same thresholds θ_k and γ_k for message k . Using the fact that best-response policies are of threshold type, we obtain an explicit expression of the success probability $p_k^i(t)$. Assuming that up to message $k-1$ only symmetric equilibria have been played, we use this simple expression of the success probability in Section 4.1 to establish the conditions under which there exists a unique symmetric Nash equilibrium.

4.1 Symmetric Nash Equilibrium

Assume that all relays have played symmetric equilibria² for messages $1, 2, \dots, k-1$, that is, $\theta_j^i = \theta_j$ and $\gamma_j^i = \gamma_j$ for $j = 1, 2, \dots, k-1$. A direct consequence of Corollary 4.1 is that if all relays play their Nash equilibrium strategies, the success probability $p_k(t)$ of a player has a very simple structure. For $y \geq x \geq 0$, let

$$v_{x,y}(s) = e^{-\lambda(x-s)} + \frac{\lambda}{\mu - \lambda} e^{-\mu y} e^{\lambda s} \left(e^{(\mu-\lambda)x} - e^{(\mu-\lambda)s} \right), \quad (9)$$

for $s < x$, and $v_{x,y}(s) = 1$ otherwise. Note that if $x = \min(\theta_k, t)$ and $y = \min(\gamma_k, t)$, $v_{x,y}(s)$ represents the probability that relay j be not able to deliver the message k by time t given that it comes into play at time s . Then, introducing

$$V_k(x, y) = \int_{\theta_{k-1}}^{\infty} \phi_k(s) v_{x,y}(s) ds, \quad (10)$$

$$= 1 - \int_{\theta_{k-1}}^x \phi_k(s) (1 - v_{x,y}(s)) ds, \quad (11)$$

the quantity

$$f_k(t) = V_k(\min(\theta_k, t), \min(\gamma_k, t)), \quad (12)$$

represents the probability that a relay fails to deliver the message to the destination by time t , either because it does not meet the source by time $\min(\theta_k, t)$, or because it meets it but does not meet the destination before $\min(\gamma_k, t)$. It then follows that the probability of success of a given relay is

$$p_k(t) = f_k^{N-1}(t) = V_k(\min(\theta_k, t), \min(\gamma_k, t))^{N-1}. \quad (13)$$

Note that $p_k(t)$ is constant after γ_k , and that $p_k(t) = V_k(\theta_k, t)^{N-1}$ for all $t \in [\theta_k, \gamma_k]$. As a consequence, defining

$$\omega = \left(\frac{C_s}{\mu \bar{R}} \right)^{1/(N-1)}, \quad (14)$$

the second threshold γ_k is the greatest value of t such that $V_k(\theta_k, t) \geq \omega$. For a given $\theta \geq 0$, let

$$\gamma(\theta) = \sup\{x : V_k(\theta, x) \geq \omega\}. \quad (15)$$

Note that the function $\gamma(\theta)$ takes its values in $[0, \infty]$. We establish in Lemma 4.1 that there exist θ_{min} and θ_{max} such that $\gamma(\theta) = \infty$ for $\theta \leq \theta_{min}$, whereas $\gamma(\theta)$ takes a uniquely defined finite value for $\theta \in (\theta_{min}, \theta_{max}]$.

²In the sequel, since we are treating the symmetric case, we shall not use superscripts to distinguish relays or player. For example, we shall use $p_k(t)$ instead of $p_k^i(t)$ for the probability of success.

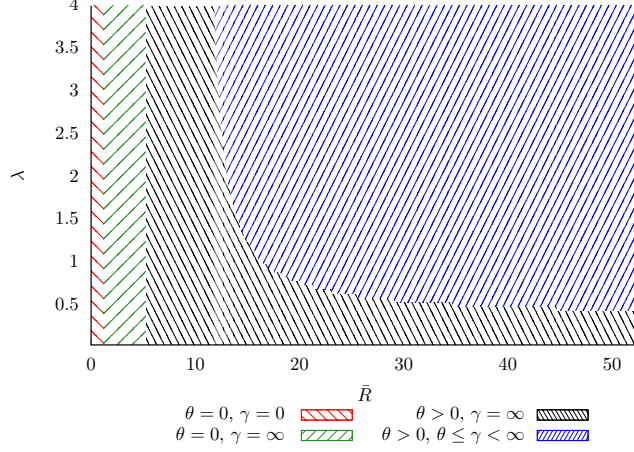


Figure 3: Existence of a symmetric Nash equilibrium for 3 relays.

Lemma 4.1. *Let θ_{min} be the solution of*

$$1 + \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left(e^{-\lambda(\theta_{min}-s)} - 1 \right) ds = \omega, \quad (16)$$

and θ_{max} be such that

$$V_k(\theta_{max}, \theta_{max}) = \omega. \quad (17)$$

Then, for θ fixed, the equation $V_k(\theta, \gamma)$ has a unique finite solution $\gamma \geq \theta$ if and only if $\theta \in (\theta_{min}, \theta_{max}]$. Moreover, $\gamma(\theta)$ is a strictly decreasing function of $\theta \in (\theta_{min}, \theta_{max}]$.

Proof. See Appendix A.3.1. \square

We use Lemma 4.1 to establish in Theorem 4.1 below the conditions under which there exists a unique symmetric Nash equilibrium.

Theorem 4.1. *There exists a symmetric Nash equilibrium with $\theta > 0$ if and only if*

$$\bar{R} \geq C_r + \frac{C_s}{\mu}. \quad (18)$$

Under this condition, the symmetric Nash equilibrium is unique. Moreover, the parameters of the equilibrium are finite, i.e., $\theta_k > 0$ and $\theta_k \leq \gamma_k < \infty$ if and only if

$$1 + \mu \frac{C_r}{C_s} < \frac{(1+b)^N - 1}{Nb} \quad (19)$$

where

$$b = \frac{1}{\sigma\omega} \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)} \right) ds, \quad (20)$$

and $\sigma = (\mu - \lambda)/\lambda$.

Proof. See Appendix A.3.2. \square

We remind the reader that $R_{min} = C_r + \frac{C_s}{\mu} + C_d$ is the minimum value that the reward R should have for a single relay to attempt the delivery of a message. Theorem 4.1 shows that for any value of R greater than this minimum value, the existence of a unique symmetric equilibrium is guaranteed. Figure 3 illustrates the condition (19) for the first message when $N = 3$, $\mu = 0.4$, $C_s = 0.5$ and $C_r = 4.0$. In that case, the minimum value of \bar{R} is $C_r + \frac{C_s}{\mu} = 5.25$. We note that for the first message we have $e^{-\lambda\theta_{min}} = \omega$ and thus $b = \sigma^{-1}(1 - \omega^\sigma)$.

Remark The condition (19) of Theorem 4.1 can be equivalently written as $f(b) > 1 + \mu \frac{C_r}{C_s}$, where

$$f(x) = \frac{(1+x)^N - 1}{Nx}.$$

Note that $b > 0$ for all $\sigma \in [-1, \infty)$ and all $\omega \in (0, 1)$. Using the binomial formula, it is easy to show that $f(x) > 1 + g(x)$ for all $x > 0$, where $g(x) := \frac{N-1}{2}x \left(1 + \frac{N-2}{3}x\right)$. Hence, a sufficient condition for θ_k and γ_k to be finite is $g(b) > \mu \frac{C_r}{C_s}$. Since $g(x)$ is strictly increasing over $[0, \infty)$, this is equivalent to $b > y$, where

$$y = \frac{3}{N-2} \left(\sqrt{\frac{1}{4} + \frac{2}{3}\mu \frac{C_r}{C_s} \frac{N-2}{N-1}} - \frac{1}{2} \right),$$

is the unique solution of $g(x) = \mu \frac{C_r}{C_s}$. The latter sufficient condition can be written as

$$\bar{R} > \frac{C_s}{\mu} \left(\frac{1}{\sigma y} \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)} \right) ds \right)^{-\frac{1}{N-1}}.$$

5 Transient analysis of the symmetric equilibrium

In the rest of the paper, we shall focus only on the symmetric equilibrium. First, we give an algorithm to compute the thresholds θ_k and γ_k for message k which will then be used to derive the probability of message delivery and expected message delay from these thresholds.

5.1 Recursive computation of the success probability

The thresholds θ_k and γ_k of a symmetric equilibrium are obtained from (5) and (6), in which $p_k^i(t) = V_k(\theta_k, t)^{N-1}$ for all $t \in [\theta_k, \gamma_k]$ and all relays i . The computation of the function $V_k(x, y)$ however requires the knowledge of the probability density function $\phi_k(t)$, $\int_a^b \phi_k(t) dt$ representing the probability that a given relay comes into play for the delivery of the k^{th} message between time instants a and b . In this section, we shall show how this probability density function can be recursively computed for symmetric equilibria.

To this end, let us define $I_k(x, t)$ as the probability that a relay that comes into play at time x will accept the k^{th} message and will not be able to deliver it to the destination by time $t \in [\theta_k, \gamma_k]$. Therefore,

$$\begin{aligned} I_k(x, t) &= \int_x^{\theta_k} \lambda e^{-\lambda(s-x)} e^{-\mu(t-s)} ds \\ &= \frac{e^{-\mu t}}{\mu - \lambda} \lambda e^{\lambda x} \left(e^{(\mu-\lambda)\theta_k} - e^{(\mu-\lambda)x} \right) \end{aligned} \quad (21)$$

Thus, $1 - I_k(x, t)$ is the probability that a relay will not have the k^{th} message at time t , either because it has not met the source, or because it has already delivered the message. Similarly, $I_k(x, t_1) - I_k(x, t_2)$ represents the probability that a relay that comes into play at time x will meet the source before θ_k and deliver the message to the destination in the time interval $(t_1, t_2]$. Finally, note also that $\frac{dI_k(x, t)}{dt} = -\mu I_k(x, t)$. We use the definition of $I_k(x, t)$ as well as its above mentioned properties to prove Lemma 5.1 below, which gives a recursion to numerically compute the density $\phi_k(t)$.

Lemma 5.1. For $t \in [\theta_k, \gamma_k]$,

$$\begin{aligned} \phi_{k+1}(t) &= h_1(\theta_k) \delta_{\theta_k}(t) + \phi_k(t) \\ &\quad + h_2(\theta_k) \left\{ \mu e^{-\mu t} + e^{-\mu \gamma_k} \delta_{\gamma_k}(t) \right\} \end{aligned} \quad (22)$$

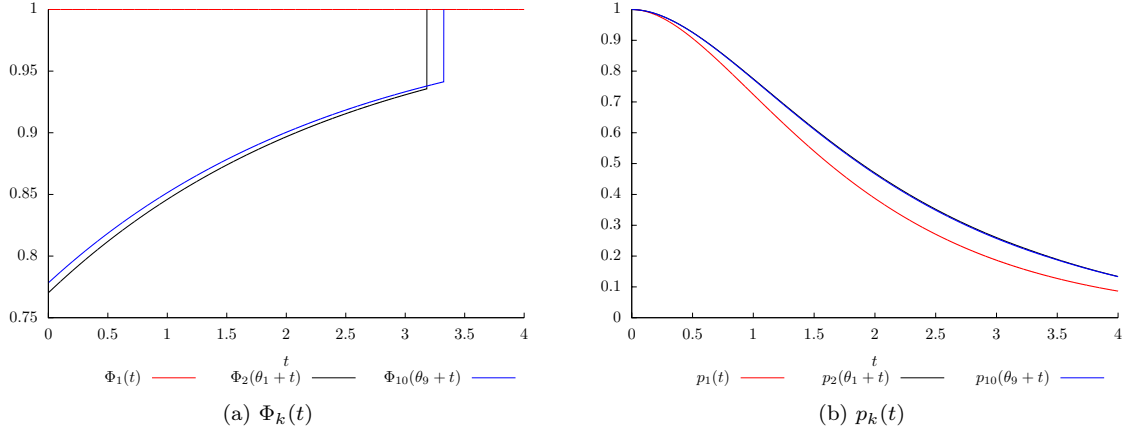


Figure 4: Evolution of $\Phi_k(t)$ and $p_k(t)$ with k .

where

$$h_1(\theta_k) = \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) \{1 - I_k(x, \theta_k)\} dx,$$

$$h_2(\theta_k) = e^{\mu\theta_k} \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) I_k(x, \theta_k) dx.$$

Proof. See Appendix A.4. □

Lemma 5.1 can be used for the recursive numerical computation of the density $\phi_k(t)$, from which we can derive the probability of success $p_k(t) = (V_k(\min(\theta_k, t), \min(\gamma_k, t)))^{N-1}$. Figures 4a and 4b show the CDF $\Phi_k(t)$ and the success probability $p_k(t)$ for $k \in \{1, 2, 10\}$, respectively, in the case $N = 3$ relays, using the following parameters: $\lambda = 1.25$, $\mu = 0.4$, $C_s = 0.5$, $C_d = C_r = 4.0$ and $R = 30$.

The thresholds θ_k and γ_k can then be obtained by solving (5)-(6) using any root finding method, such as the bisection method, as illustrated in Algorithm 1.

5.2 Performance metrics

From the point of view of the source, the main performance metrics are the probability that a message is successfully delivered and, provided that it reaches its destination, the expected time to deliver it. Our first result in this direction is on the probability of message delivery.

Proposition 5.1. *Assume that all relays play a symmetric equilibrium strategy with parameters θ_k and γ_k for the delivery of message k . Let ζ_k be the probability that this message is successfully delivered, that is, the probability that at least one copy reaches the destination by time γ_k . Then*

$$\zeta_k = 1 - \left(\frac{C_s}{\mu R} \right)^{\frac{N}{N-1}}, \quad (23)$$

if $\gamma_k < \infty$, whereas

$$\zeta_k = 1 - \left(1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - e^{-\lambda(\theta_k - s)} \right) ds \right)^N, \quad (24)$$

otherwise.

Algorithm 1 Computation of successive symmetric Nash equilibria

Require: $\phi_1(t) = \delta_0(t)$, $\theta_0 = 0$

```

1: for  $k = 1, 2, \dots$  do
2:   Compute  $\theta_{min}$  and  $\theta_{max}$  as the solutions of (16) and (17)
3:    $a = 0$ ,  $b = \theta_{max}$ 
4:   repeat
5:      $c = (a + b)/2$ 
6:     if  $c > \theta_{min}$  then
7:       Compute  $\gamma(c)$  as the solution of  $V_k(c, \gamma) = \omega$ 
8:     else
9:        $\gamma(c) = \infty$ 
10:    end if
11:     $G_k = \int_c^{\gamma(c)} \mu e^{-\mu(t-c)} \left( \frac{C_s}{\mu} - \overline{R}V_k(c, t)^{N-1} \right) dt$ 
12:    if  $G_k < -C_r$  then
13:       $a = c$ 
14:    else
15:       $b = c$ 
16:    end if
17:  until  $|G_k + C_r| < \epsilon$ .
18:   $\theta_k = c$ ,  $\gamma_k = \gamma(c)$ 
19:  Compute  $\phi_{k+1}(t)$  with (22)
20: end for

```

Proof. From (12), the probability that all relays fail to deliver the message to the destination by time γ_k is $V_k(\theta_k, \gamma_k)^N$, from which we deduce that $\zeta_k = 1 - V_k(\theta_k, \gamma_k)^N$. If $\gamma_k < \infty$, it follows from (13) and (5) that $p_k^i(\gamma_k) = V_k(\theta_k, \gamma_k)^{N-1} = C_s/(\mu \overline{R})$, from which we readily obtain (23). If on the contrary $\gamma_k = \infty$, then (24) follows from (11) and

$$\begin{aligned}
\lim_{y \rightarrow \infty} V_k(\theta_k, y) &= 1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - \lim_{y \rightarrow \infty} v_{\theta_k, y}(s) \right) ds, \\
&= 1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - e^{-\lambda(\theta_k - s)} \right) ds.
\end{aligned}$$

□

We emphasize that Proposition 5.1 can be used by the source to compute the minimum reward R allowing to achieve a target delivery probability. If in addition the source wishes the message to be delivered within a certain amount of time, it can use Proposition 5.2 below.

Proposition 5.2. *Assume that all relays play a symmetric equilibrium strategy with parameters θ_k and γ_k for the delivery of message k , and let D_k be the expected delivery time of this message. Provided that at least one copy reaches the destination by time γ_k , the expected delivery time is*

$$\mathbb{E}[D_k | D_k \leq \gamma_k] = \frac{1}{\zeta_k} \int_{\theta_{k-1}}^{\gamma_k} \left(V_k(\min(t, \hat{\theta}_k), t)^N - V_k(\theta_k, \gamma_k)^N \right) dt \quad (25)$$

Proof. From (12), we have $\mathbb{P}(D_k > t) = V_k(\min(\theta_k, t), t)^N$ for all $t \leq \gamma_k$. It yields

$$\mathbb{P}(D_k > t | D_k \leq \gamma_k) = \frac{1}{\zeta_k} \left(V_k(\min(\theta_k, t), t)^N - V_k(\theta_k, \gamma_k)^N \right),$$

and the result directly follows from

$$\mathbb{E}[D_k | D_k \leq \gamma_k] = \int_{\theta_{k-1}}^{\gamma_k} \mathbb{P}(D_k > t | D_k \leq \gamma_k) dt.$$

□

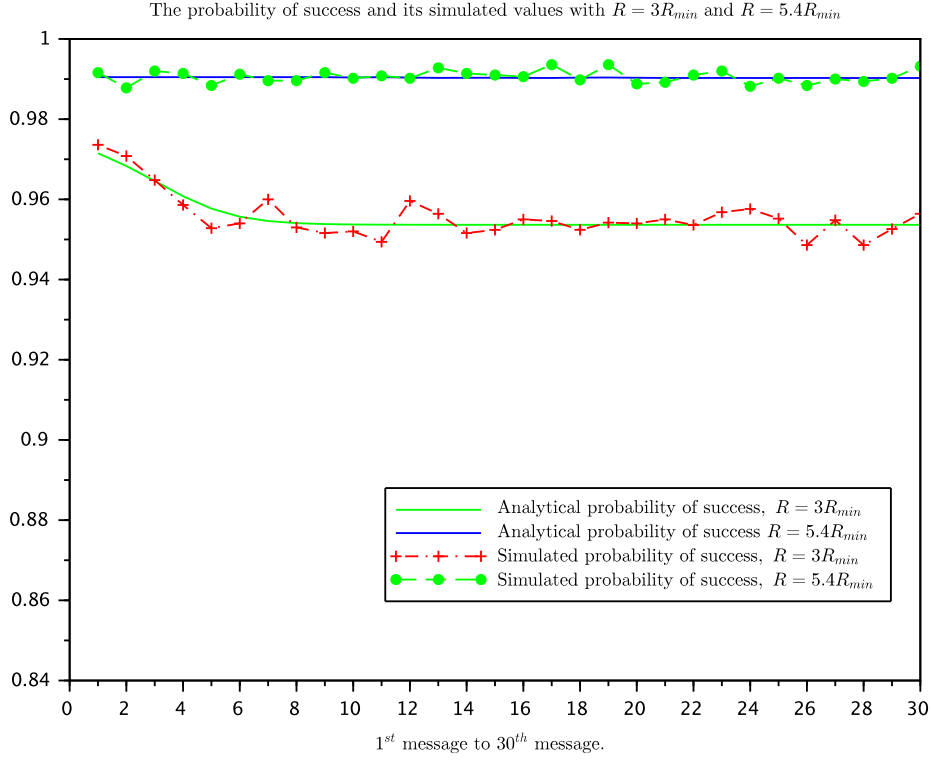


Figure 5: Theoretical values of the probability of success and its values obtained through simulations for $R = 3 \times R_{min}$ ($\gamma = \infty$) and for $R = 5.4 \times R_{min}$ ($\gamma < \infty$).

We do some simulations with different values of R to see how expected delay and probability of success change with R . We take the following values for the parameters: $C_r = 10, C_s = 0.4, C_d = 4, \lambda = 0.8, \mu = 0.4, N = 15$. Figure 5 compares the values of the probability of success obtained with Proposition 5.1 against the values obtained through simulations, for $R = 3 \times R_{min}$, which yields $\gamma = \infty$, and for $R = 5.4 \times R_{min}$ which gives a finite γ . Note from (23) that for $R = 5.4 \times R_{min}$ the success probability is the same for all messages, whereas for $R = 3 \times R_{min}$ it decreases with k . Figure 7 illustrates the convergence of the expected delay as a function of $R \in [3 \times R_{min}, 8 \times R_{min}]$ and k (messages 15 and 29 have almost the same expected delay). This figure also shows that for k large the messages have a greater expected delay than the first messages. Figure 8 shows more clearly the convergence of the expected delay with k in the case $R = 3 \times R_{min}$ and $R = 5.4 \times R_{min}$, whereas Figure 6 shows that the probability of success increases with R and approaches 1 when $R \rightarrow \infty$.

6 Conclusions and future work

We considered a fixed reward incentive mechanism for a two-hop DTN with a single-destination pair and arbitrary number of competing relays. The source was assumed to be backlogged and proposes messages in a sequential way to the relays it meets. It was shown that the equilibrium policy of the relays for each message is threshold type. That is, a relay accepts the k th message if and only if it meets the source before a given threshold, and once it accepts the message, it keeps this message until a second threshold. A recursive formula for the computation of these thresholds was presented for symmetric equilibria.

Our on-going and future work aims at obtaining the thresholds in the stationary case when the number of messages proposed goes to infinity. A mean-field approximation for this case is also

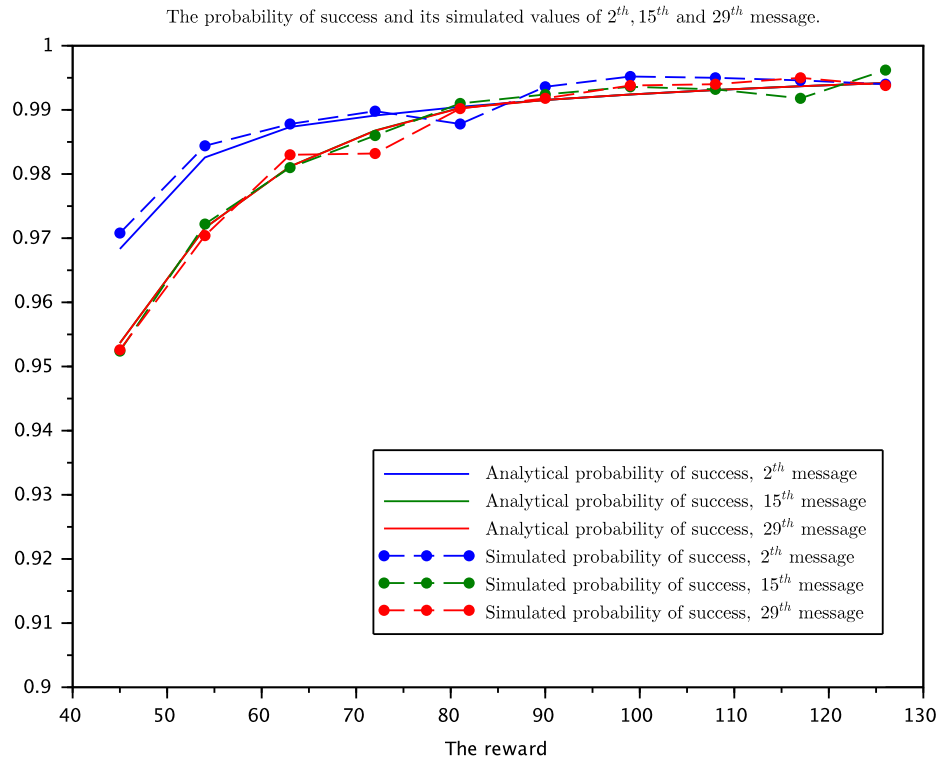


Figure 6: Comparison of the theoretical probabilities of success of 2^{nd} and 29^{th} messages against expected probabilities of success obtained through simulations.

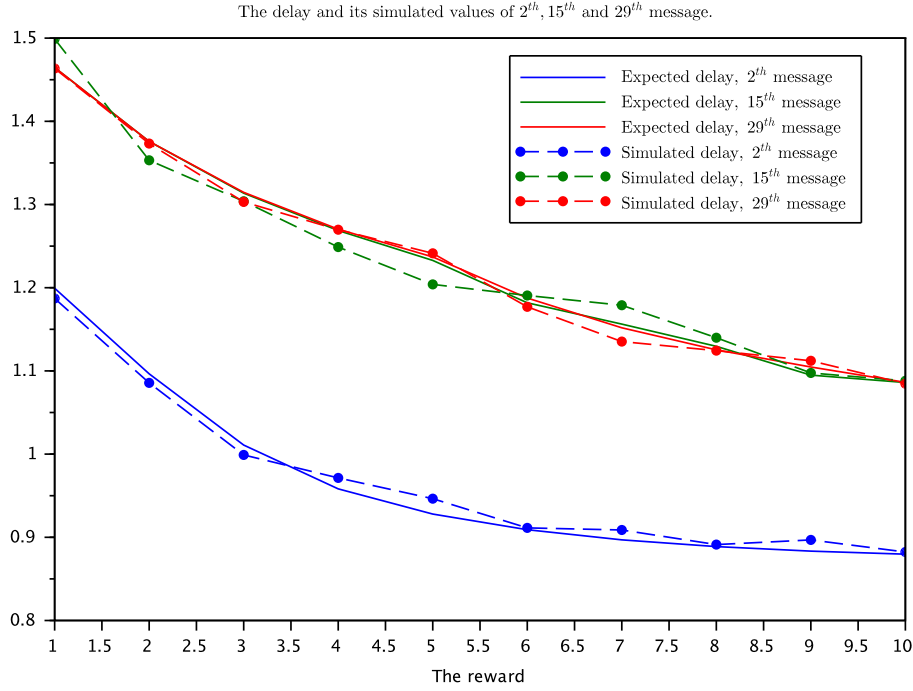


Figure 7: Comparison of the theoretical expected delays of 2^{nd} , 15^{th} and 29^{th} messages against expected delays obtained through simulations.

being investigated. Extending the above analysis to multiple sources and destination as well as allowing the possibility for the relays to drop a message and pick another one is part of our future plans.

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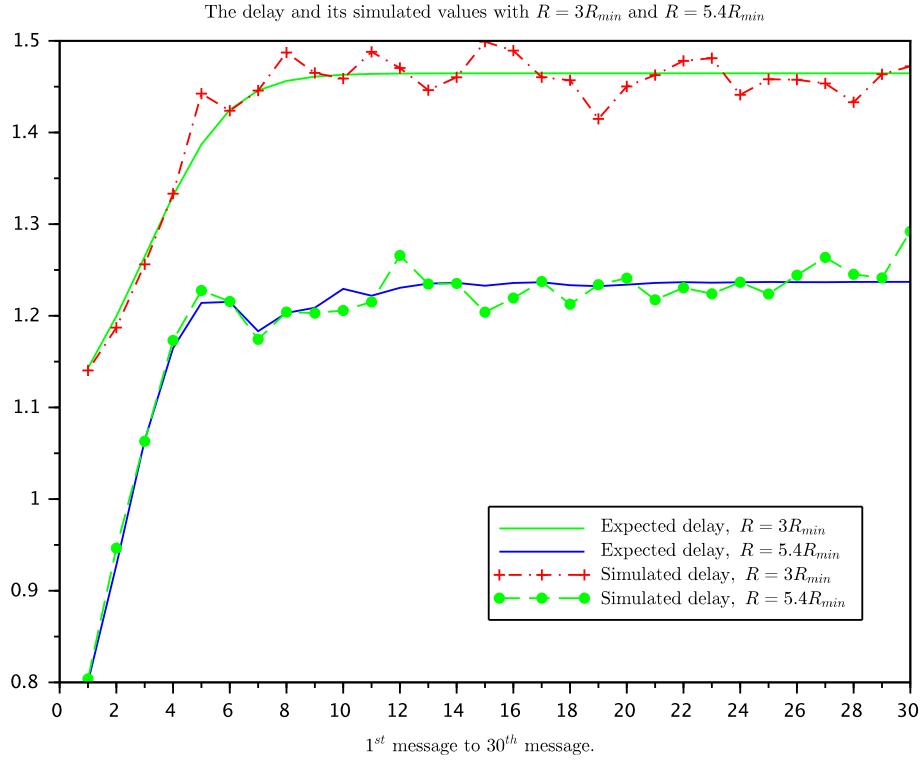


Figure 8: Theoretical expected delays of first 30 messages compared against values obtained through simulations for $R = 3 \times R_{min}$ and $R = 5.4 \times R_{min}$.

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A Proof of Results in Section 3

A.1 Proof of Proposition 3.1

We consider a given relay (say relay i) and establish the optimality equations of problem (2) for this relay in the limiting regime $M \rightarrow \infty$. The proof proceeds in two steps: (a) assuming M is large but fixed, we first use the well-know uniformization technique [18] to establish the optimality equations for an equivalent discrete-time MDP, and (b) we then establish the limits of these optimality equations when $M \rightarrow \infty$.

To simplify notations, let $q = \frac{\lambda}{M}$, $p = \frac{\mu}{M}$, $\bar{p} = 1 - p$ and $\bar{q} = 1 - q$. Denoting by \mathbf{Q} the infinitesimal generator of the controlled CTMC shown in Figure 2, the equivalent discrete-time MDP has transition matrix $\mathbf{P}(a) = \mathbf{I} + \frac{1}{M}\mathbf{Q}(a)$ under action a , that is,

$$\mathbf{P}(a) = \begin{matrix} & \begin{matrix} 0 & m_s & 1 & m_d & 2 \end{matrix} \\ \begin{matrix} 0 \\ m_s \\ 1 \\ m_d \\ 2 \end{matrix} & \left(\begin{array}{ccccc} \bar{q} & q & 0 & 0 & 0 \\ \mathbb{1}_{\{a=reject\}} & 0 & \mathbb{1}_{\{a=accept\}} & 0 & 0 \\ \mathbb{1}_{\{a=drop\}} & 0 & \bar{p}\mathbb{1}_{\{a=keep\}} & p\mathbb{1}_{\{a=keep\}} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right), \end{matrix}$$

and costs-per-stage

$$\tilde{g}(x, a, \mathbf{x}_{-i}) = \frac{1}{M}g(x, a, \mathbf{x}_{-i}), \quad \forall a \in \mathcal{A}(x), \forall x \in \mathcal{S}.$$

Let $V_n(x)$ be the optimal cost-to-go of relay i starting in state $x \in \mathcal{S}$ at time n , and let $q(n)$ denotes the probability that the relay be the first one to deliver the message to the destination at that time. Note that the latter probability depends on the policies π_{-i} of the other relays, although we do not make explicit this dependence. Lemma A.1 establishes the optimality equations for the states $x = m_s$ and $x = 1$.

Lemma A.1. *Provided that $R \geq C_r + C_d + \frac{C_s}{\mu}$, the optimal costs-to-go are given by*

$$V_n(m_s) = \min(0, C_r + V_{n+1}(1)), \quad (26)$$

$$V_n(1) = \min(0, U_{n,1}, U_{n,2}, \dots), \quad (27)$$

where

$$U_{n,m} = \frac{\mu}{M} \sum_{k=0}^{m-1} (\bar{p})^k \left\{ \frac{C_s}{\mu} + \bar{R}q(n+k+1) \right\}. \quad (28)$$

Proof. Since $V_n(m_d) = (C_d - R)q(n)$, the dynamic programming equation

$$V_n(x) = \min_{a \in \mathcal{A}(x)} \left\{ \tilde{g}(x_n, a, \mathbf{x}_{-n}) + \sum_{y \in \mathcal{S}} p_{x,y}(a) V_{n+1}(y) \right\} \quad (29)$$

yields

$$\begin{aligned} V_n(1) &= \min \left(0, \frac{C_s}{M} + p\bar{R}q(n+1) + \bar{p}V_{n+1}(1) \right) \\ &= \min \left(0, \frac{C_s}{M} + p\bar{R}q(n+1), \frac{C_s}{M}(1 + \bar{p}) + \right. \\ &\quad \left. p\bar{R} \sum_{k=0}^1 (\bar{p})^k q(n+k+1) + (\bar{p})^2 V_{n+2}(1) \right), \end{aligned}$$

which can be developed recursively to obtain

$$V_n(1) = \min(0, U_{n,1}, U_{n,2}, \dots).$$

The optimal cost-to-go in state m_s is obtained directly from the dynamic programming equation (29). \square

We note that the term $U_{n,m}$ in Lemma A.1 corresponds to the cost obtained if the action "keep" is played m consecutive times starting from the current decision epoch n , until the relay meets the destination or decides to drop the message. The optimal policy at instant n is to retain the message if either of the $U_{n,m}$ is negative. Otherwise it is optimal to drop the message.

We now turn to the second part of the proof, which is based on Lemma A.2.

Lemma A.2. *Let $s, t \in \mathbb{R}$, $s > t \geq 0$. We have*

$$\lim_{M \rightarrow \infty} U_{\lfloor Mt \rfloor, \lfloor M(s-t) \rfloor} = G_k^i(t, s). \quad (30)$$

Proof. To simplify notation, let $n = \lfloor Mt \rfloor$ and $m = \lfloor M(s-t) \rfloor$. The term $U_{n,m}$ can be rewritten as follows

$$\begin{aligned} U_{n,m} &= \frac{C_s}{\mu} (1 - \bar{p}^m) + \frac{\mu}{M} \bar{R} \sum_{k=0}^{m-1} (\bar{p})^k q(n+k+1), \\ &= \frac{C_s}{\mu} \left(1 - \left(1 - \frac{\mu}{M} \right)^m \right) + \\ &\quad \frac{\mu}{M} \bar{R} \sum_{k=0}^{m-1} \left(1 - \frac{\mu}{M} \right)^k q(n+k+1), \end{aligned} \quad (31)$$

Since $m = \lfloor M(s-t) \rfloor$, for the first term on the LHS, we have

$$\lim_{M \rightarrow \infty} \frac{C_s}{\mu} \left(1 - \left(1 - \frac{\mu}{M} \right)^m \right) = \frac{C_s}{\mu} \left(1 - e^{-\mu(t-s)} \right).$$

Besides, since the discrete-time Markov chain corresponds to the original continuous-time Markov chain observed at random times according to a Poisson process with intensity Mt , we can identify $q(n+k+1)$ with $p_k^i(t + \frac{k+1}{M})$, so that the second term on the LHS of (31) can be rewritten as follows

$$\frac{\mu}{M} \bar{R} \sum_{k=0}^{m-1} \left(1 - \frac{\mu}{M} \right)^k p_k^i(t + \frac{k+1}{M}).$$

Approximating $(1 - \frac{\mu}{M})^k$ by $e^{-\frac{k\mu}{M}}$, it yields

$$e^{\frac{\mu}{M}} \sum_{k=1}^m \mu \bar{R} e^{-\frac{k\mu}{M}} p_k^i(t + \frac{k}{M}) \frac{1}{M},$$

which can be rewritten as

$$e^{\frac{\mu}{M}} \sum_{k=1}^m f\left(t + \frac{k}{M}\right) (x_k - x_{k-1}),$$

where $f(x) = \mu \bar{R} e^{-\mu(x-t)} p_k^i(x)$ and $x_k = t + \frac{k}{M}$ for $k = 0, \dots, m$. When $M \rightarrow \infty$, the term $e^{\frac{\mu}{M}} \rightarrow 1$, whereas the Riemann sum

$$\sum_{k=1}^m f\left(t + \frac{k}{M}\right) (x_k - x_{k-1}) \xrightarrow{M \rightarrow \infty} \int_t^s \mu \bar{R} e^{-\mu(x-t)} p_k^i(x) dx.$$

In view of (1), summing the limits of the first and second terms on the LHS of (31) concludes the proof. \square

We are now in position to prove Proposition 3.1.

Proof of Proposition 3.1. The proof directly follows from Lemmata A.1 and A.2 since

$$\begin{aligned} J_i^*(t, 1; \pi_{-i}) &= \lim_{M \rightarrow \infty} V_{\lfloor Mt \rfloor}(1) \\ &= \min \left(0, \min_{k \geq 1} \lim_{M \rightarrow \infty} U_{\lfloor Mt \rfloor, k} \right) \\ &= \min \left(0, \inf_s G_k^i(t, s) \right), \end{aligned}$$

and the result on $J_i^*(t, m_s; \pi_{-i})$ is obtained similarly. \square

A.2 Proof of Theorem 3.1

Define $g_k^i(t, s) = \mu e^{-\mu(s-t)} (\frac{C_s}{\mu} - \bar{R} p_k^i(s))$. Note that $G_k^i(t, s) = \int_t^s g_k^i(t, x) dx$. That is, $g_k^i(t, x)$ is the marginal cost of keeping the message at time x given that it was accepted at time t . The crucial observation is that the sign of $g_k^i(t, s)$ depends only on s :

$$g_k^i(t, s) < 0 \iff \frac{C_s}{\mu \bar{R}} < p_k^i(s), \quad \forall t, \forall s \geq t. \quad (32)$$

We use this observation in Lemma A.3 below.

Lemma A.3. Define $\phi(t) = \inf_{s \geq t} G_k^i(t, s)$ for all $t \geq 0$, and let γ_k^i be defined as in Theorem 3.1. Then: (a) $\phi(t) \geq 0$ for all $t \geq \gamma_k^i$ and $\phi(t) < 0$ for all $t < \gamma_k^i$, (b) $\phi(t) = G_k^i(t, \gamma_k^i)$ for all $t < \gamma_k^i$, and (c) $\phi(t)$ is strictly increasing in t in the interval $[0, \gamma_k^i]$.

Proof. We first prove assertion (a). By definition of γ_k^i , $p_k^i(y) \leq \frac{C_s}{\mu \bar{R}}$ for all $y \geq \gamma_k^i$. According to (32), it yields $g_k^i(t, y) \geq 0$ for all t and y such that $y \geq t$ and $y \geq \gamma_k^i$. Hence

$$G_k^i(t, s) = \int_t^s g_k^i(t, y) dy \geq 0, \quad \forall s \geq t, \forall t \geq \gamma_k^i,$$

implying that $\phi(t) = \inf_{s \geq t} G_k^i(t, s) \geq 0$ for all $t \geq \gamma_k^i$. Similarly, for $y < \gamma_k^i$, we have $p_k^i(y) > \frac{C_s}{\mu \bar{R}}$. With (32), it implies that $g_k^i(t, y) < 0$ for all t and y such that $t \leq y < \gamma_k^i$. Hence

$$\phi(t) = \inf_{s \geq t} G_k^i(t, s) \leq G_k^i(t, \gamma_k^i) = \int_t^{\gamma_k^i} g_k^i(t, y) dy < 0.$$

We thus conclude that $\phi(t) \geq 0$ for all $t \geq \gamma_k^i$ and $\phi(t) = G_k^i(t, \gamma_k^i) < 0$ for all $t < \gamma_k^i$, as claimed.

We now prove assertion (b). We know that $g_k^i(t, y) < 0$ for all t and y such that $t \leq y < \gamma_k^i$. This implies that for $t < \gamma_k^i$ fixed, $G_k^i(t, y)$ is a strictly decreasing function of y on the interval $[t, \gamma_k^i]$, so that $G_k^i(t, y) > G_k^i(t, \gamma_k^i)$. Moreover, $g_k^i(t, y) \geq 0$ for all t and y such that $y \geq t$ and $y \geq \gamma_k^i$. This implies that, for $t < \gamma_k^i$ fixed, $G_k^i(t, y)$ is a non-decreasing function of y on the interval $[\gamma_k^i, \infty]$, so that $G_k^i(t, y) \geq G_k^i(t, \gamma_k^i)$. As a consequence, $\phi(t) = \inf_{s \geq t} G_k^i(t, s) = G_k^i(t, \gamma_k^i)$ for all $t < \gamma_k^i$.

Finally, in order to prove assertion (c), we note that from assertion (b), we have $\phi(t) = G_k^i(t, \gamma_k^i)$ for all $t < \gamma_k^i$, so that $\phi'(t) = -g_k^i(t, t)$. Since $g_k^i(t, y) < 0$ for all t and y such that $t \leq y < \gamma_k^i$, we have $g_k^i(t, t) < 0$ for all $t < \gamma_k^i$, and thus $\phi'(t) > 0$ for all $t < \gamma_k^i$. \square

The proof of Theorem 3.1 now readily follows from Proposition 3.1 and Lemma A.3.

Proof of Theorem 3.1. We know from Proposition 3.1 that $J_i^*(t, 1; \pi_{-i}) = \min(0, \phi(t))$. As proven in Lemma A.3, $\phi(t)$ is negative if and only if $t < \gamma_k^i$. Therefore, the best-response policy of the tagged relay is such that if it is in state 1 at time t , it should keep the message if $t < \gamma_k^i$, and drop it otherwise. Similarly, according to Proposition 3.1, we have $J_i^*(t, m_s; \pi_{-i}) = \min(0, C_r + \phi(t))$. Therefore, if the relay is in state m_s at time t , it should accept the message if and only if $C_r + \phi(t) < 0$. If $t \geq \gamma_k^i$, the relay should reject the message since, as proven in Lemma A.3, $\phi(t) \geq 0$. If on the contrary $t < \gamma_k^i$, we have $C_r + \phi(t)$ so the relay should accept the message if and only if $C_r + G_k^i(t, \gamma_k^i) < 0$, that is, if and only if $t \leq \theta_k^i$, as claimed. \square

A.3 Proofs of results in Section 4.1

A.3.1 Proof of Lemma 4.1

We shall first establish some properties of the function $V_k(x, y)$ in Lemma A.4, before proving Lemma 4.1.

Lemma A.4. *For y fixed, the function $V_k(x, y)$ is strictly decreasing in x in the interval $[0, y)$, and for $x > 0$ fixed, it is strictly decreasing in y in the interval $[0, \infty]$.*

Proof. The proof directly follows from

$$\begin{aligned} \frac{\partial V_k}{\partial x}(x, y) &= \int_{\theta_{k-1}}^x \phi_k(s) \frac{\partial v_{x,y}}{\partial x}(s) ds, \\ &= \lambda e^{-\lambda x} \left(e^{-\mu(y-x)} - 1 \right) \int_{\theta_{k-1}}^x \phi_k(s) e^{\lambda s} ds, \end{aligned}$$

which is negative for all $y > x$, and

$$\begin{aligned} \frac{\partial V_k}{\partial y}(x, y) &= \int_{\theta_{k-1}}^x \phi_k(s) \frac{\partial v_{x,y}}{\partial y}(s) ds, \\ &= -\frac{\lambda \mu e^{-\mu y}}{\mu - \lambda} \int_{\theta_{k-1}}^x \phi_k(s) e^{\lambda s} \left(e^{(\mu-\lambda)x} - e^{(\mu-\lambda)s} \right) ds, \end{aligned}$$

which is also negative for all $y, x \geq 0$. \square

These properties of the function $V_k(x, y)$ are now used to prove Lemma 4.1.

Proof of Lemma 4.1. Since according to Lemma A.4 the continuous function $V_k(x, y)$ is strictly decreasing in y for x fixed, the equation $V_k(\theta, \gamma) = \omega$ has a solution $\gamma \geq \theta$ if and only if

$$\lim_{y \rightarrow \infty} V_k(\theta, y) < \omega \leq V_k(\theta, \theta).$$

With (9) and (10), the LHS inequality directly leads to $\theta > \theta_{min}$, whereas the RHS one yields $\theta \leq \theta_{max}$. Hence, for θ fixed, the equation $V_k(\theta, \gamma) = \omega$ has a solution $\theta \leq \gamma < \infty$ if and only if $\theta \in (\theta_{min}, \theta_{max}]$.

To show that $\gamma(\theta)$ is decreasing on $(\theta_{min}, \theta_{max}]$, note that

$$\frac{d}{dx} V_k(x, \gamma(x)) = \frac{\partial}{\partial x} V_k(x, \gamma(x)) + \gamma'(x) \frac{\partial}{\partial y} V_k(x, \gamma(x)).$$

On the interval $(\theta_{min}, \theta_{max}]$, the function $V_k(\theta, \gamma(\theta))$ is a constant, and its derivative is thus 0. From Lemma A.4, both the partial derivatives of V_k are negative, from which we conclude that derivative of $\gamma(\theta)$ is strictly negative. \square

A.3.2 Proof of Theorem 4.1

Define the function

$$\hat{G}_k(\theta) = G_k(\theta, \gamma(\theta)). \quad (33)$$

The value of θ at an equilibrium is determined by a solution of $\hat{G}_k(\theta) = -C_r$. Thus, the number of equilibria will depend upon the number of roots of the equation $\hat{G}_k + C_r = 0$ on the positive real line.

The next result gives some properties of \hat{G}_k that are then sufficient to conclude the unicity of the symmetric equilibrium.

Lemma A.5. *On the interval $[0, \theta_{max}]$, the function \hat{G}_k is*

- (a) *continuous;*
- (b) *strictly increasing; with*
- (c)

$$\hat{G}_k(0) = \frac{C_s}{\mu} - \bar{R}, \quad (34)$$

$$\hat{G}_k(\theta_{max}) = 0. \quad (35)$$

Proof. (a) The continuity of \hat{G}_k on the open interval $[0, \theta_{min}) \cup (\theta_{min}, \theta_{max}]$ follows from the definition of G_k . In order to show the continuity of \hat{G}_k it is thus sufficient to show that

$$\lim_{\theta \rightarrow \theta_{min}^-} \hat{G}_k(\theta) = \lim_{\theta \rightarrow \theta_{min}^+} \hat{G}_k(\theta).$$

In order to prove this, observe that we can write

$$\hat{G}_k(\theta) = \int_{\theta}^{\gamma(\theta)} g_k(\theta, t) dt, \quad (36)$$

where $g_k(\theta, t) = \mu e^{-\mu(t-\theta)} (\frac{C_s}{\mu} - \bar{R} V_k(\theta, t)^{N-1})$ is the marginal cost of keeping a message at time t given that it was accepted at time θ . Since $V_k(\theta, t) \in [0, 1]$, we have $|g_k(\theta, t)| \leq m_1 \mu e^{-\mu(t-\theta)}$ for all θ and t , where $m_1 = \max \left(\bar{R} - \frac{C_s}{\mu}, \frac{C_s}{\mu} \right)$. It yields

$$\begin{aligned} \left| \int_{\theta_{min}}^{\infty} g_k(\theta, t) dt - \hat{G}_k(\theta) \right| &\leq \int_{\theta_{min}}^{\theta} |g_k(\theta, t)| dt + \\ &\quad \int_{\gamma(\theta)}^{\infty} |g_k(\theta, t)| dt \\ &\leq m_1 \left\{ e^{-\mu(\theta_{min}-\theta)} - 1 + \right. \\ &\quad \left. e^{-\mu(\gamma(\theta)-\theta)} \right\}, \end{aligned}$$

from which we conclude that

$$\begin{aligned}
\lim_{\theta \rightarrow \theta_{min}^+} \hat{G}_k(\theta) &= \lim_{\theta \rightarrow \theta_{min}^+} \int_{\theta_{min}}^{\infty} g_k(\theta, t) dt, \\
&= \int_{\theta_{min}}^{\infty} g_k(\theta_{min}, t) dt.
\end{aligned}$$

where the last equality is obtained using the dominated convergence theorem. Similar arguments can be used to establish that $\hat{G}_k(\theta)$ converges to the same limit when $\theta \rightarrow \theta_{min}^-$.

(b) We have

$$\begin{aligned}
\frac{dG_k}{d\theta}(\theta, \gamma(\theta)) &= g_k(\theta, \gamma(\theta))\gamma'(\theta) - g_k(\theta, \theta) + \\
&\quad \int_{\theta}^{\gamma(\theta)} \frac{\partial g_k}{\partial \theta}(\theta, x) dx
\end{aligned} \tag{37}$$

$$\begin{aligned}
&= g_k(\theta, \gamma(\theta)) - g_k(\theta, \theta) + \\
&\quad \int_{\theta}^{\gamma(\theta)} \frac{\partial g_k}{\partial \theta}(\theta, x) dx \\
&= \int_{\theta}^{\gamma(\theta)} \left(\frac{\partial g_k}{\partial x}(\theta, x) + \frac{\partial g_k}{\partial \theta}(\theta, x) \right) dx,
\end{aligned} \tag{38}$$

where (38) is obtained from (37) by observing that, for $\theta > \theta_{min}$, $v(\theta, \gamma(\theta)) = \omega$ implies that $g_k(\theta, \gamma(\theta)) = 0$, whereas for $\theta \leq \theta_{min}$, $\gamma(\theta) = \infty$ also implies $g_k(\theta, \gamma(\theta)) = 0$. Since

$$\begin{aligned}
\frac{\partial g_k}{\partial x}(\theta, x) + \frac{\partial g_k}{\partial \theta}(\theta, x) &= -\bar{R}(N-1)\mu e^{-\mu(x-\theta)} V_k(\theta, x)^{N-2} \\
&\quad \times \left(\frac{\partial V_k}{\partial x}(\theta, x) + \frac{\partial V_k}{\partial \theta}(\theta, x) \right),
\end{aligned}$$

we conclude from Lemma A.4 that $G_k(\theta, \gamma(\theta))$ is strictly increasing in θ .

(c) Equality (34) follows from noting that $V_k(0, x) = 1$, and using this in (36). Similarly, (35) is obtained by noting that $\gamma(\theta_{max}) = \theta_{max}$ (from Lemma 4.1), and using this in (36). \square

An immediate consequence of Lemma A.5 is stated in Corollary A.1.

Corollary A.1. *There is a unique solution to $\hat{G}_k(\theta) = -C_r$ in the interval $[0, \theta_{max}]$ if and only if*

$$C_r + \frac{C_s}{\mu} \leq \bar{R}$$

We are now in position to prove Theorem 4.1.

Proof of Theorem 4.1. From Lemma 4.1 there is a unique γ for a given $\theta > 0$ that satisfies (5). Also, from Corollary A.1 there is unique $\theta > 0$ that satisfies (6) if and only if $C_r + \frac{C_s}{\mu} \leq \bar{R}$. Thus, this last inequality is necessary and sufficient for the existence of a unique symmetric equilibrium.

From Lemma A.5 and Lemma 4.1, we deduce that, for γ to be finite the necessary and sufficient condition is

$$\hat{G}_k(\theta_{min}) < -C_r.$$

From (11),

$$\begin{aligned}
V_k(\theta_{min}, x + \theta_{min}) &= 1 + \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left\{ \left(e^{-\lambda(\theta_{min}-s)} - 1 \right) + \right. \\
&\quad \left. \frac{e^{-\mu x}}{\sigma} \left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)} \right) \right\} ds \\
&= \omega \left(1 + e^{-\mu x} b \right),
\end{aligned}$$

where

$$b = \frac{1}{\sigma\omega} \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)} \right) ds.$$

Then,

$$\begin{aligned} \hat{G}_k(\theta_{min}) &= \frac{C_s}{\mu} - \bar{R} \int_0^\infty \mu e^{-\mu x} v(\theta_{min}, x + \theta_{min})^{N-1} dx \\ &= \frac{C_s}{\mu} - \bar{R} \frac{\omega^{N-1}}{Nb} ((1+b)^N - 1). \end{aligned}$$

where the last equality follows from the binomial formula.

Thus, $\hat{G}_k(\theta_{min}) < -C_r$ if and only if

$$\frac{C_s}{\mu} - \bar{R} \frac{\omega^{N-1}}{Nb} ((1+b)^N - 1) < -C_r,$$

which, since $\omega^{N-1} = C_s/(\mu\bar{R})$, is equivalent to

$$1 + \mu \frac{C_r}{C_s} < \frac{(1+b)^N - 1}{Nb},$$

as claimed. \square

A.4 Proof of Lemma 5.1

Proof of Lemma 5.1. Let us first consider the probability that the relay be ready for competing for the delivery of the $(k+1)^{th}$ message at time θ_k . This is only possible if it was ready for competing for the k^{th} message at some time $x \in [\theta_{k-1}, \theta_k]$, and has not the message at time θ_k . As a consequence

$$\Phi_{k+1}(\theta_k) = \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) [1 - I_k(x, \theta_k)] dx = h_1(\theta_k).$$

Consider now the probability that T_{k+1} be in the interval $(\theta_k, t]$ for some $t < \gamma_k$. This can occur if the relay was ready for competing for the k^{th} message at some time $x \in [\theta_{k-1}, \theta_k]$, took this message from the source at some time $s \in [x, \theta_k]$ and deliver it to the destination in $y \in (\theta_k - s, t - s]$ units of time. Another possibility is that the relay comes into play for the delivery of the k^{th} message after θ_k but before t , in which case it will be proposed directly the $(k+1)^{th}$ message. As a consequence

$$\begin{aligned} \Phi_{k+1}(t) - \Phi_{k+1}(\theta_k) &= \int_{\theta_k}^t \phi_k(x) dx \\ &\quad + \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) (I_k(x, \theta_k) - I_k(x, t)) dx \end{aligned}$$

which upon derivation with respect to t yields

$$\begin{aligned} \phi_{k+1}(t) &= \phi_k(t) + \mu \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) I_k(x, t) dx \\ &= \phi_k(t) + \mu e^{-\mu t} h_2(\theta_k) \end{aligned}$$

Finally, the only possibility for the relay to come into play at time γ_k is that it was ready for competing for the k^{th} message at some time $x \in [\theta_{k-1}, \theta_k]$, took the message from the source but was not able to meet the destination by γ_k . Therefore

$$\mathbb{P}(T_{k+1} = \gamma_k) = \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) I_k(x, \gamma_k) dx = e^{-\mu\gamma_k} h_2(\theta_k),$$

\square