Performance of a fixed reward incentive scheme for two-hop DTNs with competing relays

Thi Thu Hang Nguyen, Olivier Brun, Balakrishna Prabhu

Performance of a fixed reward incentive scheme for two-hop DTNs with competing relays

T. T. Hang Nguyen[1,2] Olivier Brun[1] Balakrishna J. Prabhu[1]

[1]LAAS-CNRS, Université de Toulouse, CNRS, Toulouse, France
[2]Université de Toulouse, INSA, Toulouse, France

{thi-thu-hang.nguyen, brun, balakrishna.prabhu}@laas.fr

Abstract

We analyse the performance of an incentive scheme for two-hop DTNs in which a backlogged source proposes a fixed reward to the relays to deliver a message. Only one message at a time is proposed by the source. For a given message, only the first relay to deliver it gets the reward corresponding to this message thereby inducing a competition between the relays. The relays seek to maximize the expected reward for each message whereas the objective of the source is to satisfy a given constraint on the probability of message delivery. We show that the optimal policy of a relay is of threshold type: it accepts a message until a first threshold and then keeps the message until it either meets the destination or reaches the second threshold. Formulas for computing the thresholds as well as probability of message delivery are derived for a backlogged source.

1 Introduction

The technology of Delay-Tolerant Networks (DTN) has been designed to support communications in environments where end-to-end paths between a source and a destination may not be available at all time. This technology is in particular used to enable the communication between mobile nodes scattered in outermost and sparsely populated regions. In these challenging environments, when a source node wants to transmit a message to a destination node, it can only rarely transmit directly its message to that destination, and therefore a different approach from the one used in traditional communication networks has to be used. The approach used in DTN is based on the so-called *store-carry-and-forward* paradigm. In this approach, the source node transmits its message to each and every mobile node that it meets. The latter nodes play the role of relays. They store the message and carry it, in the hope that they will eventually reach the destination and be able to deliver the message. A source can also play the role of a relay and vice versa.

Due to random node mobility and uncertainty in connectivity, DTN routing schemes usually replicate many copies of the message . In particular, *Epidemic routing* is flooding-based in nature, as relays continuously replicate and transmit messages to newly discovered contacts that do not already possess a copy of the message [VB00]. The advantage is that it can be guaranteed with high probability that some copy will reach the destination, and with a minimum delivery delay; but the downside is that it floods the network with message copies, leading to a significant energy consumption. Other routing schemes have been proposed as well (see Chapter 2 of Ph.D. thesis [Che12] for a detailed description of the different schemes). For instance, in the *two-hop routing* scheme, which is known to provide a good tradeoff between message delivery time and energy consumption [AHNA08], a relay cannot forward the message to another relay, so it stores and carries the message until the relay is in radio range of the destination.

The delivery of a message incurs a certain number of costs for a relay, in particular in terms of energy – a scarce resource in mobile networks. It can therefore be expected that, even though some nodes will cooperate out of altruism, many mobile nodes will behave selfishly, acting as "free riders" that profit from the resources of others for transmitting their own messages without offering their own resources in exchange. Clearly, if there are too many selfish mobiles nodes, the network collapses and it is no more possible to communicate between nodes. It is therefore crucial to design incentive mechanisms to persuade mobile nodes to serve as relays.

Contributions: In the present work, we analyze the incentive mechanism for message delivery in two-hop DTNs assuming a *backlogged* source (a source with infinite number of messages to send) and a *fixed reward* (may depend upon the message). When the source wants to send a message, it proposes a fixed reward to each relay it meets. The reward may vary from message to message but for a given message, the same reward is proposed to each relay. The first relay to deliver gets the reward. The relay can decide to accept or to reject the message depending on the time at which it meets the source. The cost of delivering the message for a relay consists of a linear term which depends upon the duration the relay carries this message as well as constant terms for receiving and transmitting the message.

One of the main questions in incentive mechanisms is to determine the value of reward that an agent should propose. The main aim of this paper is to give the precise relationship between the performance measures and the reward when multiple relays are competing for message delivery. This, in turn, will help the source providing an adequate reward in order to achieve a target delivery probability. Towards this end:

- We model the strategic message delivery game described above as a Bayesian game. The meeting time of a relay with the source is known to this relay only and can be seen as the private information of this relay. The other relays have a belief distribution on this time but do not know its exact value. Such games with asymmetric information and beliefs on the unknown information are modelled by Bayesian games.

- For the Bayesian game described above, we show that any pure Bayesian Nash equilibria (pure BNE) policy of a relay is of threshold type: a relay accepts a message until a first threshold and then keeps it until it either meets the destination or reaches the second threshold. Once a message is no longer accepted by the relays, the source starts giving out the following message. The thresholds of a message depend upon its index and the reward proposed.

- A pure BNE may not be unique. It could be symmetric, that is, each relay has the same two thresholds (depends upon the message) for a given message, or asymmetric. We give examples of scenarios with multiple pure BNEs. However, we shall show that any symmetric NE is unique.

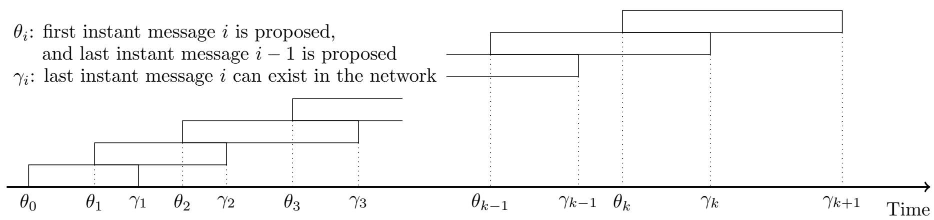


Figure 1: Message injection policy of the source at the symmetric Bayesian Nash equilibrium.

- For pure symmetric BNE, for each message, formulas for the thresholds as well as for performance measures such as the probability of delivery and expected delay are derived as a function of the reward proposed for this message. This analysis will be called *transient* analysis, that is, for message k the quantities will depend upon k.

Figure 1 illustrates how, at the symmetric BNE, messages will be injected by the source into the network. At time θ_0, the source will start proposing message 1, which the relays will accept if they meet the source after θ_0 and before θ_1. At θ_1, the source will stop proposing message 1 because it knows that none of the relays will accept it, and start proposing message 2. A relay which accepted message 1 will keep it until γ_1 or until it meets the destination, whichever occurs earlier, after which it will drop this message.

Related work: There has been a large body of literature on incentive mechanisms for DTNs. These mechanisms can be broadly classified into three categories: reputation- based schemes [MGLB00, ZXL$^+$11], barter-based schemes [BDFV07] and credit-based schemes [ZLL$^+$09, CC10, ZCY03, SBEP17].

Reputation-based schemes, such as SORI [HWK04], MobiGame [WCZ11], CONFIDANT [BB02] and RELICS [UGA10], are based on a simple principle: a node's message is forwarded only if it has forwarded messages originated from others. This however requires each node to monitor the traffic information of all encountered nodes and keep track of their reputation values. In addition, these reputation values should be updated and propagated to all other nodes efficiently and effectively, which is clearly impractical due to the intermittent connectivity between nodes.

Barter-based incentive mechanisms have also been considered to enforce fair cooperation of all nodes. For example, the authors of [SSQZ08] propose an incentive-aware routing protocol which is based on the Tit-for-Tat (TFT) strategy, in which each node forwards as much traffic for an encountered node as the latter forwards for it. In [BDFV07] and [BDFV10], Buttyan *et al.* propose a mechanism which is based on the principle of barter: a node relays the message of a neighbor if the latter relays a message of the former in return. One of the issues with this scheme is that a message might be not delivered to its destination if the destination has no message to forward in return.

Finally, in credit-based schemes, the credits earned by nodes from forwarding the messages of other nodes can be used to pay for the delivery of their own ones. As compared to reputation-based schemes, these schemes do not require global information sharing, but they assume the existence of a Trusted Third Party to manage the rewarding procedure. Credit-based incentive schemes are often designed using concepts from Game Theory, such as Vickrey-Clarke-Groves (VCG) auctions [ZLL$^+$05, LWX$^+$08] or Minority Games [CSEA$^+$13]. Other examples of credit-based schemes are Mobicent [CC10], SMART [ZLL$^+$09], PIS [MS10], INPAC [CZ10] and FRAME [LW09], among others.

For most of these schemes, it has been difficult to obtain performance measures such as probability of message delivery and mean time to deliver a message with the exception of [Alt09, SBEP17].

In [Alt09], a simple reward-based mechanism was proposed in which the first relay to deliver gets the reward. The author provided the expression of the success delivery probability of a packet within a fixed time τ with the assumption that a relay i will participate in delivering the packet until a certain time. It was shown that the equilibrium policy is of threshold type: relays participate until a certain time after which they are deactivated. All the computations and results are for a single message. Our setting is different from [Alt09] in the following ways. In that work, the relays decided how long they participate in the network and during this time they accepted the message with certain probability and did not drop it. In our work, the relays can decide how long they accept they message and then how long they keep it. This gives more freedom to the relays to make their choice. Our cost structure is also different from that in [Alt09]. The linear term in our work depends only upon the duration the relay stores a message whereas in [Alt09] this term depends upon the time the relay is participating. These two are different because in the latter case, relays accrue a cost even if they do not have a message. Furthermore, there is no cost of receiving the message from the source in [Alt09]. The inclusion of this cost leads to a policy with two thresholds in our case as opposed to a single threshold in [Alt09]. We also show how the performance measures depend upon the reward offered by the source thereby giving the source an explicit way to compute the reward so as to achieve its targeted performance. Finally, we consider a backlogged source as opposed to a single message in [Alt09]. This induces dependence between the policies of messages which was not there in that work.

In [SBEP17], the source offers rewards that depend upon the meeting time with the condition that only the first one to deliver the message receives its reward. Since the mobility model is random, a relay that meets the source later has lower probability of being the first to deliver the message and hence receiving the reward. The reward proposed to a relay is inversely proportional to its success probability, and is such that a relay always accepts the message. The analysis relies heavily on the assumption that the relays do not discard a message once they accept it from the source. This assumption may be realistic in participative networks in which nodes are altruistic. On the other hand, when nodes are selfish, as is the case in the present paper, they could decide to throw away a message once it is not longer profitable to keep it (because the probability of success is too small) and reduce their costs. This possibility to reject or drop the messages is the main difference of our work with [SBEP17], in which no strategic interaction between relays was considered.

A model similar to the one studied in this work was first considered in [SBP18] (which is based on Chapter 5 of [Ser15]) in which the competition was modelled as a stochastic game. That model was in discrete-time, restricted to two relays and a single message, and had partial results on the optimal policy. The model studied in the present work is in continuous time, for an arbitrary number of relays, and a backlogged (with possibly infinite number of messages) source which sequentially proposes messages. The passage from a discrete-time to a continuous-time model introduces some technical difficulties such as decisions that can be made at arbitrary time instants instead of just at Poisson ones. Thus, our problem cannot be modelled directly as a continuous-time Markov Decision Processes (MDP) which would be the natural analog of the discrete-time MDP in [SBP18]. To circumvent this problem, we use the framework of Bayesian games which imposes certain restriction on the actions but at the same time removes the constraint on decisions at Poisson instants. In the appendix, we give nonetheless a continuous-time MDP formulation and a uniformization based informal argument that shows that same results can also be expected using this alternative method. Preliminary results of this work without the Bayesian game formulation but with the informal MDP one have appeared in [NBP16]. The results in that paper were limited to a game with just one message for the source. Here, we generalize the results to a backlogged source. Also, we give conditions for the existence and uniqueness of a symmetric equilibrium, which were not given in that paper.

Organization: The rest of this paper is organised as follows. Section 2 is devoted to model description. In Section 3, the Bayesian game is formally defined and the structure of the best response policy of a relay is shown to be of threshold type. Section 4 gives the conditions for the existence and uniqueness of the symmetric BNE. In Section 5, we present a method for recursively computing the thresholds of the symmetric equilibrium as well as the probabilities of message delivery for a backlogged source. Section 6 is devoted to simulation results. Some conclusions are drawn in Section 7.

2 Model Description

Consider a network of one source, one destination, and N relays. The source and the destination are assumed to be fixed, whereas the relays move according to a given mobility model. It is assumed that the mobility pattern of any two relays are independent, and that the inter-contact times between relay i and the source (resp. the destination) are independent and identically distributed according to an exponential distribution of rate λ_i (resp. μ_i). The inter-contact processes of different relays with the source as well as with the destination are assumed to be statistically identical. We note that the assumption of exponentially distributed inter-contact times is satisfied under the Random Waypoint Mobility model [SM04a, GNK04, SM04b, CE07] and has been observed to hold in real motion traces [ZFX$^+$10].

When it meets the source, a relay is offered a fixed reward, R_k, to deliver message k. The reward is fixed in the sense that, for a given message, each relay is offered the same reward irrespective of their meeting times. The relay has a choice to either accept the message or not. There is no cost associated with rejecting the message. If it accepts the message, the relay can decide to drop the message at any time in the future at no additional cost. If during this time the relay meets the destination, then it can transmit the message to the destination and claim the reward only if it is the first one to do so for this message.

The various costs incurred for accepting and storing a message are assumed to be as follows: *(i)* C_r is the cost of receiving the message from the source; *(ii)* C_d is the cost of transmitting the message to the destination; and *(iii)* C_s is the cost per unit time for storing the message. These costs are all the same for all relays.

We illustrate the cost structure with an example. Suppose that relay i meets the source at time instant a, accepts the message k and decides to keep it until time b, then the expected total cost of keeping the message in the interval (a, b) will be

$$\begin{aligned} C_r + &\int_a^b \mu_i e^{-\mu_i(\tau-a)}(C_s(x-a) + (C_d - R_k)p_k^i(\tau))d\tau \\ &+ e^{-\mu_i(b-a)}C_s(b-a) =: C_r + G_k^i(a,b), \end{aligned} \tag{1}$$

where $p_k^i(\tau)$ is the probability that the relay is the first one to deliver this message when it meets the destination at time τ. A one-time cost of C_r is incurred for accepting the message at a. From then on, a storage cost of C_s is incurred per unit of time either until b (that is, for a duration $b-a$) or until it meets the destination. If the relay meets the destination at time $t < b$ and it is the first one to meet the destination with this message then it will transmit the message to the destination and get the reward thereby incurring a net cost of $C_s(t-a) + (C_d - R_k)p_k^i(\tau)$. So, the second term is the expected cost incurred if the relay meets the destination before b. Finally, the last term is the storage cost incurred if the relay does not meet the destination before b. The sum of the last two terms will be denoted by $G_k(a,b)$ which is the expected cost of keeping the message in the interval (a,b). Note that this cost depends on the strategies of the other relays through the success probability $p_k^i(\tau)$.

The analysis in the paper will be done for $R_k = R, \forall k$. This is done to make the notation less cumbersome. The results carry over to the case when the reward depends upon the index of the message. We shall sometimes use the notation $\overline{R} = R - C_d$. In addition, we shall assume that $R \geq R_{min} := \max(R^i_{min})$, where $R^i_{min} = C_r + \frac{C_s}{\mu_i} + C_d$. Note that R^i_{min} is the average cost of relay i if it were to be the only one to be competing for the message. It is therefore natural that the reward should be larger than this average cost for any relay to participate in forwarding. Then R_{min} is the minimum required cost in order to have all relays participate in the game.

It shall be assumed that a relay can store only one message at a time. Further, if a relay already has a message in its buffer, then it does not seek a new (or the same) message until it either meets the destination or drops the message. A message can be dropped only because it is no longer profitable to store this message due to a small probability of success but not because the relay meets the source. Once it has delivered or dropped the message, the relay can seek a new one from the source.

The source has an unlimited number of messages to send to the destination, each of which it proposes sequentially. That is, to each message the source associates an interval of time during which it proposes this message to any relay it meets. We shall denote this interval for message k by $[\theta_{k-1}, \theta_k)$, where θ_{k-1} is the last time message $k-1$ was proposed.

For a given relay (called tagged relay) when it meets the source, we shall assume that the decision to accept and the duration can depend upon its history of contacts and previous decisions but not upon the history of the other relays. While the exact history of the other relays is not available to the tagged relay, we shall assume that it can compute a belief (or a probability distribution) on when the other relays will meet the source for this message. This belief will be computed based upon the statistics of the mobility model, and will be denoted by $\hat{\Phi}_k^i(t)$. An auxiliary quantity is the probability that relay i will enter into competition for message k on or before time t, which will be denoted by $\Phi_k^i(t)$. By entering into competition on or before time t, we mean that that there was time instant before t at which relay i did not have any message with index smaller than k. We shall denote by $\phi_k^i(t)$ the probability density function corresponding to $\Phi_k^i(t)$. Note that $\hat{\Phi}_k^i(t)$ is the convolution of $\Phi_k^i(t)$ and an exponential distribution of rate λ_i.

Let us take an example to illustrate the notion of competing for a message. Suppose there are two relays. Relay 1 meets the source for the first time at some instant between θ_1 and θ_2 when the source is proposing message 2. If relay 2 had the message 1 at time θ_1 then we say that it is not in competition for message 2 until it has this message because even if it meets the source it cannot accept message 2. Now suppose that relay 2 meets the destination at some time $t_2 \in [\theta_1, \theta_2)$. At t_2, we say that relay 2 enters into competition with relay 1 for message 2. Of course, relay 1 does not have exact knowledge of the contact history of relay 2 but, using the statistics of the mobility model, it can compute the probability that relay 2 entered into competition at time $t \in [\theta_1, \theta_2)$. This probability distribution will be denoted by $\Phi_2^2(t)$.

A further assumption we shall make is that the relays do not know whether there will be any more messages in the future[1]. Hence, they treat each message as though it were the last one. The policy for a message thus does not depend upon the future messages but could depend upon the policy for the previous messages.

3 A Bayesian game approach

In this section, we shall model the strategic game between the relays as a Bayesian game [FT91]. These games specifically treat models in which each player has some private information (also called its type) and a belief over the private information of the other players. In our game, each relay knows the time instant it meets the source but does not know the meeting times of the other relays with the source. It however has a belief on the meeting times of the other relays which is captured by the belief function $\hat{\Phi}_k^i$ for message k and relay i. This asymmetry in information between relays means that Bayesian games are an appropriate model for finding the equilibrium strategies of the relays.

A Bayesian game is defined by the type space, the strategy space, the common belief, and the utility function for the relays. For message k, the set of types of relay i, T_i, is its meeting time with the source relative to the generation instant of this message[2]. Thus, $T_i = [0, \infty)$. The strategy, S_i, is assumed to be $[0, \infty)$, where the strategy $s = 0$ implies that the message is refused whereas $s > 0$ implies the message is accepted and kept until time s or until it meets the destination (whichever occurs first). We shall assume that the type of player i is a random variable with distribution $\hat{\Phi}_i$ and is independent of the types of the other relays. This defines the joint belief structure for the types of the relays.

Remark The structure of the Bayesian game is such that the dependence on message k is only through the belief distributions Φ_k^i. In the following, we shall be present a solution of the game by assuming an arbitrary belief structure. Later on, we shall explain how to compute the belief for message k recursively and then compute the solution for this message using its belief. Therefore, for the sake of clarity, we shall drop the dependence on k.

We now define the remaining quantity which is the utility function of the relay i, u_i. It is a mapping from $\prod_i S_i \times T_i \to \mathbb{R}$, that is, for every possible actions and types of the relays, it defines the reward (or the cost) of relay i. Let $\rho_i(\tau; s_i, s_{-i}, t_i, t_{-i})$ be the probability of relay i winning the reward when it meets the destination at τ given s_i (resp. t_i), the strategy (resp. type) of relay i, and s_{-i} (resp. (t_{-i})), the strategy (resp. type) of the other relays. Then, the utility of relay i can be defined by:

$$u_i(s_i, s_{-i}; t_i, t_{-i}) = \begin{cases} 0 & \text{if } s_i = 0; \\ F_i(s_i, s_{-i}; t_i, t_{-i}) & \text{if } s_i > 0. \end{cases} \tag{2}$$

where

$$F_i(s_i, s_{-i}; t_i, t_{-i}) = C_r + \int_{t_i}^{t_i+s_i} \mu_i e^{-\mu_i(\tau - t_i)} \left(C_s(\tau - t_i) + \bar{R}\rho_i(\tau; s_i, s_{-i}, t_i, t_{-i}) \right) d\tau + e^{-\mu_i s_i} C_s s_i,$$

[1] The source is assumed to be backlogged but this information is not known to the relays.

[2] This set does not depend on the message. Hence, we drop the dependence on k.

is the cost of accepting and keeping the message for duration s_i given t_i, s_{-i}, t_{-i}. It is the cost of action $s_i > 0$ conditioned on t_i, s_{-i}, t_{-i}.

With this we have all the inputs (types, strategies,beliefs and utilities) necessary to define the Bayesian game and compute its equilibrium. We note that our game is one with a continuum of types and of actions.

3.1 Pure Bayesian Nash equilibrium

A strategy profile for relay i is the function $\pi_i : T_i \to S_i$ that determines the strategy for each possible type of player i. Let π_i be a strategy profile for relay i. Then, the strategy profile vector (π_i^*, π_{-i}^*) is a pure strategy BNE if [Mei03]

$$\pi_i^*(t_i) = \arg\min_{s_i} \int u_i(s_i, \pi_{-i}^*(t_{-i})); t_i, t_{-i}) d\hat{\Phi}_{-i}(t_{-i}|t_i), \tag{3}$$

for all i. From the definition of u_i and F_i, and noting that

$$p^i(\tau) = \int \rho_i(\tau; s_i, s_{-i}, t_i, t_{-i}) d\hat{\Phi}_{-i}(t_{-i}|t_i),$$

(3) can be rewritten as:

$$\pi_i^*(t_i) = \arg\min_{s_i} \bar{u}_i(s_i, \pi_{-i}^*; t_i), \tag{4}$$

where

$$\bar{u}_i(s_i, \pi_{-i}; t_i) = \begin{cases} 0 & \text{if } s_i = 0; \\ C_r + G_i(t_i, t_i + s_i) & \text{if } s_i > 0. \end{cases} \tag{5}$$

The existence of an equilibrium for games with continuum of types and actions is not known in general. In [Mei03], sufficient conditions requiring the continuity of the utility functions are presented. These conditions are not satisfied in our setting because of a discontinuity at $s = 0$ which arises due to the fixed cost, C_r of accepting a message. Nonetheless, we are able to obtain the existence of an equilibrium using some properties of the utility functions.

First, we give the structure of the best-response policy of a given relay and for a fixed strategy of the other relays. The structure of the best-response policy will then determine the structure of the pure BNE.

Theorem 3.1. *Given the strategies of the other relays, the best-response policy $\pi_i^*(t; \pi_{-i})$ of relay i is a threshold-type policy: there exists a θ^i and $\gamma^i > \theta^i$ such that $\pi_i^*(t; \pi_{-i}) = \gamma^i - t$ if and only if $t \le \theta^i$, and $\pi_i^*(t; \pi_{-i}) = 0$ if and only if $t > \theta^i$. Moreover, (θ^i, γ^i) is the solution of:*

$$\gamma^i = \sup\{x : p^i(x) > \frac{C_s}{\mu_i(R - C_d)}\}, \tag{6}$$

$$\theta^i = \sup\{x : C_r + G^i(x, \gamma^i) < 0\}, \tag{7}$$

where by convention the supremum of the empty set is 0.

Proof. See Appendix A. □

Theorem 3.1 says that, for a fixed policy of the other relays, there exists a θ^i such that if the type of relay i is less than θ^i (i.e., if it meets the source before θ^i) then its optimal strategy will be to accept the message and keep it until $\gamma^i > t$ (unless it meets the destination first). If it meets the source after θ^i then its optimal strategy will be to refuse the message.

Note that in Theorem 3.1, nothing precludes that $\gamma^i = \infty$, or even that $\theta^i = \gamma^i = \infty$, as shown in the following example. Consider the case where the strategy of the other relays is to never accept the message. In that case, the success probability of relay i is $p^i(x) = 1$ for all $x \geq 0$, and it follows from the assumption $R \geq R^i_{min}$ that $\gamma^i = \infty$. Moreover, since in that case $G^i(t, \infty) = \frac{C_s}{\mu_i} - \overline{R} < 0$ for all $t \geq 0$, we also have $\theta^i = \infty$. Hence, the best-response policy of player i is to always accept the message and to keep it forever. This shows in particular that, under the assumption $R \geq R^i_{min}$, the vector of policies in which all relays always reject the message cannot be an equilibrium of our game.

A direct consequence of Theorem 3.1 is the following structural result of any pure BNE.

Corolary 3.1. *At a pure BNE, if any, all players use a threshold-type policy, that is, there exist vectors $\boldsymbol{\theta}$ and $\boldsymbol{\gamma}$ such that relay i uses a threshold-type strategy with parameters (θ^i, γ^i).*

A pure BNE can be asymmetric or symmetric. An asymmetric equilibrium can be of the form: relay 1 always accepts and keeps the message until it meets the destination and relay 2 never accepts. For an example of this type of asymmetric equilibrium, assume that $\lambda_i = \mu_i = 1$ for all i. Under the given policy of relay 1, the probability of success of relay 2 at time t will be

$$p^2(t) = e^{-t}(1 + t). \tag{8}$$

From the above equation and (6), it follows that γ^2 will be finite. Suppose that relay 2 meets the source at time 0. This is the most favorable scenario for relay 2. If it is not profitable to accept the message at time 0, then it will never be so later on. From (1), if the relay meets the source at time 0, then its total cost to go if it accepts the message and keeps it until time γ^2 will be

$$C_r + G^2(0, \gamma^2) > C_r - \bar{R} \int_0^{\gamma^2} (1 + t)e^{-2t} dt > C_r - \bar{R} \int_0^{\infty} (1 + t)e^{-2t} dt = C_r - \frac{3\bar{R}}{4}$$

That is, if $\bar{R} < \frac{4}{3} C_r$, then relay 2 will always have a positive cost of accepting and its best response will be never to accept. Of course, if $\bar{R} > C_r + C_s$ then relay 1 will always accept if it knows that relay 2 will never accept because this reward is greater than the average cost incurred by one relay. Thus, we have the claimed asymmetric equilibrium.

In the sequel, we let $\lambda_i = \lambda$, and $\mu_i = \mu$ for all i. We shall study the existence and uniqueness of only the symmetric BNE, that is, equilibria in which all relays use the same thresholds θ_k and γ_k for message k. Using the fact that best-response policies are of threshold type, we obtain an explicit expression of the success probability $p^i_k(t)$. Assuming that up to message $k - 1$ only symmetric equilibria have been played, we use this simple expression of the success probability in the following to establish the conditions under which there exists a unique symmetric BNE.

4 Symmetric Bayesian Nash Equilibrium

Assume that all relays have played symmetric equilibria[3] for messages $1, 2, \ldots, k-1$, that is, $\theta_j^i = \theta_j$ and $\gamma_j^i = \gamma_j$ for $j = 1, 2, \ldots, k-1$. A direct consequence of Corollary 3.1 is that if all relays play their equilibrium strategies, the success probability $p_k(t)$ of a player has a very simple structure. For $y \geq x \geq 0$, let

$$v_{x,y}(s) = e^{-\lambda(x-s)} + \frac{\lambda}{\mu - \lambda} e^{-\mu y} e^{\lambda s} \left(e^{(\mu-\lambda)x} - e^{(\mu-\lambda)s} \right), \tag{9}$$

for $s < x$, and $v_{x,y}(s) = 1$ otherwise. Note that if $x = \min(\theta_k, t)$ and $y = \min(\gamma_k, t)$, $v_{x,y}(s)$ represents the probability that relay j be not able to deliver the message k by time t given that it comes into play at time s. Then, introducing

$$\begin{aligned} V_k(x, y) &= \int_{\theta_{k-1}}^{\infty} \phi_k(s) v_{x,y}(s) ds, & (10) \\ &= 1 - \int_{\theta_{k-1}}^{x} \phi_k(s)(1 - v_{x,y}(s)) ds, & (11) \end{aligned}$$

the quantity

$$f_k(t) = V_k \left(\min(\theta_k, t), \min(\gamma_k, t) \right), \tag{12}$$

represents the probability that a relay fails to deliver the message to the destination by time t, either because it does not meet the source by time $\min(\theta_k, t)$, or because it meets it but does not meet the destination before $\min(\gamma_k, t)$. It then follows that the probability of success of a given relay is

$$p_k(t) = f_k^{N-1}(t) = V_k \left(\min(\theta_k, t), \min(\gamma_k, t) \right)^{N-1}. \tag{13}$$

Note that $p_k(t)$ is constant after γ_k, and that $p_k(t) = V_k(\theta_k, t)^{N-1}$ for all $t \in [\theta_k, \gamma_k]$. As a consequence, defining

$$\omega = \left(\frac{C_s}{\mu \overline{R}} \right)^{1/(N-1)}, \tag{14}$$

the second threshold γ_k is the greatest value of t such that $V_k(\theta_k, t) \geq \omega$. For a given $\theta \geq 0$, let

$$\gamma(\theta) = \sup\{x : V_k(\theta, x) \geq \omega\}. \tag{15}$$

Note that the function $\gamma(\theta)$ takes its values in $[0, \infty]$. We establish in Lemma 4.1 that there exist θ_{min} and θ_{max} such that $\gamma(\theta) = \infty$ for $\theta \leq \theta_{min}$, whereas $\gamma(\theta)$ takes a uniquely defined finite value for $\theta \in (\theta_{min}, \theta_{max}]$.

Lemma 4.1. *Let θ_{min} be the solution of*

$$1 + \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left(e^{-\lambda(\theta_{min} - s)} - 1 \right) ds = \omega, \tag{16}$$

[3] In the sequel, since we are treating the symmetric case, we shall not use superscripts to distinguish relays or player. For example, we shall use $p_k(t)$ instead of $p_k^i(t)$ for the probability of success.

and θ_{max} be such that

$$V_k(\theta_{max}, \theta_{max}) = \omega. \tag{17}$$

Then, for θ fixed, the equation $V_k(\theta, \gamma)$ has a unique finite solution $\gamma \geq \theta$ if and only if $\theta \in (\theta_{min}, \theta_{max}]$. Moreover, $\gamma(\theta)$ is a strictly decreasing function of $\theta \in (\theta_{min}, \theta_{max}]$.

Proof. See Appendix B.1. □

We use Lemma 4.1 to establish in Theorem 4.1 below the conditions under which there exists a unique symmetric BNE.

Theorem 4.1. *There exists a symmetric BNE with $\theta > 0$ if and only if*

$$\overline{R} \geq C_r + \frac{C_s}{\mu}. \tag{18}$$

Under this condition, the symmetric BNE is unique. Moreover, the parameters of the equilibrium are finite, i.e., $\theta_k > 0$ and $\theta_k \leq \gamma_k < \infty$ if and only if

$$1 + \mu \frac{C_r}{C_s} < \frac{(1+b)^N - 1}{N\, b} \tag{19}$$

where

$$b = \frac{1}{\sigma\omega} \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)} \right) ds, \tag{20}$$

and $\sigma = (\mu - \lambda)/\lambda$.

Proof. See Appendix B.2. □

We remind the reader that $R_{min} = C_r + \frac{C_s}{\mu} + C_d$ is the minimum value that the reward R should have for a single relay to attempt the delivery of a message. Theorem 4.1 shows that for any value of R greater than this minimum value, the existence of a unique symmetric equilibrium is guaranteed. Figure 2 illustrates the condition (19) for the first message when $N = 3$, $\mu = 0.4$, $C_s = 0.5$ and $C_r = 4.0$. In that case, the minimum value of $\bar{R}$ is $C_r + \frac{C_s}{\mu} = 5.25$. We note that for the first message we have $e^{-\lambda\theta_{min}} = \omega$ and thus $b = \sigma^{-1}(1 - \omega^\sigma)$.

Remark The condition (19) of Theorem 4.1 can be equivalently written as $f(b) > 1 + \mu \frac{C_r}{C_s}$, where

$$f(x) = \frac{(1+x)^N - 1}{Nx}.$$

Note that $b > 0$ for all $\sigma \in [-1, \infty)$ and all $\omega \in (0, 1)$. Using the binomial formula, it is easy to show that $f(x) > 1 + g(x)$ for all $x > 0$, where $g(x) := \frac{N-1}{2} x \left(1 + \frac{N-2}{3} x\right)$. Hence, a sufficient condition for θ_k and γ_k to be finite is $g(b) > \mu \frac{C_r}{C_s}$. Since $g(x)$ is strictly increasing over $[0, \infty)$, this is equivalent to $b > y$, where

$$y = \frac{3}{N-2} \left(\sqrt{\frac{1}{4} + \frac{2}{3} \mu \frac{C_r}{C_s} \frac{N-2}{N-1}} - \frac{1}{2} \right),$$

is the unique solution of $g(x) = \mu \frac{C_r}{C_s}$. The latter sufficient condition can be written as

$$\overline{R} > \frac{C_s}{\mu} \left(\frac{1}{\sigma y} \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)} \right) ds \right)^{-\frac{1}{N-1}}.$$

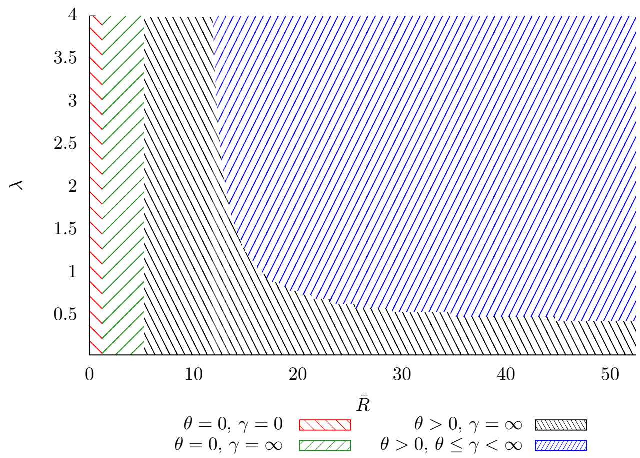


Figure 2: Different possible types of symmetric BNE for 3 relays.

5 Transient analysis of the symmetric BNE

In the rest of the paper, we shall focus only on the symmetric equilibrium. First, we give an algorithm to compute the thresholds θ_k and γ_k for message k which will then be used to derive the probability of message delivery and expected message delay from these thresholds.

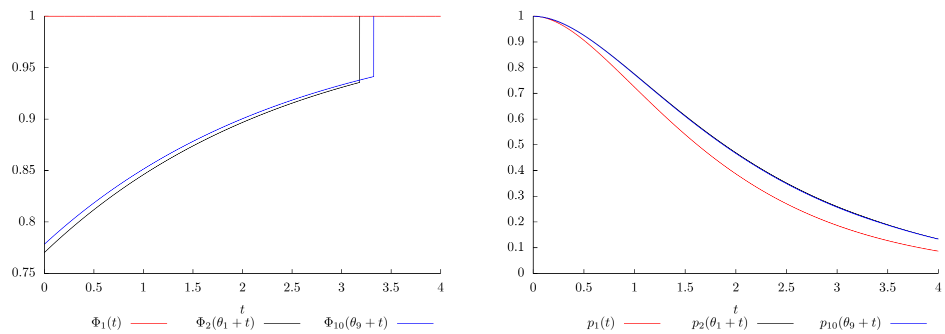
5.1 Recursive computation of the success probability

The thresholds θ_k and γ_k of a symmetric equilibrium are obtained from (6) and (7), in which $p_k^i(t) = V_k(\theta_k, t)^{N-1}$ for all $t \in [\theta_k, \gamma_k]$ and all relays i. The computation of the function $V_k(x, y)$ however requires the knowledge of the probability density function $\phi_k(t)$, $\int_a^b \phi_k(t)dt$ representing the probability that a given relay comes into play for the delivery of the k^{th} message between time instants a and b. In this section, we shall show how this probability density function can be recursively computed for symmetric equilibria .

To this end, let us define $I_k(x, t)$ as the probability that a relay that comes into play at time x will accept the k^{th} message and will not be able to deliver it to the destination by time $t \in [\theta_k, \gamma_k]$. Therefore,

$$\begin{aligned} I_k(x,t) &= \int_x^{\theta_k} \lambda e^{-\lambda(s-x)} e^{-\mu(t-s)} ds \\ &= \frac{e^{-\mu t}}{\mu - \lambda} \lambda e^{\lambda x} \left(e^{(\mu-\lambda)\theta_k} - e^{(\mu-\lambda)x} \right) \end{aligned} \tag{21}$$

Thus, $1 - I_k(x, t)$ is the probability that a relay will not have the k^{th} message at time t, either because it has not met the source, or because it has already delivered the message. Similarly, $I_k(x, t_1) - I_k(x, t_2)$ represents the probability that a relay that comes into play at time x will meet the source before θ_k and deliver the message to the destination in the time interval $(t_1, t_2]$. Finally, note also that $\frac{dI_k(x,t)}{dt} = -\mu I_k(x, t)$. We use the definition of



(a) $\Phi_k(t)$ (b) $p_k(t)$

Figure 3: Evolution of $\Phi_k(t)$ and $p_k(t)$ with k.

$I_k(x,t)$ as well as its above mentioned properties to prove Lemma 5.1 below, which gives a recursion to numerically compute the density $\phi_k(t)$.

Lemma 5.1. *For $t \in [\theta_k, \gamma_k]$,*

$$\phi_{k+1}(t) \quad = \quad h_1(\theta_k)\delta_{\theta_k}(t) + \phi_k(t) + h_2(\theta_k)\left\{\mu e^{-\mu t} + e^{-\mu\gamma_k}\delta_{\gamma_k}(t)\right\} \tag{22}$$

where

$$\begin{aligned} h_1(\theta_k) &= \int_{\theta_{k-1}}^{\theta_k} \phi_k(x)\left\{1 - I_k(x,\theta_k)\right\} dx, \\ h_2(\theta_k) &= e^{\mu\theta_k} \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) I_k(x,\theta_k) dx. \end{aligned}$$

Proof. See Appendix B.3. □

Lemma 5.1 can be used for the recursive numerical computation of the density $\phi_k(t)$, from which we can derive the probability of success $p_k(t) = \left(V_k\left(\min(\theta_k,t), \min(\gamma_k,t)\right)\right)^{N-1}$. Figures 3a and 3b show the CDF $\Phi_k(t)$ and the success probability $p_k(t)$ for $k \in \{1,2,10\}$, respectively, in the case $N=3$ relays, using the following parameters: $\lambda = 1.25$, $\mu = 0.4$, $C_s = 0.5$, $C_d = C_r = 4.0$ and $R = 30$.

The thresholds θ_k and γ_k can then be obtained by solving (6)-(7) using any root finding method, such as the bisection method, as illustrated in Algorithm 1.

5.2 Performance metrics

From the point of view of the source, the main performance metrics are the probability that a message is successfully delivered and, provided that it reaches its destination, the expected time to deliver it. Our first result in this direction is on the probability of message delivery.

ALGORITHM 1: Computation of successive symmetric BNE

Require: $\phi_1(t) = \delta_0(t)$, $\theta_0 = 0$
for $k = 1, 2, \ldots$ **do**
  Compute θ_{min} and θ_{max} as the solutions of (16) and (17)
  $a = 0$, $b = \theta_{max}$
  repeat
    $c = (a+b)/2$
    if $c > \theta_{min}$ **then**
      Compute $\gamma(c)$ as the solution of $V_k(c, \gamma) = \omega$
    else
      $\gamma(c) = \infty$
    end if
    $G_k = \int_c^{\gamma(c)} \mu e^{-\mu(t-c)} \left(\frac{C_s}{\mu} - \overline{R} V_k(c,t)^{N-1} \right) dt$
    if $G_k < -C_r$ **then**
      $a = c$
    else
      $b = c$
    end if
  until $|G_k + C_r| < \epsilon$.
  $\theta_k = c$, $\gamma_k = \gamma(c)$
  Compute $\phi_{k+1}(t)$ with (22)
end for

Proposition 5.1. *Assume that all relays play a symmetric equilibrium strategy with parameters θ_k and γ_k for the delivery of message k. Let ζ_k be the probability that this message is successfully delivered, that is, the probability that at least one copy reaches the destination by time γ_k. Then*

$$\zeta_k = 1 - \left(\frac{C_s}{\mu \overline{R}} \right)^{\frac{N}{N-1}}, \tag{23}$$

if $\gamma_k < \infty$, whereas

$$\zeta_k = 1 - \left(1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - e^{-\lambda(\theta_k - s)} \right) ds \right)^N, \tag{24}$$

otherwise.

Proof. From (12), the probability that all relays fail to deliver the message to the destination by time γ_k is $V_k(\theta_k, \gamma_k)^N$, from which we deduce that $\zeta_k = 1 - V_k(\theta_k, \gamma_k)^N$. If $\gamma_k < \infty$, it follows from (13) and (6) that $p_k^i(\gamma_k) = V_k(\theta_k, \gamma_k)^{N-1} = C_s/(\mu\overline{R})$, from which we readily obtain (23). If on the contrary $\gamma_k = \infty$, then (24) follows from (11) and

$$\begin{aligned} \lim_{y \to \infty} V_k(\theta_k, y) &= 1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - \lim_{y \to \infty} v_{\theta_k, y}(s) \right) ds, \\ &= 1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - e^{-\lambda(\theta_k - s)} \right) ds. \end{aligned}$$

□

We emphasize that Proposition 5.1 can be used by the source to compute the minimum reward R allowing to achieve a target delivery probability. If in addition the source wishes the message to be delivered within a certain amount of time, it can use Proposition 5.2 below.

Proposition 5.2. *Assume that all relays play a symmetric equilibrium strategy with parameters θ_k and γ_k for the delivery of message k, and let D_k be the expected delivery time of this message. Provided that at least one copy reaches the destination by time γ_k, the expected delivery time is*

$$\mathbb{E}\left[D_k|D_k \leq \gamma_k\right] = \frac{1}{\zeta_k}\int_{\theta_{k-1}}^{\gamma_k} \left(V_k(\min\left(t,\hat{\theta}_k\right),t)^N - V_k(\theta_k,\gamma_k)^N\right) dt \tag{25}$$

Proof. From (12), we have $\mathbb{P}\left(D_k > t\right) = V_k(\min(\theta_k, t), t)^N$ for all $t \leq \gamma_k$. It yields

$$\mathbb{P}\left(D_k > t|D_k \leq \gamma_k\ \right) = \frac{1}{\zeta_k}\left(V_k(\min(\theta_k,t),t)^N - V_k(\theta_k,\gamma_k)^N\right),$$

and the result directly follows from

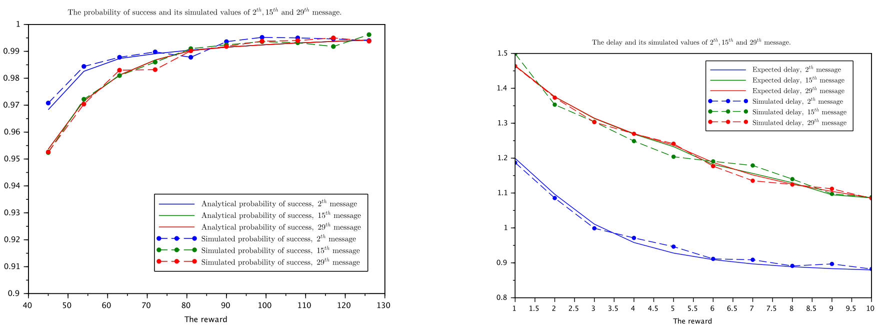
$$\mathbb{E}\left[D_k|D_k \leq \gamma_k\right] = \int_{\theta_{k-1}}^{\gamma_k} \mathbb{P}\left(D_k > t|D_k \leq \gamma_k\ \right) dt.$$

□

We do some simulations with different values of R to see how expected delay and probability of success change with R. We take the following values for the parameters: $C_r = 10, C_s = 0.4, C_d = 4, \lambda = 0.8, \mu = 0.4, N = 15$. Figure 4b illustrates the convergence of the expected delay as a function of $R \in [3 \times R_{min}, 8 \times R_{min}]$ and k (messages 15 and 29 have almost the same expected delay). This figure also shows that for k large the messages have a greater expected delay than the first messages, whereas Figure 4a shows that the probability of success increases with R and approaches 1 as $R \to \infty$. We do another simulation with $C_r = 2, C_s = 0.4, C_d = 2, \lambda = 0.2, \mu = 0.1, N = 10$. Figure 5a compares the values of the probability of success obtained with Proposition 5.1 against the values obtained through simulations, for $R = 10$, which yields $\gamma = \infty$, and for $R = 15.33$ which gives a finite γ. Note from (23) that for $R = 5.4 \times R_{min}$ the success probability is the same for all messages, whereas for $R = 3 \times R_{min}$ it decreases with k. Figure 5b shows more clearly the convergence of the expected delay with k in the case $R = 3 \times R_{min}$ and $R = 5.4 \times R_{min}$.

6 Simulation results

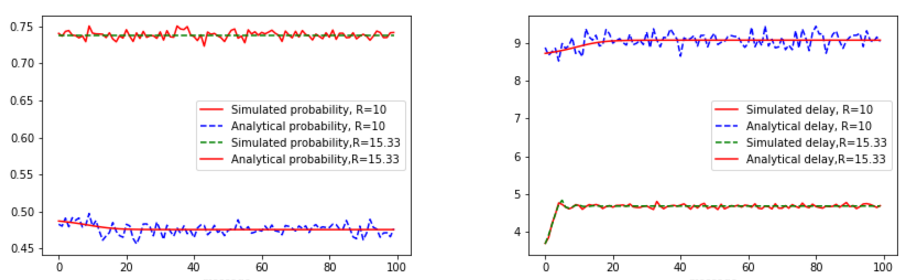
In this section, we assess the applicability of our results in scenarios where some of our assumptions are violated using event-driven simulations. We first present the results obtained with synthetic traces in Section 6.1, beforevalidating our approach against real mobility traces in Section 6.2.



(a) Delivery probability

(b) Expected delivery time

Figure 4: Comparison of the theoretical performance metrics of 2^{nd}, 15^{th} and 29^{th} messages against those obtained through simulations.



(a) Delivery probability

(b) Expected delivery time

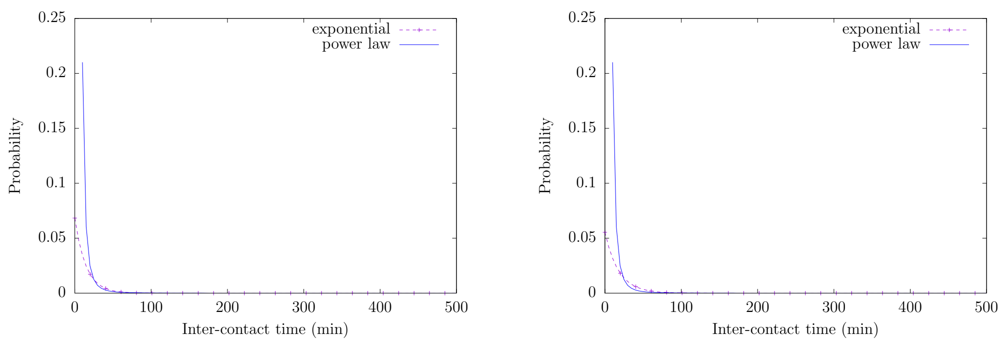
Figure 5: Comparison of the theoretical performance metrics against those obtained through simulations for $R = 10$ ($\gamma = \infty$) and for $R = 15.33$ ($\gamma < \infty$).

6.1 Synthetic traces

In this section, we investigate the effect of non-exponential inter-contact time distributions on the accuracy of our results. For the inter-contact times, we use a truncated power-law distribution whose probability density function is

$$f(x) = \frac{\alpha + 1}{x_{max}^{\alpha+1} - x_{min}^{\alpha+1}} \, x^{\alpha},$$

for x in the interval $[x_{min}, x_{max}]$. For the source node, we use $x_{min} = 15$, $x_{max} = 100$ and $\alpha = -3.5$, whereas for the destination we use $x_{min} = 10$, $x_{max} = 500$ and $\alpha = -3.1$. The corresponding probability densities are shown in Fig. 6.



(a) pdf of inter-contact times with the source node

(b) pdf of inter-contact times with the destination node

Figure 6: Probability density functions of the inter-contact times with the source and the destination.

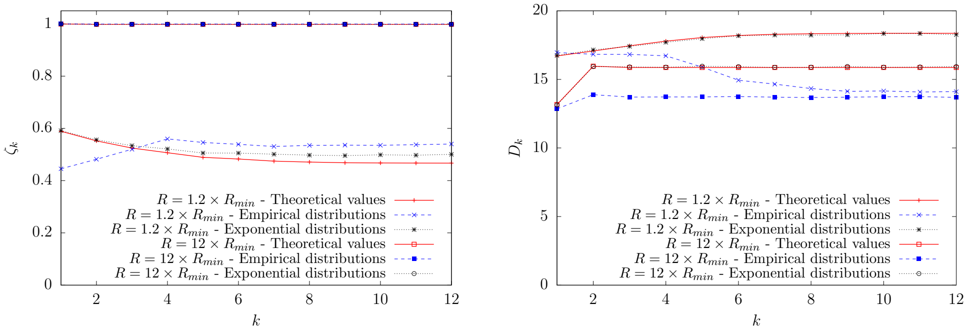
Since inter-contact times are not exponentially distributed, we choose $\frac{1}{\lambda}$ (resp. $\frac{1}{\mu}$) as the mean residual inter-contact time with the source (resp. destination), which yields $\frac{1}{\lambda} = 14.63$ minutes and $\frac{1}{\mu} = 18.05$ minutes. The values of the thresholds θ_k and γ_k are then computed for $k = 1, \ldots, 12$, using the above values of λ and μ and for 4 relays. It was assumed that $C_s = 0.01$ and that $C_r = C_d = 10$. The thresholds were computed for two different values of the reward R, $R = 1.2 \times R_{min} = 24.22$ and $R = 12 \times R_{min} = 242.16$.

The delivery ratios of the first twelve messages obtained through event-driven simulations (with $100,000$ sample paths) are compared to their theoretical values in Fig. 7a. For $R = 12 \times R_{min}$, simulation results are in perfect agreement with theoretical values, but for $R = 2 \times R_{min}$ the discrepancy is larger and can be as high as 25% for the first message. The error is lower for subsequent messages. The results obtained for the expected delivery time are shown in Fig 7b and we observe similar relative errors.

6.2 Validation against real mobility traces

To assess the applicability of our results against a real-world scenario, in which mobile nodes are not perfectly homogeneous and inter-contact times are not necessarily exponentially distributed, we have used GPS-based mobility traces collected by the cabspotting project[4]

[4] `http://cabspotting.org`



(a) (b)

Figure 7: (a) Delivery ratio and (b) expected delivery time of the first twelve messages for inter-contact times following a power-law distribution.

in May 2008 in the San Francisco Bay Area, USA [PSDG09]. In this 30-day experiment, 536 taxi cabs were outfitted with a GPS tracking device and were sending regularly a location-update (timestamp, identifier, geo-coordinates) to a central receiving station.

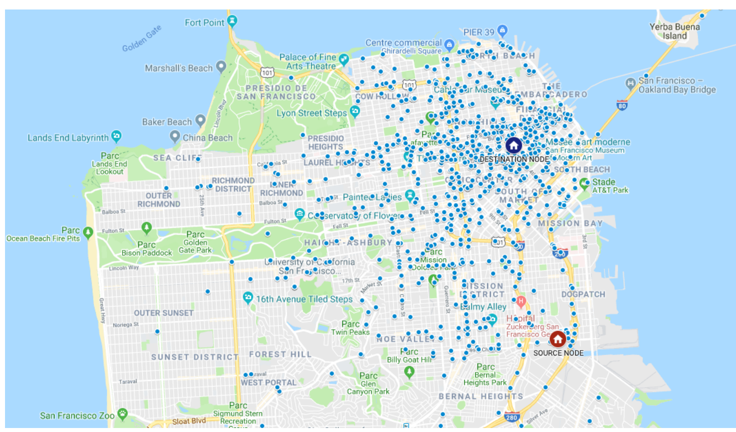


Figure 8: The source node is located at position $37°45'0.72''N$, $122°23'45.6''W$, whereas the destination node is located 4.5 Km away at position $37°47'18.24''N$, $122°24'25.2'W'$. Some positions occupied by cab 47 during the experiment are also shown ($1,000$ out of the $25,000$ points recorded).

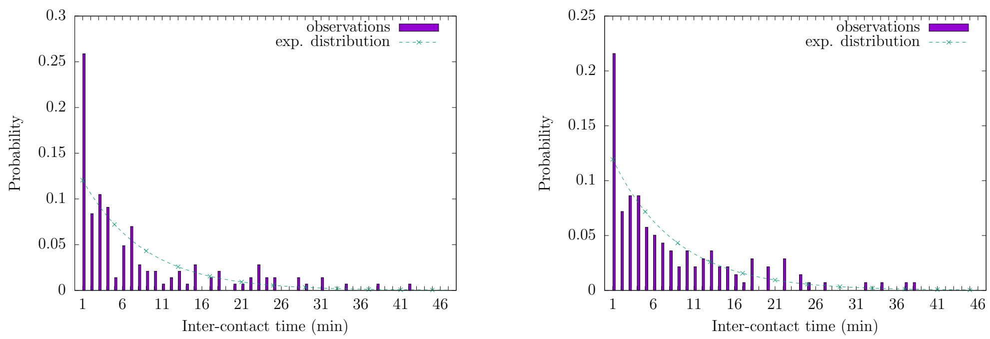
We first chose a source node and a destination node in San Francisco, which are located as shown in Fig. 8. Out of the 218 taxi cabs which are frequently enough in radio range of the source and destination nodes, we identified 4 taxi cabs which have more or less similar mean inter-contact times with the source, as well as with the destination. It was assumed that the transmission range of a taxi is 250m. Fig. 8 shows some of the points visited by one of these taxi cabs, taxi 47.

The total number of contacts of one of the selected taxi cab with the source (resp. destination) node is 93 ± 3 (resp. 143 ± 4). In Fig. 9, we show the probability distribution of

Cab	47	106	117	217
$1/\lambda_i$	359.1	345.3	362.6	347.0
$1/\mu_i$	233.8	234.6	236.5	236.1

Table 1: Mean inter-contact times in minutes with the source and with the destination for the selected taxi cabs.

inter-contact times of taxis 47 and 217 with the destination. It can be seen that these probability distributions are slightly different from each other and that they do not correspond to an exponential distribution. Fig. 10 provides a similar information, but for inter-contact times with the source, and for taxi cabs 106 and 117. The mean inter- contact times between the source or the destination and the taxis are shown in Table 1.



(a) (b)

Figure 9: Probability that the inter-contact time with the destination be greater than $k \times 30$ minutes and strictly lower than $(k+1) \times 30$ minutes as a function of k, for (a) taxi cab 47 and (b) taxi cab 217.

The aggregate mean inter-contact time with the source (resp. destination) is 353.5 minutes (resp. 235.3 minutes), whereas it second order moment is 260657 (resp. 151995). Since inter-contact times with the source (resp. destination) are not exponentially distributed, we choose $\frac{1}{\lambda}$ (resp. $\frac{1}{\mu}$) as the mean residual inter-contact time with the source (resp. destination), that is,

$$\frac{1}{\lambda} = \frac{260657}{2 \times 353.5} = 368.7 \text{ and } \frac{1}{\mu} = \frac{151995}{2 \times 235.3} = 323.0.$$

The values of the thresholds θ_k and γ_k were then computed for $k = 1, \ldots, 12$, using the above values of λ and μ and for 4 relays. It was assumed that $C_s = 0.01$ and that $C_r = C_d = 10$. The thresholds were computed for two different values of the reward R, $R = 2 \times R_{min} = 46.46$ and $R = 12 \times R_{min} = 278.76$. The former value of R leads to an infinite value of γ_k for the first twelve messages, whereas all γ_k are finite for the latter value of R.

The simulations then consist of generating meeting times of taxi cabs with the source and the destination, then each relay deciding whether to accept or not the message and when to drop it depending on the time at which it meets the source and the destination, and then

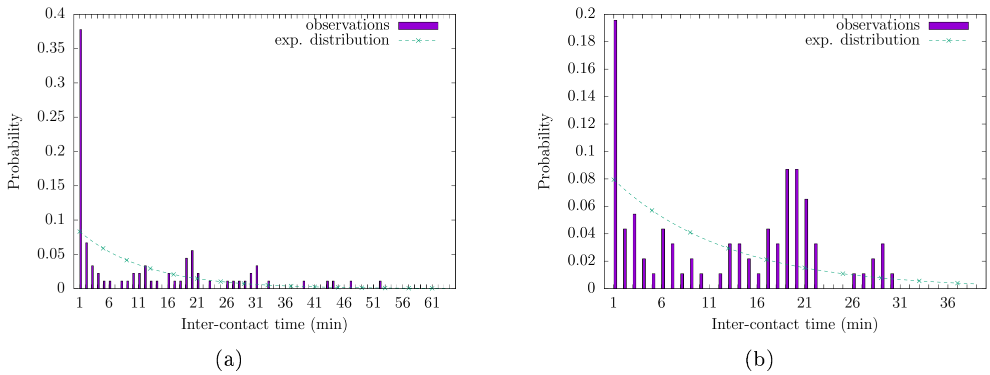


Figure 10: Probability that the inter-contact time with the source node be greater than $k \times 30$ minutes and strictly lower than $(k+1) \times 30$ minutes as a function of k, for (a) taxi cab 106 and (b) taxi cab 117.

determining which relay wins the reward. Note that since there were not enough contact times with the source and the destination, inter-contact times were randomly drawn from the empirical distributions obtained from the mobility traces. The value of the delivery ratio ζ_k and of the expected delivery time D_k were then averaged over $100,000$ sample paths. The simulation was performed twice for each value of R: first using the empirical distributions derived from the dataset, and second using exponential inter-contact distributions with parameters λ and μ.

The results obtained for the delivery probabilities of the first twelve messages are shown in Fig. 11a. For $R = 12 \times R_{min}$, simulation results are in perfect agreement with theoretical values. For $R = 2 \times R_{min}$, we notice that the discrepancy is larger, even though it is always lower than 4.3%. Fig. 11b shows the results obtained for the expected delivery time. Again, the results obtained for $R = 12 \times R_{min}$ are far more accurate than those obtained for $R = 2 \times R_{min}$. However, the maximum error on the expected delivery time is below 5.4 %, which might be deemed reasonable, given that the contact processes of the relays with the source and the destination are heterogeneous and do not follow an exponential distribution.

7 Conclusions and future work

We considered a fixed reward incentive mechanism for a two-hop DTN with a single-destination pair and arbitrary number of competing relays. The source was assumed to be backlogged and proposes messages in a sequential way to the relays it meets. it was shown that the equilibrium policy of the relays for each message is threshold type. That is, a relay accepts the kth message if and only if it meets the source before a given threshold, and once it accepts the message, it keeps this message until a second threshold. A recursive formula for the computation of these thresholds was presented for symmetric equilibria.

Our results were obtained under a number of crucial assumptions. One of our key assumption is that of exponentially distributed inter-contact times. Although satisfied under the Random Waypoint Mobility model, this assumption is not always met in practice and it would be natural to relax it. Although the Bayesian game approach is in principle not

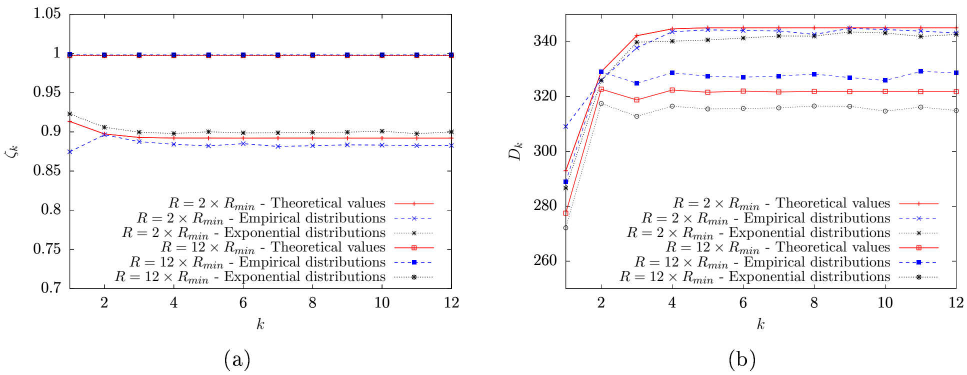


Figure 11: (a) Delivery ratio and (b) expected delivery time of the first twelve messages.

limited to exponential inter-contact time distributions, the analysis becomes then much more complex, in particular for a backlogged source. This avenue is being currently explored.

Another important assumption for some of our results is that the relays have homogeneous contact processes with the source and with the destination. In practice, it often happens that nodes are more or less heterogeneous, with diverse behaviours per each group of nodes. While it was proven that even in a heterogeneous setting all relays use a threshold strategy at a BNE, it can be expected that in this case all equilibria will be asymmetric. As discussed in Section 3, the characterisation of asymmetric equilibria is much more involved than that of symmetric ones.

With memory space becoming cheap for modern devices, another natural generalisation would be to assume that a relay can store more than one message. This extension however gives rise to non-trivial questions. In particular, it is not clear which message a relay should transmit when it meets the destination, assuming that it can give only one. It is not necessarily optimal to transmit the most recent message. Also, the analysis of the probability of success for a relay would be more complicated since it has to take into account which messages other relays transmit when they meet with the destination.

Another possible direction is to introduce a message arrival process at the source, for example the messages could arrive according to a Poisson process.

One of the assumptions required the relays to know the parameters of the game such as inter-contact distributions and the number of players, that is these quantities are common knowledge. In practice, it is possible that these quantities are unknown, and it may be useful to design learning algorithms that converge to the desired equilibrium.

Extending the above models to multiple sources and destination as well as allowing the possibility for the relays to drop a message and pick another one are also part of our future plans.

References

[AHNA08] A. Al-Hanbali, P. Nain, and E. Altman. Performance of ad hoc networks with two-hop relay routing and limited packet lifetime. *Performance Evaluation*, 65(6-7):463–483, June 2008.

[Alt09] Eitan Altman. *Competition and Cooperation between Nodes in Delay Tolerant Networks with Two Hop Routing*, pages 264–278. Springer Berlin Heidelberg, Berlin, Heidelberg, 2009.

[BB02] S. Buchegger and J. L. Boudec. Performance analysis of the confidant protocol: Cooperation of nodes-fairness in distributed ad-hoc networks. In *Proceedings of ACM MobiHoc*, 2002.

[BDFV07] L. Buttyan, L. Dora, M. Felegyhazi, and I. Vajda. Barter-based cooperation in delay-tolerant personal wireless networks. In *IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks*, 2007.

[BDFV10] L. Buttyan, L. Dora, M. Felegyhazi, and I. Vajda. Barter trade improves message delivery in opportunistic networks. *Ad Hoc Networks*, 8(1):1–14, January 2010.

[CC10] B. B. Chen and M. C. Chan. Mobicent: a credit-based incentive system for disruption tolerant network. In *INFOCOM, 2010 Proceedings IEEE*, pages 1–9. IEEE, 2010.

[CE07] H. Cai and D.Y. Eun. Crossing over the bounded domain: From exponential to power-law inter-meeting time in manet. In *Proceedings of ACM/IEEE MOBICOM*, 2007.

[Che12] H. Chen. *Routing Strategies and Cooperation Incentive Mechanisms for Delay Tolerant Networks*. PhD thesis, The Hong Kong Polytechnic University, April 2012.

[CSEA+13] W. Chahin, H. B. Sidi, R. El-Azouzi, F. De Pellegrini, and J. Walrand. Incentive mechanisms based on minority games in heterogeneous DTNs. In *Proceedings of 25th ITC conference*, Sanghai, China, Sept. 10-12 2013.

[CZ10] T. Chen and S. Zhong. INPAC: An enforceable incentive scheme for wireless networks using network coding. In *Proceedings of IEEE INFOCOM*, 2010.

[FT91] D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, 1991.

[GNK04] R. Groenevelt, P. Nain, and G. Koole. Message delay in MANET. In *Proceedings of ACM SIGMETRICS*, 2004.

[HWK04] Q. He, D. Wu, and P. Khosla. SORI: A secure and objective reputation-based incentive scheme for ad hoc networks. In *Proceedings of IEEE WCNC*, 2004.

[LW09] F. Li and J. Wu. FRAME: An innovative incentive scheme in vehicular networks. In *Proceedings of IEEE International Conference on Communications*, 2009.

[LWX+08] X. Y. Li, Y. Wu, P. Xu, G. Chen, and M. Li. Hidden information and actions in multi-hop wireless ad hoc networks. In *Proceedings of ACM MobiHoc*, 2008.

[Mei03] Adam Meirowitz. On the existence of equilibria to bayesian games with non-finite type and action spaces. *Economics Letters*, 78(2):213 – 218, 2003.

[MGLB00] S. Marti, T. J. Giuli, K. Lai, and M. Baker. Mitigating routing misbehavior in mobile ad hoc networks. *In Proc. of MobiCom*, pages 255–265, 2000.

[MS10] M. E. Mahmoud and X. Shen. PIS: A practical incentive system for multi-hop wireless networks. *IEEE Transactions on Vehicular Technology*, pages 4012–4025, 2010.

[NBP16] T. T. H. Nguyen, O. Brun, and B. Prabhu. Performance of a fixed reward incentive scheme for two-hop DTNs with competing relays. *ACM SIGMETRICS Performance Evaluation Review*, 44(3):39–39, December 2016.

[PSDG09] Michal Piorkowski, Natasa Sarafijanovoc-Djukic, and Matthias Grossglauser. A Parsimonious Model of Mobile Partitioned Networks with Clustering. In *The First International Conference on COMmunication Systems and NETworkS (COMSNETS)*, January 2009.

[Put94] Martin L. Puterman. *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. John Wiley & Sons, Inc., New York, NY, USA, 1st edition, 1994.

[SBEP17] T. Seregina, O. Brun, R. Elazouzi, and B. Prabhu. On the design of a reward-based incentive mechanism for delay tolerant networks. *IEEE Transactions on Mobile Computing*, 16(2):453–465, 2017.

[SBP18] T. Seregina, O. Brun, and B.J. Prabhu. A threshold type policy of a dtn node under fixed reward incentive mechanisms. Technical report, 2018.

[Ser15] T. Seregina. *Applications of Game Theory to Distributed Routing and Delay Tolerant Networking*. PhD thesis, INSA de Toulouse, Université de Toulouse, 2015.

[SM04a] G. Sharma and R. Mazumdar. Scaling laws for capacity and delay in wireless ad hoc networks with random mobility. In *Proceedings of IEEE International Conference on Communication (ICC)*, 2004.

[SM04b] G. Sharma and R. R. Mazumdar. Delay and capacity trade-off in wireless ad hoc networks with random mobility. *ACM/Kluwer Journal on Mobile Networks and Applications (MONET)*, 2004.

[SSQZ08] U. Shevade, H.H. Song, L. Qiu, and Y. Zhang. Incentive-aware routing in DTNs. In *IEEE International Conference on Network Protocols (ICNP)*, pages 238–247, 2008.

[UGA10] M. Y. S. Uddinand, B. Godfrey, and T. Abdelzaher. RELICS: In-network realization of incentives to combat selfishness in DTNs. In *Proceedings of IEEE International Conference on Network Protocols (ICNP)*, pages 203–212, 2010.

[VB00] A. Vahdat and D. Becker. Epidemic routing for partially connected ad hoc networks. *Technical Report, Duke University*, 2000.

[WCZ11] L. Wei, Z. Cao, and H. Zhu. MobiGame: A user-centric reputation based incentive protocol for delay/disruption tolerant networks. In *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, pages 1–5, 2011.

[ZCY03] S. Zhong, J. Chen, and Y. R. Yang. Sprite, a simple, cheat-proof, credit-based system for mobile ad-hoc networks. In *Proceedings of INFOCOM 2003*, pages 1987–1997, San Francisco, CA, USA, April 2003.

[ZFX+10] H. Zhu, L. Fu, G. Xue, Y. Zhu, M. Li, and L. M. Ni. Recognizing exponential inter-contact time in vanets. In *2010 Proceedings IEEE INFOCOM*, pages 1–5, March 2010.

[ZLL+05] S. Zhong, L. Li, Y. G. Liu, , and Y. R. Yang. On designing incentive-compatible routing and forwarding protocols in wireless ad-hoc networks - an integrated approach using game theoretical and cryptographic techniques. In *Proceedings of ACM MobiCom.*, 2005.

[ZLL+09] H. Zhu, X. Lin, R. Lu, Y. Fan, and X. Shen. SMART: A secure multilayer credit-based incentive scheme for delay-tolerant networks. *IEEE Transactions on Vehicular Technology*, 58(8):4628–4639, 2009.

[ZXL+11] X. Zhang, X.Wang, A. Liu, Q. Zhang, and C. Tang. Reputation-based schemes for delay tolerant networks. In *Proceedings of 2011 International Conference on Computer Science and Network Technology*, 2011.

A Proof of Theorem 3.1

Define $g^i(t,s) = \mu_i e^{-\mu_i(s-t)}\left(\frac{C_s}{\mu_i} - \overline{R}\, p^i(s)\right)$. Note that $G^i(t,s) = \int_t^s g^i(t,x)dx$. That is, $g^i(t,x)$ is the marginal cost of keeping the message at time x given that it was accepted at time t. The crucial observation is that the sign of $g^i(t,s)$ depends only on s:

$$g^i(t,s) < 0 \iff \frac{C_s}{\mu_i \overline{R}} < p^i(s), \quad \forall t, \forall s \geq t. \tag{26}$$

We use this observation in Lemma A.1 below.

Lemma A.1. *Define $\beta(t) = \inf_{s\geq t} G^i(t,s)$ for all $t \geq 0$, and let γ^i be defined as in Theorem 3.1. Then: (a) $\beta(t) \geq 0$ for all $t \geq \gamma^i$ and $\beta(t) < 0$ for all $t < \gamma^i$, (b) $\beta(t) = G^i(t,\gamma^i)$ for all $t < \gamma^i$, and (c) $\beta(t)$ is strictly increasing in t in the interval $[0,\gamma^i]$.*

Proof. We first prove assertion (a). By definition of γ^i, $p^i(y) \leq \frac{C_s}{\mu_i \overline{R}}$ for all $y \geq \gamma^i$. According to (26), it yields $g^i(t,y) \geq 0$ for all t and y such that $y \geq t$ and $y \geq \gamma^i$. Hence

$$G^i(t,s) = \int_t^s g^i(t,y)\,dy \geq 0, \quad \forall s \geq t, \forall t \geq \gamma^i,$$

implying that $\beta(t) = \inf_{s\geq t} G^i(t,s) \geq 0$ for all $t \geq \gamma^i$. Similarly, for $y < \gamma^i$, we have $p^i(y) > \frac{C_s}{\mu_i \overline{R}}$. With (26), it implies that $g^i(t,y) < 0$ for all t and y such that $t \leq y < \gamma^i$. Hence

$$\beta(t) = \inf_{s\geq t} G^i(t,s) \leq G^i_k(t,\gamma^i) = \int_t^{\gamma^i} g^i(t,y)\,dy < 0.$$

We thus conclude that $\beta(t) \geq 0$ for all $t \geq \gamma^i$ and $\beta(t) = G^i(t,\gamma^i) < 0$ for all $t < \gamma^i$, as claimed.

We now prove assertion (b). We know that $g^i(t,y) < 0$ for all t and y such that $t \leq y < \gamma^i$. This implies that for $t < \gamma^i$ fixed, $G^i(t,y)$ is a strictly decreasing function of y on the interval $[t,\gamma^i]$, so that $G^i(t,y) > G^i(t,\gamma^i)$. Moreover, $g^i(t,y) \geq 0$ for all t and y such that $y \geq t$ and $y \geq \gamma^i$. This implies that, for $t < \gamma^i$ fixed, $G^i(t,y)$ is a non-decreasing function of y on the interval $[\gamma^i,\infty)$, so that $G^i(t,y) \geq G^i(t,\gamma^i)$. As a consequence, $\beta(t) = \inf_{s\geq t} G^i(t,s) = G^i(t,\gamma^i)$ for all $t < \gamma^i$.

Finally, in order to prove assertion (c), we note that from assertion (b), we have $\phi(t) = G^i(t,\gamma_k^i)$ for all $t < \gamma^i$, so that $\beta'(t) = -g^i(t,t)$. Since $g^i(t,y) < 0$ for all t and y such that $t \leq y < \gamma^i$, we have $g^i(t,t) < 0$ for all $t < \gamma^i$, and thus $\beta'(t) > 0$ for all $t < \gamma^i$. □

The proof of Theorem 3.1 now readily follows from Lemma A.1.

Proof of Theorem 3.1. As proven in Lemma A.1, γ^i is the time until which it is optimal to keep a message conditioned on the fact that the relay decides to accept it. The decision of a relay of type t can then be summarized as a choice between two options, $s = 0$ (reject the message) or accept and keep it until γ^i. That is, $\pi^*(t;\boldsymbol{\pi}_{-i}) = \arg\min\big(0, C_r + G_i(t,\gamma^i))\big)$. The best-response of type t, will therefore be to accept the message if and only if $C_r + G_i(t,\gamma^i) < 0$. Again, from Lemma A.1, $\beta(t) = G_i(t,\gamma^i)$ is increasing in t. Thus, there exists a θ^i (possible infinity) such that it is optimal to accept if $t \leq \theta^i$ and reject if $t > \theta^i$. If the message is accepted, then it is optimal to keep until γ^i. □

B Proofs of results in Section 4 and Section 5

B.1 Proof of Lemma 4.1

We shall first establish some properties of the function $V_k(x,y)$ in Lemma B.1, before proving Lemma 4.1.

Lemma B.1. *For y fixed, the function $V_k(x,y)$ is strictly decreasing in x in the interval $[0,y)$, and for $x > 0$ fixed, it is strictly decreasing in y in the interval $[0,\infty]$.*

Proof. The proof directly follows from

$$\begin{aligned}
\frac{\partial V_k}{\partial x}(x,y) &= \int_{\theta_{k-1}}^{x} \phi_k(s)\frac{\partial v_{x,y}}{\partial x}(s)\,ds,\\
&= \lambda\, e^{-\lambda x}\left(e^{-\mu(y-x)}-1\right)\int_{\theta_{k-1}}^{x}\phi_k(s)e^{\lambda s}\,ds,
\end{aligned}$$

which is negative for all $y > x$, and

$$\begin{aligned}
\frac{\partial V_k}{\partial y}(x,y) &= \int_{\theta_{k-1}}^{x} \phi_k(s)\frac{\partial v_{x,y}}{\partial y}(s)\,ds,\\
&= -\frac{\lambda\mu e^{-\mu y}}{\mu-\lambda}\int_{\theta_{k-1}}^{x}\phi_k(s)e^{\lambda s}\left(e^{(\mu-\lambda)x}-e^{(\mu-\lambda)s}\right)ds,
\end{aligned}$$

which is also negative for all $y, x \geq 0$. □

These properties of the function $V_k(x,y)$ are now used to prove Lemma 4.1.

Proof of Lemma 4.1. Since according to Lemma B.1 the continuous function $V_k(x,y)$ is strictly decreasing in y for x fixed, the equation $V_k(\theta,\gamma)=\omega$ has a solution $\gamma \geq \theta$ if and only if

$$\lim_{y\to\infty} V_k(\theta,y) < \omega \leq V_k(\theta,\theta).$$

With (9) and (10), the LHS inequality directly leads to $\theta > \theta_{min}$, whereas the RHS one yields $\theta \leq \theta_{max}$. Hence, for θ fixed, the equation $V_k(\theta,\gamma)=\omega$ has a solution $\theta \leq \gamma < \infty$ if and only if $\theta \in (\theta_{min},\theta_{max}]$.

To show that $\gamma(\theta)$ is decreasing on $(\theta_{min},\theta_{max}]$, note that

$$\frac{d}{dx}V_k(x,\gamma(x)) = \frac{\partial}{\partial x}V_k(x,\gamma(x)) + \gamma'(x)\frac{\partial}{\partial y}V_k(x,\gamma(x)).$$

On the interval $(\theta_{min},\theta_{max}]$, the function $V_k\left(\theta,\gamma(\theta)\right)$ is a constant, and its derivative is thus 0. From Lemma B.1, both the partial derivatives of V_k are negative, from which we conclude that derivative of $\gamma(\theta)$ is strictly negative. $\square$

B.2 Proof of Theorem 4.1

Define the function

$$\hat{G}_k(\theta) = G_k(\theta,\gamma(\theta)). \tag{27}$$

The value of θ at an equilibrium is determined by a solution of $\hat{G}_k(\theta) = -C_r$. Thus, the number of equilibria will depend upon the number of roots of the equation $\hat{G}_k + C_r = 0$ on the positive real line.

The next result gives some properties of $\hat{G}_k$ that are then sufficient to conclude the unicity of the symmetric equilibrium.

Lemma B.2. *On the interval* $[0,\theta_{max}]$*, the function* $\hat{G}_k$ *is*

(a) continuous;

(b) strictly increasing; with

(c)

$$\hat{G}_k(0) = \frac{C_s}{\mu} - \overline{R}, \tag{28}$$

$$\hat{G}_k(\theta_{max}) = 0. \tag{29}$$

Proof. **(a)** The continuity of $\hat{G}_k$ on the open interval $[0,\theta_{min}) \cup (\theta_{min},\theta_{max}]$ follows from the definition of G_k. In order to show the continuity of $\hat{G}_k$ it is thus sufficient to show that

$$\lim_{\theta\to\theta_{min}^-} \hat{G}_k(\theta) = \lim_{\theta\to\theta_{min}^+} \hat{G}_k(\theta).$$

In order to prove this, observe that we can write

$$\hat{G}_k(\theta) = \int_{\theta}^{\gamma(\theta)} g_k(\theta,t)dt, \tag{30}$$

where $g_k(\theta,t) = \mu e^{-\mu(t-\theta)}(\frac{C_s}{\mu} - \overline{R}\, V_k(\theta,t)^{N-1})$ is the marginal cost of keeping a message at time t given that it was accepted at time θ. Since $V_k(\theta,t) \in [0,1]$, we have $|g_k(\theta,t)| \leq m_1 \mu e^{-\mu(t-\theta)}$ for all θ and t, where $m_1 = \max\left(\overline{R} - \frac{C_s}{\mu}, \frac{C_s}{\mu}\right)$. It yields

$$\begin{aligned}\left|\int_{\theta_{min}}^{\infty} g_k(\theta,t)dt - \hat{G}_k(\theta)\right| &\leq \int_{\theta_{min}}^{\theta} |g_k(\theta,t)|\, dt + \int_{\gamma(\theta)}^{\infty} |g_k(\theta,t)| dt \\ &\leq m_1 \left\{ e^{-\mu(\theta_{min}-\theta)} - 1 + e^{-\mu(\gamma(\theta)-\theta)} \right\},\end{aligned}$$

from which we conclude that

$$\begin{aligned}\lim_{\theta \to \theta_{min}^{+}} \hat{G}_k(\theta) &= \lim_{\theta \to \theta_{min}^{+}} \int_{\theta_{min}}^{\infty} g_k(\theta,t)dt, \\ &= \int_{\theta_{min}}^{\infty} g_k(\theta_{min},t)dt.\end{aligned}$$

where the last equality is obtained using the dominated convergence theorem. Similar arguments can be used to establish that $\hat{G}_k(\theta)$ converges to the same limit when $\theta \to \theta_{min}^{-}$.
(b) We have

$$\begin{aligned}\frac{dG_k}{d\theta}(\theta,\gamma(\theta)) &= g_k(\theta,\gamma(\theta))\gamma'(\theta) - g_k(\theta,\theta) + \int_{\theta}^{\gamma(\theta)} \frac{\partial g_k}{\partial \theta}(\theta,x)dx \qquad (31) \\ &= g_k(\theta,\gamma(\theta)) - g_k(\theta,\theta) + \int_{\theta}^{\gamma(\theta)} \frac{\partial g_k}{\partial \theta}(\theta,x)dx \qquad (32) \\ &= \int_{\theta}^{\gamma(\theta)} \left(\frac{\partial g_k}{\partial x}(\theta,x) + \frac{\partial g_k}{\partial \theta}(\theta,x) \right) dx,\end{aligned}$$

where (32) is obtained from (31) by observing that, for $\theta > \theta_{min}$, $v(\theta,\gamma(\theta)) = \omega$ implies that $g_k(\theta,\gamma(\theta)) = 0$, whereas for $\theta \leq \theta_{min}$, $\gamma(\theta) = \infty$ also implies $g_k(\theta,\gamma(\theta)) = 0$. Since

$$\frac{\partial g_k}{\partial x}(\theta,x) + \frac{\partial g_k}{\partial \theta}(\theta,x) = -\overline{R}(N-1)\mu e^{-\mu(x-\theta)} V_k(\theta,x)^{N-2} \times \left(\frac{\partial V_k}{\partial x}(\theta,x) + \frac{\partial V_k}{\partial \theta}(\theta,x) \right),$$

we conclude from Lemma B.1 that $G_k(\theta,\gamma(\theta))$ is strictly increasing in θ.
(c) Equality (28) follows from noting that $V_k(0,x) = 1$, and using this in (30). Similarly, (29) is obtained by noting that $\gamma(\theta_{max}) = \theta_{max}$ (from Lemma 4.1), and using this in (30). □

An immediate consequence of Lemma B.2 is stated in Corolary B.1.

Corolary B.1. *There is a unique solution to $\hat{G}_k(\theta) = -C_r$ in the interval $[0, \theta_{max}]$ if and only if*

$$C_r + \frac{C_s}{\mu} \leq \overline{R}$$

We are now in position to prove Theorem 4.1.

Proof of Theorem 4.1. From Lemma 4.1 there is a unique γ for a given $\theta > 0$ that satifies (6). Also, from Corollary B.1 there is unique $\theta > 0$ that satisfies (7) if and only if $C_r + \frac{C_s}{\mu} \leq \overline{R}$. Thus, this last inequality is necessary and sufficient for the existence of a unique symmetric equilibrium.

From Lemma B.2 and Lemma 4.1, we deduce that, for γ to be finite the necessary and sufficient condition is

$$\hat{G}_k(\theta_{min}) < -C_r.$$

From (11),

$$\begin{aligned} V_k(\theta_{min}, x + \theta_{min}) &= 1 + \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left\{ \left(e^{-\lambda(\theta_{min}-s)} - 1 \right) + \right. \\ &\quad \left. \frac{e^{-\mu x}}{\sigma} \left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)} \right) \right\} ds \\ &= \omega \left(1 + e^{-\mu x} b \right), \end{aligned}$$

where

$$b = \frac{1}{\sigma\omega} \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)} \right) ds.$$

Then,

$$\begin{aligned} \hat{G}_k(\theta_{min}) &= \frac{C_s}{\mu} - \overline{R} \int_0^\infty \mu e^{-\mu x} v(\theta_{min}, x + \theta_{min})^{N-1} dx \\ &= \frac{C_s}{\mu} - \overline{R} \frac{\omega^{N-1}}{Nb} \left((1+b)^N - 1 \right). \end{aligned}$$

where the last equality follows from the binomial formula.

Thus, $\hat{G}_k(\theta_{min}) < -C_r$ if and only if

$$\frac{C_s}{\mu} - \overline{R} \frac{\omega^{N-1}}{Nb} \left((1+b)^N - 1 \right) < -C_r,$$

which, since $\omega^{N-1} = C_s/(\mu\overline{R})$, is equivalent to

$$1 + \mu \frac{C_r}{C_s} < \frac{(1+b)^N - 1}{Nb},$$

as claimed. ∎

B.3 Proof of Lemma 5.1

Proof of Lemma 5.1. Let us first consider the probability that the relay be ready for competing for the delivery of the $(k+1)^{th}$ message at time θ_k. This is only possible if it was ready for competing for the k^{th} message at some time $x \in [\theta_{k-1}, \theta_k]$, and has not the message at time θ_k. As a consequence

$$\Phi_{k+1}(\theta_k) = \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) \left[1 - I_k(x, \theta_k) \right] dx = h_1(\theta_k).$$

Consider now the probability that T_{k+1} be in the interval $(\theta_k, t]$ for some $t < \gamma_k$. This can occur if the relay was ready for competing for the k^{th} message at some time $x \in [\theta_{k-1}, \theta_k]$, took this message from the source at some time $s \in [x, \theta_k]$ and deliver it to the destination in $y \in (\theta_k - s, t - s]$ units of time. Another possibility is that the relay comes into play for the delivery of the k^{th} message after θ_k but before t, in which case it will be proposed directly the $(k+1)^{th}$ message. As a consequence

$$\Phi_{k+1}(t) - \Phi_{k+1}(\theta_k) = \int_{\theta_k}^{t} \phi_k(x)dx + \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) \left(I_k(x,\theta_k) - I_k(x,t)\right) dx$$

which upon derivation with respect to t yields

$$\begin{aligned} \phi_{k+1}(t) &= \phi_k(t) + \mu \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) I_k(x,t) dx \\ &= \phi_k(t) + \mu e^{-\mu t} h_2(\theta_k) \end{aligned}$$

Finally, the only possibility for the relay to come into play at time γ_k is that it was ready for competing for the k^{th} message at some time $x \in [\theta_{k-1}, \theta_k]$, took the message from the source but was not able to meet the destination by γ_k. Therefore

$$\mathbb{P}\left(T_{k+1} = \gamma_k\right) = \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) I_k(x,\gamma_k) dx = e^{-\mu \gamma_k} h_2(\theta_k),$$

$\square$

C An MDP approach for the DTN game

In this section, we shall give an alternative approach based on Markov Decision Processes (MDP) that can also be used to arrive at threshold-type equilibrium.

Let $\mathcal{S} = \{0, m_s, 1, m_d, 2\}$ be the set of possible states for relay i. If, at some decision epoch, relay i is in state $x_i \in \mathcal{S}$, it may choose action a from the set of allowable actions in that state, $\mathcal{A}(x_i)$. The interpretation of states as well as the actions available in each state are summarized in Table 2. When message k is proposed for the first time by the source, it may happen that relay i still has a previous message. In this case, the relay i is not competing for message k until it either drops the previous message or meets the destination. When this happens, relay i enters state 0 and now has to calculate its optimal policy.

In the following, we shall denote by $x_i(t)$ the state of relay i at time t, and by $\mathbf{x}_{-i}(t)$ the state of the other relays. We shall refer to $\mathbf{x}(t) = (x_i(t), \mathbf{x}_{-i}(t))$ as the state of the system at time t. We emphasize that relay i does not know the state of the other relays at time 0.

The main difficulty in modelling the decision problem faced by relay i is that some actions (namely, rejecting or dropping the message) lead to an immediate change of state, or, in other words, correspond to an infinite transition rate. To circumvent this difficulty, we shall temporarily assume that when the relay makes such a decision, it stays an exponentially distributed amount of time of mean $\frac{1}{M}$ in the same state, where M is some large constant. Under this assumption, it turns out that the optimal decision-making problem of relay i can be cast as an MDP, as we now explain.

Table 2: State, action sets and costs for a relay for message k.

State	Significance	Action set	Cost
0	relay is competing for message k	$\emptyset$	0
m_s	relay is in contact with the source	$\{accept,\ reject\}$	$C_r\,\mathbb{1}_{\{a=accept\}}$
1	relay has the packet	$\{drop,\ keep\}$	$C_s\,\mathbb{1}_{\{a=keep\}}$
m_d	relay is in contact with the destination	$\emptyset$	$(C_d - R)\,\mathbb{1}_{\{x_j \neq 2, \forall j \neq i\}}$
2	relay quits the game	$\emptyset$	0

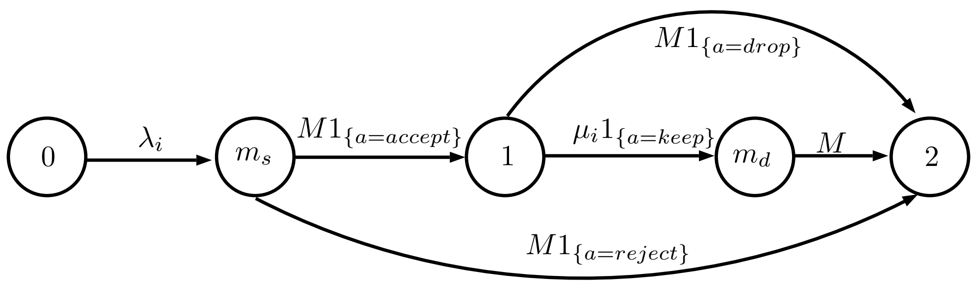


Figure 12: Controlled Markov Chain for relay i.

It is clear that the stochastic process $x_i(t)$ corresponds to a controlled continuous-time Markov chain, as shown in Figure 12. The cost incurred by the relay depends on its current state, on the action it takes, as well as on the state of the other relays. In the following, $g(x_i, a, \mathbf{x}_{-i})$ denotes the cost incurred by relay i if it takes action a when the system is in state $\mathbf{x}$. The possible values of the costs are shown in the last column of Table 2.

We define a control law (or policy) as a function $\pi : \mathbb{R} \times \mathcal{S} \to \mathcal{A}$ such that $\pi(t, x) \in \mathcal{A}(x)$ for all $x \in \mathcal{S}$. Given the policies $\boldsymbol{\pi}_{-i} = (\pi_1, \ldots, \pi_{i-1}, \pi_{i+1}, \ldots, \pi_N)$ of the other relays, the goal of relay i is to minimize the expected cost

$$J_i(\tau, 0; \pi, \boldsymbol{\pi}_{-i}) = \mathbb{E}\left\{ \int_{\tau}^{\infty} g(x_i(t), \pi(s, x_i(t), \mathbf{x}_{-i}(t))\, dt \right\}, \tag{33}$$

over all policies π. In the above equation, $J_i(s, x; \pi, \boldsymbol{\pi}_{-i})$ represents the expected cost-to-go for relay i under policy π if it is in state x at time s, and τ is the first time relay i enters state 0. The cost for relay i depends upon the states and the policies of the other relays only through p_k^i (see (1)).

Let

$$J_i^*(t, x; \boldsymbol{\pi}_{-i}) = \lim_{M \to \infty} \inf_{\pi} J_i(t, x; \pi, \boldsymbol{\pi}_{-i}),$$

be the optimal cost-to-go for the tagged relay if it is in state $x \in \mathcal{S}$ at time t when $M \to \infty$.

Proposition C.1. *When $M \to \infty$, the optimality equations read as follows*

$$J_i^*(t, 1; \boldsymbol{\pi}_{-i}) = \min\left(0, \inf_{s \geq t} G_k^i(t,s)\right), \tag{34}$$

$$J_i^*(t, m_s; \boldsymbol{\pi}_{-i}) = \min\left(0, C_r + \inf_{s \geq t} G_k^i(t,s)\right), \tag{35}$$

where $G_k^i(t,s)$ is defined in (1).

From now on, we shall only consider the limiting regime $M \to \infty$. In words, Proposition C.1 says that if relay i has the message at time t, its best-response is to keep it if and only if there exists $s \geq t$ such that the expected cost $G_k^i(t,s)$ of keeping the message in the interval (t,s) is negative. Similarly, if relay i meets the source at time t, its best-response is to accept the message if and only if there exists $s \geq t$ such that the expected reward $-G_k^i(t,s)$ offsets the cost of receiving the message from the source. Using the optimality equations stated in Proposition C.1, we can obtain the same structure for the best-response policy as in Theorem 3.1.

C.1 Proof of Proposition C.1

We consider a given relay (say relay i) and establish the optimality equations of problem (33) for this relay in the limiting regime $M \to \infty$. The proof proceeds in two steps: (a) assuming M is large but fixed, we first use the well-know uniformization technique [Put94] to establish the optimality equations for an equivalent discrete-time MDP, and (b) we then establish the limits of these optimality equations when $M \to \infty$.

To simplify notations, let $q = \frac{\lambda_i}{M}$, $p = \frac{\mu_i}{M}$, $\bar{p} = 1 - p$ and $\bar{q} = 1 - q$. Denoting by $\mathbf{Q}$ the infinitesimal generator of the controlled CTMC shown in Figure 12, the equivalent discrete-time MDP has transition matrix $\mathbf{P}(a) = \mathbf{I} + \frac{1}{M}\mathbf{Q}(a)$ under action a, that is,

$$\mathbf{P}(a) = \begin{array}{c} \\ 0 \\ m_s \\ 1 \\ m_d \\ 2 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & m_s & 1 & m_d & 2 \end{array} \\ \begin{pmatrix} \bar{q} & q & 0 & 0 & 0 \\ 0 & 0 & \mathbb{1}_{\{a=accept\}} & 0 & \mathbb{1}_{\{a=reject\}} \\ 0 & 0 & \bar{p}\,\mathbb{1}_{\{a=keep\}} & p\,\mathbb{1}_{\{a=keep\}} & \mathbb{1}_{\{a=drop\}} \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array},$$

and costs-per-stage

$$\tilde{g}(x, a, \mathbf{x}_{-i}) = \frac{1}{M} g(x, a, \mathbf{x}_{-i}), \quad \forall a \in \mathcal{A}(x), \forall x \in \mathcal{S}.$$

Let $V_n(x)$ be the optimal cost-to-go of relay i starting in state $x \in \mathcal{S}$ at time n, and let $q(n)$ denotes the probability that the relay be the first one to deliver the message to the destination at that time. Note that the latter probability depends on the policies $\boldsymbol{\pi}_{-i}$ of the other relays, although we do not make explicit this dependence. Lemma C.1 establishes the optimality equations for the states $x = m_s$ and $x = 1$.

Lemma C.1. *Provided that $R \geq C_r + C_d + \frac{C_s}{\mu_i}$, the optimal costs-to-go are given by*

$$V_n(m_s) = \min\left(0, C_r + V_{n+1}(1)\right), \tag{36}$$

$$V_n(1) = \min\left(0, U_{n,1}, U_{n,2}, \ldots\right), \tag{37}$$

where

$$U_{n,m} = \frac{\mu_i}{M} \sum_{k=0}^{m-1} (\bar{p})^k \left\{ \frac{C_s}{\mu_i} - \overline{R}\, q(n+k+1) \right\}. \tag{38}$$

Proof. Since $V_n(m_d) = (C_d - R)\, q(n)$, the dynamic programming equation

$$V_n(x) = \min_{a \in \mathcal{A}(x)} \left\{ \tilde{g}(x_n, a, \mathbf{x}_{-n}) + \sum_{y \in \mathcal{S}} p_{x,y}(a) V_{n+1}(y) \right\} \tag{39}$$

yields

$$\begin{aligned} V_n(1) &= \min\left(0, \frac{C_s}{M} - p\overline{R}q(n+1) + \bar{p}V_{n+1}(1)\right) \\ &= \min\Bigg(0, \frac{C_s}{M} - p\overline{R}q(n+1), \frac{C_s}{M}(1+\bar{p}) - \\ &\qquad p\overline{R}\sum_{k=0}^{1}(\bar{p})^k q(n+k+1) + (\bar{p})^2 V_{n+2}(1))\Bigg), \end{aligned}$$

which can be developed recursively to obtain

$$V_n(1) = \min\left(0, U_{n,1}, U_{n,2}, \ldots\right).$$

The optimal cost-to-go in state m_s is obtained directly from the dynamic programming equation (39). ☐

We note that the term $U_{n,m}$ in Lemma C.1 corresponds to the cost obtained if the action "keep" is played m consecutive times starting from the current decision epoch n, until the relay meets the destination or decides to drop the message. The optimal policy at instant n is to retain the message if either of the $U_{n,m}$ is negative. Otherwise it is optimal to drop the message.

We now turn to the second part of the proof, which is based on Lemma C.2.

Lemma C.2. *Let $s, t \in \mathbb{R}$, $s > t \geq 0$. We have*

$$\lim_{M \to \infty} U_{\lfloor Mt \rfloor, \lfloor M(s-t) \rfloor} = G_k^i(t, s). \tag{40}$$

Proof. To simplify notation, let $n = \lfloor Mt \rfloor$ and $m = \lfloor M(s-t) \rfloor$. The term $U_{n,m}$ can be rewritten as follows

$$\begin{aligned} U_{n,m} &= \frac{C_s}{\mu_i}\left(1 - \bar{p}^m\right) - \frac{\mu_i}{M}\overline{R}\sum_{k=0}^{m-1}(\bar{p})^k q(n+k+1), \\ &= \frac{C_s}{\mu_i}\left(1 - \left(1 - \frac{\mu_i}{M}\right)^m\right) - \frac{\mu_i}{M}\overline{R}\sum_{k=0}^{m-1}\left(1 - \frac{\mu_i}{M}\right)^k q(n+k+1), \end{aligned} \tag{41}$$

Since $m = \lfloor M(s-t) \rfloor$, for the first term on the LHS, we have

$$\lim_{M\to\infty} \frac{C_s}{\mu_i}\left(1-\left(1-\frac{\mu_i}{M}\right)^m\right) = \frac{C_s}{\mu_i}\left(1-e^{-\mu_i(t-s)}\right).$$

Besides, since the discrete-time Markov chain corresponds to the original continuous-time Markov chain observed at random times according to a Poisson process with intensity $M\,t$, we can identify $q(n+k+1)$ with $p_k^i(t+\frac{k+1}{M})$, so that the second term on the LHS of (41) can be rewritten as follows

$$\frac{\mu_i}{M}\overline{R}\sum_{k=0}^{m-1}\left(1-\frac{\mu_i}{M}\right)^k p_k^i(t+\frac{k+1}{M}).$$

Approximating $\left(1-\frac{\mu_i}{M}\right)^k$ by $e^{-\frac{k\mu_i}{M}}$, it yields

$$e^{\frac{\mu_i}{M}}\sum_{k=1}^{m}\mu_i\overline{R}e^{-\frac{k\mu_i}{M}}p_k^i(t+\frac{k}{M})\frac{1}{M},$$

which can be rewritten as

$$e^{\frac{\mu_i}{M}}\sum_{k=1}^{m} f\left(t+\frac{k}{M}\right)(x_k - x_{k-1}),$$

where $f(x) = \mu_i\overline{R}e^{-\mu_i(x-t)}p_k^i(x)$ and $x_k = t+\frac{k}{M}$ for $k=0,\ldots,m$. When $M\to\infty$, the term $e^{\frac{\mu_i}{M}} \to 1$, whereas the Riemann sum

$$\sum_{k=1}^{m} f\left(t+\frac{k}{M}\right)(x_k - x_{k-1}) \xrightarrow[M\to\infty]{} \int_t^s \mu_i\overline{R}e^{-\mu_i(x-t)}p_k^i(x)dx.$$

In view of (1), summing the limits of the first and second terms on the LHS of (41) concludes the proof. □

We are now in position to prove Proposition C.1.

Proof of Proposition C.1. The proof directly follows from Lemmata C.1 and C.2 since

$$\begin{aligned} J_i^*(t,1;\boldsymbol{\pi}_{-i}) &= \lim_{M\to\infty} V_{\lfloor Mt \rfloor}(1) \\ &= \min\left(0, \min_{k\geq 1}\lim_{M\to\infty} U_{\lfloor Mt \rfloor,k}\right) \\ &= \min\left(0, \inf_s G_k^i(t,s)\right), \end{aligned}$$

and the result on $J_i^*(t,m_s;\boldsymbol{\pi}_{-i})$ is obtained similarly. □