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1 Original Article

2 **Modeling as a Froude and Strouhal Dimensionless Numbers**
3 **Combination for Dynamic Similarity in Running**

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25 **Modela-r as a Froude and Strouhal Dimensionless Numbers Combination**
26 **for Dynamic Similarity in Running**

27

28 **Abstract**

29 The aim of this study was to test the hypothesis that running at fixed fractions of
30 Froude (Nfr) and Strouhal (Str) dimensionless numbers combinations induce dynamic
31 similarity between humans of different sizes. Nineteen subjects ran in three experimental
32 conditions, i) constant speed, ii) similar speed (Nfr) and iii) similar speed and similar step
33 frequency (Nfr and Str combination). In addition to anthropometric data, temporal, kinematic
34 and kinetic parameters were assessed at each stage to measure dynamic similarity informed
35 by dimensional scale factors and by the decrease of dimensionless mechanical parameter
36 variability. Over a total of 54 dynamic parameters, dynamic similarity from scale factors was
37 met for 16 (mean $r = 0.51$), 32 (mean $r = 0.49$) and 52 (mean $r = 0.60$) parameters in the first,
38 the second and the third experimental conditions, respectively. The variability of the
39 dimensionless preceding parameters was lower in the third condition than in the others. This
40 study shows that the combination of Nfr and Str, computed from the dimensionless energy
41 ratio at the center of gravity (Modela-r) ensures dynamic similarity between different-sized
42 subjects. The relevance of using similar experimental conditions to compare mechanical
43 dimensionless parameters is also proved and will highlight the study of running techniques,
44 or equipment, and will allow the identification of abnormal and pathogenic running patterns.
45 Modela-r may be adapted to study other abilities requiring bounces in human or animal
46 locomotion or to conduct investigations in comparative biomechanics.

47 **Keywords:** Spring Mass Model; Dimensionless Parameters; Center of Mass; Similar Speed;
48 Similar Frequency

49 **1. Introduction**

50 Originally used in the fluid mechanics field, the concept of dynamic similarity enables
51 two different-sized systems to be considered as scaled models by setting them in equivalent
52 experimental conditions. It suggests that when two systems are dynamically similar, one
53 could be identical to the other by multiplying (i) all lengths (L dimension) by one scale factor
54 C_L , (ii) all masses (M dimension) by another scale factor C_M , and (iii) all times (T dimension)
55 by a third scale factor C_T . Furthermore, scale factor for all other mechanical parameters
56 depending on the three preceding dimensions, such as speed, force, and impulse, can be
57 computed from C_L , C_M , and C_T . The concept was originally applied in fluid mechanics, and
58 more recently in biology, ecology, and biomechanics considering that, if isometric, a small
59 subject is a scaled model of a tall one. This concept has also been applied to compare
60 locomotion between different species (Alexander, 1989; Minetti et al., 1994; Vaughan and
61 Blaszczyk, 2008) and to study similarities between human of different sizes during walking
62 and running (Moretto et al., 2007; Delattre and Moretto, 2008; Delattre et al., 2009).

63 A Spring Mass Model (SMM, Fig. 1) is commonly used to compare locomotion
64 between animals and humans as it takes into account an elastic component and modelizes the
65 rebound occurring during jumping and running (Alexander, 1989). It consists in a body mass
66 represented at the Centre of Mass (CoM) oscillating at the end of a massless spring. This
67 model is commonly used to represent the CoM mechanical behavior of human running
68 (Blickhan, 1989; McMahon and Cheng, 1990). Its kinematic depends on seven physical
69 variables: gravity (g), mass (m), stiffness (k), initial spring length (l_0), initial spring angle
70 (θ_0), initial landing velocity (v_0), and the angle of the initial landing velocity (β_0).

71 An approach to compare similar locomotion and to ensure dynamic similarity between
72 specimens is based on the dimensionless approach focusing on locomotion models like
73 SMM. Part of this approach rests in the π theorem stated by Buckingham (Buckingham,

1914). It reduces the number of variables by considering dimensionless numbers computed from the characteristic variables of a specific problem. This theorem states that a physical equation using N_P physical variables, that are dependant of N_D base dimensions, necessitates $N_P - N_D$ dimensionless numbers (π) to describe the mechanical behavior of a system. Applying the π theorem to the SMM, the seven aforementioned physical variables ($N_P = 7$) are dependent on three base dimensions ($N_D = 3$), L (m), M (kg), and T (s). Thus, four dimensionless numbers are necessary to completely describe the movement of both systems. These four dimensionless numbers given by the theorem come from the seven physical variables as presented in table 1. Each of them can be expressed in terms of Nfr or Str. Consequently, the four dimensionless numbers are Str, Nfr, β_0 , and θ_0 (Tab 1). Nfr (v_0^2 / gl_0) is the Froude number representing the dimensionless speed and Str (fl_0 / v_0) is the Strouhal number corresponding to the dimensionless oscillatory frequency, i.e. the dimensionless form of the step frequency f ($f = \sqrt{k/m}$). The SMM modelizes the behavior of the CoM. To be in accordance with the fundamental physic principle, the Nfr and Str computation should take the position of the CoM into account rather than the leg length. This is why “l” refers to CoM height.

Nfr and Str dimensionless numbers have been used to determine experimental running conditions. Delattre et al. (Delattre et al., 2009) showed that neither Nfr nor Str were sufficient to characterize running mechanics or to establish inter-subject dynamic similarities, but each leads its own contributions. Indeed, Nfr contributes to observe similarities of antero-posterior kinetic events while Str contributes to the temporal organization. Very recently, a link has been highlighted between Nfr and Str during running (Villegier et al., 2012). According to Alexander (Alexander, 1989), these authors suggested a concomitant use of these dimensionless numbers for running gait. To this end, the Modelar dimensionless number has been developed from mechanical simulation of SMM (Delattre and Moretto,

99 2008). It is equal to the combination of Nfr and Str, which equals the ratio of Kinetic (E_K) and
100 Potential (E_P) Energies over Elastic Energy (E_E) with $E_K = 0.5mv^2$ (m the mass, g the gravity,
101 and v the speed), $E_P = mgh$ (h the CoM height) and $E_E = 0.5k\Delta l^2$ (k the stiffness and Δl the
102 variation of spring length)(Eq. 1). The ratio $(E_K+E_P)/E_E$ would be theoretically constant for
103 a SMM and would correspond to a witness of the energy transfer at the CoM. As mentioned
104 by Wannop et al. (2012), Modela-r has never been experimentally validated.

105

$$106 \quad \text{Modela-}r = \frac{E_K+E_P}{E_E} = \frac{1}{Str^2} \left(\frac{2}{Nfr} + 1 \right) \quad (\text{Eq. 1})$$

107

108 Inspired by these recent works, our study aims to ensure dynamic similarity to
109 different-sized subjects using a combination of Nfr and Str for running through the
110 introduction of Modela-r as a dimensionless number issued from the energy transfer at the
111 CoM.

112

113 **2. Methods**

114 **2.1. Population**

115 Nineteen subjects (n = 19) took part in this study after signing an informed consent
116 document. Their characteristics were (mean \pm sd [min; max]): age 23 ± 5 [18; 36] years,
117 height 1.79 ± 0.07 [1.68; 1.94] m, and mass 80.7 ± 11 [64; 102.9] kg. They were chosen so as
118 that the tallest was the heaviest, and vice versa. The experimentation was approved by the
119 ethical committee of the University of Toulouse.

120

121 **2.2. Experimental conditions**

122 **2.2.1. General procedure**

123 For 3-dimensional analysis, 42 reflective markers were fixed on subject bone

124 landmarks (Wu et al., 2002, 2005). Participants performed running tests barefoot with speed
 125 and/or step frequency determined from Nfr and Str. Experimentation was conducted on a
 126 treadmill (PF 500 CX, PRO FORM, Villepreux, FRANCE) mounted on a large forceplate
 127 sampled at 1 kHz (AMTI, Watertown, MA, USA). The positions of reflective marker were
 128 recorded by twelve optoelectronic cameras sampled at 200 Hz (VICON, Oxford's metrics,
 129 Oxford, UK). After a familiarization period, the subjects had to perform three trials per
 130 running test (Hamill and Mcniven, 1990) that were repeated in different experimental
 131 settings. The CoM height (l_i) was determined from the i^{th} subject's anatomic position
 132 ($i \in [1, n]$) with the anthropometric model of De Leva (de Leva, 1996). The center of rotation
 133 of the hip was determined using the SCoRE method (Ehrig et al., 2006).

134

135 **2.2.2. Experimental steps**

136 The experimentation was separated into the three steps detailed below and in fig. 2.

137

138 *EC_{SPEED}*

139 The subjects performed six stages of running with speeds set at 1.67, 2.22, 2.78, 3.33, 3.89,
 140 and 4.44 $\text{m}\cdot\text{s}^{-1}$ (Eq. 2). These six speed stages were indexed as $k \in [1, 6]$. The first
 141 experimental condition consisted in setting the same constant speed for all the subjects. At
 142 speed stage k :

$$143 \quad v_{ik} = 1.111 + 0.556 \times k = v_k \quad (\text{Eq. 2})$$

144

145 *EC_{NFR}*

146 The second experimentation time consisted of imposing six stages of running with similar
 147 velocities. A mean $\overline{\text{Nfr}}_k$ was computed from EC_{SPEED} for each stage of speed (Eq. 3). Then,
 148 similar velocities at speed k were determined from $\overline{\text{Nfr}}_k$ (Eq. 4) for each subject.

149 $\overline{Nfr_k} = (1/n) \sum_{i=1}^n Nfr_{ik} = (1/n) \sum_{i=1}^n \frac{v_k^2}{gl_i}$ (Eq. 3)

150 $vsim_{ik} = \sqrt{Nfr_k gl_i}$ (Eq. 4)

151

152 EC_{MOD}

153 The third experimentation time consisted of imposing six stages of running with similar

154 velocities (Eq. 4) and similar frequencies. A mean $\overline{Str_k}$ was computed from EC_{NFR} for each

155 stage of speed (Eq. 5). Then, similar frequencies at speed k for each subject were determined

156 from $\overline{Str_k}$ (Eq. 6)

157 $\overline{Str_k} = (1/n) \sum_{i=1}^n Str_{ik} = \frac{1}{n} \sum_{i=1}^n \frac{f_{ik} l_i}{vsim_{ik}}$ (Eq. 5)

158 $f_{sim_{ik}} = \overline{Str_k} \frac{vsim_{ik}}{l_i}$ (Eq. 6)

159

160 **2.3. Parameters assessed**

161 4th order zero lag Butterworth filters were applied to kinematic and kinetic data with a

162 cut off frequency set at 6 Hz and 10 Hz respectively (Goldberg and Stanhope, 2013). Then, 5

163 consecutive cycles were averaged at each stage of speed.

164 The ground reaction forces (GRF) were measured by a large force platform under the

165 treadmill. A threshold of 10 N was used to detect the contact phase in running. The kinetic

166 parameters suggested by Delattre et al. (Delattre et al., 2009) to study the GRF similarities

167 during running were adapted. Indeed, eight parameters were studied aiming at reader

168 comprehension of the results (Fig. 3). The different parameters are detailed in Fig. 3 legend.

169 The flexion extension angles at the ankle, the knee, and the hip were also considered

170 and expressed in radian to respect the international unity system and a dimensionless form. In

171 order to compare angle variability, the averaged cycle was normalized to 100 points wherein

172 each corresponded to a percentage of the cycle.

173 The mass (m), the CoM height (l), and the CoM oscillation frequency (f), were
174 considered to compute the dimensionless values of the kinetic parameters and to normalize
175 them with respect to the basic dimensions [M , L , and T^{-1}] (Table 2). A “D” has been added as
176 an exponent of the parameter acronym to differentiate the dimensionless value from the real
177 one. Thus, the relative stride length, the relative contact time (duty factor), and the relative
178 peak of force were noted as SL^D , TC^D , and VPF^D for running, respectively.

179

180 **2.4. Analysis to consider similarity**

181 The similarity analysis was a two step procedure. The first step was based on the
182 correlation between the scale factors predicted from basis scale factors and measures. The
183 second step was to verify the decrease of variance of the dimensionless parameters.
184 Experimental setups that enable concomitantly the increase in the scale factors correlation
185 and the decrease in the variability will be considered as successful means to induce dynamic
186 similarity between different subjects.

187 A scale factor was a ratio of a mechanical parameter of one subject to another. With
188 19 subjects, 171 scale factors were built for each parameter. Basis scale factors (C_L , C_M and
189 C_T) were derived from the three basis dimensions of any system (length, mass and time,
190 respectively). C_L was calculated by subject height ratios, predicted C_M was computed from
191 $C_M = C_L^3$ because the subjects had theoretically the same density index, and predicted C_T
192 depended on the experimental conditions. Predicted scale factors were developed from the
193 basis scale factors (Table 1) and represented how the individuals' parameters should be
194 related if the conditions of dynamic similarity were met. Measured scale factors were those
195 developed from the measurements of the mechanical parameters. For instance, the predicted
196 scale factor between two subjects S_i and S_j for the braking peak was
197 $C_{BPF} = C_{FORCE} = C_M C_L C_T^{-2}$ whereas the measured scale factor was $C_{BPF} = BPF_i / BPF_j$ with

198 BPF the measured values. When for a given parameter all predicted scale factors equaled all
 199 measured scale factors, it could be stated that the parameter was similar or proportional from
 200 one subject to another. We reiterate that C_L and $C_M (= CL^3)$ were given by anthropometry;
 201 however, C_T was dependent on experimental conditions and is presented thereafter.

202

203 *EC_{SPEED}*

204 At constant speed k , the speed scale factor (table 2) between subjects (i and j) was
 205 $C_{SPEED} = v_{ik}/v_{jk} = C_L C_T^{-1} = 1$, thus $C_T = C_L$ with $j \in [1, n]$ and $i \neq j$.

206

207 *EC_{NFR}*

208 The speed scale factor between two similar velocities ($C_L C_T^{-1}$) was equal to $C_L^{0.5}$ (Eq. 7) that
 209 induced a $C_T = C_L^{0.5}$ time scale factor.

$$210 \frac{Vsim_{ik}}{Vsim_{jk}} = \frac{\sqrt{Nfr_k g l_i}}{\sqrt{Nfr_k g l_j}} = \sqrt{\frac{l_i}{l_j}} = C_L^{0.5}, \text{ thus } C_T = C_L^{0.5} \quad (\text{Eq. 7})$$

211

212 *EC_{MOD}*

213 The frequency scale factor between two similar frequencies (C_T^{-1}) was equal to $C_L^{-0.5}$ (Eq. 8)
 214 that induced the time scale factor of $C_T = C_L^{0.5}$.

$$215 \frac{f_{sim_{ik}}}{f_{sim_{jk}}} = \frac{\overline{Str_k} Vsim_{ik}/l_i}{\overline{Str_k} Vsim_{jk}/l_j} = \frac{Vsim_{ik} l_j}{Vsim_{jk} l_i} = C_L^{0.5} C_L^{-1} = C_L^{-0.5}, \text{ thus } C_T = C_L^{0.5} \quad (\text{Eq. 8})$$

216

217 It should be noted that the decrease of variance of dimensionless parameters signifies
 218 a more similar behavior (Pierrynowski and Galea, 2001).

219

220 **2.5. Statistical analysis**

221 All statistical analyses were performed with the STATISTICA software (STATISTICA

222 V6, Statsoft, Maison-Alfort, FRANCE). For all statistical tests, normality was checked using
223 the Kolmogorov-Smirnov test. For normal distributions, parametric tests were performed
224 other than non parametric tests were used.

225 Statistical analysis performed on kinetic parameter scale factors was divided into two
226 steps. First, a Spearman coefficient was computed between predicted and measured scale
227 factors under each experimental condition. Only significant correlations ($p < 0.05$) were taken
228 into account. Then, Wilcoxon paired tests were conducted to identify if there were significant
229 difference between the predicted and the measured scale factors. If the Spearman correlation
230 coefficients were significant and the Wilcoxon test did not reveal significant difference
231 between predicted and measured scale factors for a kinetic parameter, then the parameter was
232 considered as similar from one subject to another. In addition to the kinetic parameters, the
233 same tests were repeated on mass (C_M) and on step time (C_T).

234 3 repeated factors ANOVA (EC_{SPEED} , EC_{NFR} , and EC_{MOD}) was conducted for ankle,
235 knee, and hip angles at each stage of speed ($p < 0.05$) to detect the significant effect of the
236 experimental conditions on the inter-subject variance. A Tukey post hoc comparison enabled
237 a refinement of the analysis.

238 The homogeneity of variance of the dimensionless gait parameters SL^D , TC^D , $TPPF^D$,
239 VPF^D , BPF^D , VI^D , BI^D , PI^D , and LR^D between the three experimental conditions was tested
240 with a Levene test ($p < 0.05$). Then, the Fisher and Snedecor F-test ($p < 0.05$) was performed as
241 a post hoc test to highlight which variance was significantly different from the others. It was
242 repeated for the six speed stages.

243

244 **3. Results**

245 For kinetic parameter scale factors, two criteria were taken into account to determine
246 if one experimental condition produced more dynamic similarities than the others: first, the

247 numbers of parameters for which the measured and predicted scale factors were correlated
248 and non-statistically different from each other; then, the mean of the correlation value for
249 these parameters. The dynamic similarity results are presented below and in Fig. 4. They
250 were met for 16, 32, and 52 parameters out-of 54 dynamic parameters in EC_{SPEED}, EC_{NFR}, and
251 EC_{MOD}, respectively. No similarities were found on C_T (step time) in EC_{SPEED} and EC_{NFR}. The
252 mean coefficients of correlation for all parameters were 0.51, 0.49, and 0.60 in EC_{SPEED},
253 EC_{NFR}, and EC_{MOD}.

254 The variances of ankle, knee, and hip angles are presented in table 3. The lowest
255 variability of angles of knee and hip was met in EC_{MOD} for all speeds. In EC_{MOD}, the
256 variability of ankle angles was the highest at the 2.22 m.s⁻¹ stage whereas it was the lowest at
257 the last three speed stages. Moreover, EC_{NFR} generated more variability of ankle angles than
258 the other conditions at the two last stages.

259 Referring to table 3, EC_{NFR} allowed a reduction of the variability of a total of 13
260 dimensionless parameters compared to EC_{SPEED}. The variability of 64 dimensionless
261 parameters was decreased in EC_{MOD} compared to EC_{SPEED}. EC_{MOD} enabled a reduction of the
262 variability of 52 dimensionless parameters compared to EC_{NFR}.

263

264 **4. Discussion**

265 This study aimed to ensure dynamic similarity between different-sized subjects using
266 a new dimensionless number, Modela-r. As a combination of Nfr and Str, Nmodela-r accounts
267 for the energy transfer at the CoM during running.

268 The increase of correlations between predicted and measured mechanical scale factors
269 associated with the decrease of the dimensionless parameter variability highlights the interest
270 of the association of Nfr and Str to induce dynamic similarity. In our study, EC_{MOD} leads to
271 more dynamic similarity than the other conditions at each stage of speed. Thus, in order of

272 importance, EC_{MOD} and EC_{NFR} lead to more similar gait parameters than EC_{SPEED} . Our results
273 are in line with those of Delattre et al. (Delattre et al., 2009) and Alexander (Alexander, 1989)
274 who suggested using a combination of Nfr and Str dimensionless numbers to obtain
275 similarities on running patterns. Moreover, we have shown that Nfr alone brings similarities
276 and its combination with Str leads to further similarities. As defined in 2.4. (EC_{NFR} and
277 EC_{MOD}), the time constraint generated a theoretical relationship between C_L and C_T as $C_T =$
278 $C_L^{0.5}$. Thus, the correlation between measured and predicted scale factors of time was higher
279 (0.94) in EC_{MOD} . However, the C_T dynamic similarity was met only in EC_{MOD} with a
280 correlation of 0.94. Thus, in EC_{NFR} the spontaneous frequency was not proportional (different
281 from $C_L^{-0.5}$) in our study. This is in accordance with Delattre et al. (Delattre et al., 2009).
282 Indeed, they reported correlations of -0.27 and 0.99 between predicted and measured scale
283 factors of stride frequency (or stride time) in experimental conditions which respected the
284 same Nfr and Str, respectively. A non-proportional spontaneous frequency in EC_{NFR} could be
285 an explanation of the effect of the additional use of Str on dynamic similarity in EC_{MOD} .

286 Based on robust physic theory as π theorem, four dimensionless numbers (Nfr, Str, β_0 ,
287 and θ_0) are necessary to describe the behavior of the SMM which modelizes the CoM
288 movement in running gaits. Our model enabled the computation of Nfr and Str at the CoM
289 and the determination of similar speeds and similar step frequencies from the CoM height and
290 the CoM oscillation frequency. In this study, only two of the four dimensionless numbers are
291 necessary to describe the movement of the spring mass models. As Bullimore and Donelan
292 (Bullimore and Donelan, 2008) have shown with two unconstrained simulations of SMM,
293 two dimensionless numbers are not sufficient to ensure dynamic similarities. Indeed, from the
294 same values of two dimensionless numbers they have simulated different SL^D (~ 2.96 and
295 ~ 5.52), TC^D (~ 0.31 and ~ 0.2), and VPF^D (~ 2.4 and ~ 4). Referring to our data, the variability
296 of the dimensionless parameters (VPF^D , TC^D , and SL^D) from the same dimensionless

297 numbers (Nfr and Str in EC_{MOD}) was very low. This discrepancy suggests that human
298 locomotion in our case cannot be summarized as unconstrained simulations. Indeed, the
299 organization of the movement suggests that for an association of Nfr and Str, a constrained
300 behavior corresponds. This can be an explanation of the lower variability of our measured
301 dimensionless parameter in EC_{MOD} . Moreover, the variability of SL^D was close to 0 in
302 EC_{MOD} . SL^D is the inverse of Str (Alexander, 1989) and explains its zero variability in EC_{MOD}
303 wherein Str is taken into account.

304 The locomotion model used in this study is constrained by the gravity and an elastic
305 phenomenon. The gravity constraint is taken into account in Nfr and the elastic phenomenon
306 is strongly dependent on the general stiffness (k), which is introduced in Str. The elastic
307 phenomenon (Cavagna et al., 1964) during running is taken into account in Modela-r
308 (Delattre and Moretto, 2008). Modela-r is a witness of the energy transfer that occurs at the
309 CoM and can be expressed as a combination of Nfr and Str (Eq. 1). Thus, subjects, who move
310 at the same Modela-r number, move similarly. More precisely, the use of Modela-r as a
311 combination of Nfr and Str allows the researcher to generate similar experimental conditions
312 that constrain energy transfer occurring at the CoM. Moreover, its development being based
313 on the SMM behavior, Modela-r could be applied to the whole of locomotion characterized
314 like SMM. Thus, Modela-r should be useful in comparative biomechanics between species
315 (Alexander, 1989; Farley et al., 1993; Srinivasan and Holmes, 2008) and could be a means to
316 construct a dimensionless database of running.

317 Many studies compare mechanical parameters between different populations that are
318 not homogeneous among themselves (ex. A small specimen versus a tall one), especially
319 normalizing the parameters by individual anthropometrical characteristics (i.e. height and
320 mass). Besides population characteristics like heights and masses, many studies compare
321 normalized mechanical parameters under dissimilar conditions. It means they compare

322 parameters relative to individual characteristics under experimental conditions which
323 themselves are not relative to individual characteristics. Indeed, small and tall subjects
324 running at the same speed is not comparable, this is like comparing children and adults
325 running at the same speed. In these conditions, Modela-r allows scientists to put different-
326 sized specimens in similar conditions which makes the comparison of dimensionless
327 parameters relevant. Indeed, if two specimens move similarly they would have the same
328 dimensionless mechanical parameters. Then, the identification of unequal parameters could
329 highlight abnormal running, such as expertise, lack of practice, long-distance training or
330 fatigue.

331 Furthermore, a part of the inter-individual variability under similar experimental
332 conditions is a matter of biological system variability. Indeed, two mechanical systems have
333 to move similarly in similar conditions, or else the differences between both should come
334 from the part of biological variability of the bio-mechanics field. Hence, similar conditions,
335 such as EC_{MOD} , allows one to study and identify the role of significant subjects like gender
336 (Ferber et al., 2003), stiffness (Blum et al., 2009), prostheses (Hobara et al., 2013), and ability
337 of elastic energy storage/recoiling in running more accurately.

338 Finally, the movement of the CoM in running can be characterized like a SMM.
339 Hence, the concomitant use of Nfr and Str ensures dynamic similarities between different-
340 sized subjects. Constraining locomotion by Str and Nfr allows researchers to constrain energy
341 transfer occurring at the CoM (Modela-r), and thus, estimate the elastic energy origin and its
342 function more accurately. So, this study highlights the importance of using similar
343 experimental conditions by removing the individual anthropometrical characteristics effect to
344 compare mechanical parameters and to more accurately study serious topics in running.
345 Modela-r has been experimentally validated and shows its usefulness in i) establishing similar
346 experimental conditions and ii) constraining the energy transfer at the CoM. Further studies

347 that involve application of SMM to locomotion patterns like bouncing, trotting, and running
348 in animals would enlightened the interest of Modela-r in comparative biomechanics.

349

350

351 **Conflict of interest statement**

352 None.

353

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355 None.

356 **References**

- 357 Alexander, R., 1989. Optimization and Gaits in the Locomotion of Vertebrates. *Physiol. Rev.*
 358 69, 1199–1227.
- 359 Blickhan, R., 1989. The Spring Mass Model for Running and Hopping. *J. Biomech.* 22,
 360 1217–1227.
- 361 Blum, Y., Lipfert, S.W., Seyfarth, A., 2009. Effective leg stiffness in running. *Journal of*
 362 *Biomechanics* 42, 2400–2405.
- 363 Buckingham, E., 1914. On physically similar systems. Illustration of the use of dimensional
 364 equations. *Physical Review* 4, 345–376.
- 365 Bullimore, S.R., Donelan, J.M., 2008. Criteria for dynamic similarity in bouncing gaits. *J.*
 366 *Theor. Biol.* 250, 339–348.
- 367 Cavagna, G.A., Saibene, F.P., Margaria, R., 1964. Mechanical Work in Running. *Journal of*
 368 *applied physiology* 19, 249–56.
- 369 de Leva, P., 1996. Adjustments to Zatsiorsky-Seluyanov’s segment inertia parameters. *Journal*
 370 *of Biomechanics* 29, 1223–1230.
- 371 Delattre, N., Lafortune, M.A., Moretto, P., 2009. Dynamic similarity during human running:
 372 About Froude and Strouhal dimensionless numbers. *J. Biomech.* 42, 312–318.
- 373 Delattre, N., Moretto, P., 2008. A new dimensionless number highlighted from mechanical
 374 energy exchange during running. *J. Biomech.* 41, 2895–2898.
- 375 Ehrig, R.M., Taylor, W.R., Duda, G.N., Heller, M.O., 2006. A survey of formal methods for
 376 determining the centre of rotation of ball joints. *J. Biomech.* 39, 2798–2809.
- 377 Farley, C., Glasheen, J., McMahon, T., 1993. Running Springs - Speed and Animal Size. *J.*
 378 *Exp. Biol.* 185, 71–86.
- 379 Ferber, R., McClay Davis, I., Williams III, D.S., 2003. Gender differences in lower extremity
 380 mechanics during running. *Clinical Biomechanics* 18, 350–357.
- 381 Goldberg, S.R., Stanhope, S.J., 2013. Sensitivity of joint moments to changes in walking
 382 speed and body-weight-support are interdependent and vary across joints. *J Biomech*
 383 46, 1176–1183.
- 384 Hamill, J., Mcniven, S., 1990. Reliability of Selected Ground Reaction Force Parameters
 385 During Walking. *Hum. Mov. Sci.* 9, 117–131.
- 386 Hobara, H., Baum, B.S., Kwon, H.-J., Miller, R.H., Ogata, T., Kim, Y.H., Shim, J.K., 2013.
 387 Amputee locomotion: Spring-like leg behavior and stiffness regulation using running-
 388 specific prostheses. *Journal of Biomechanics* 46, 2483–2489.
- 389 McMahon, T., Cheng, G., 1990. The Mechanics of Running - How Does Stiffness Couple
 390 with Speed. *Journal of Biomechanics* 23, 65–78.
- 391 Minetti, A.E., Saibene, F., Ardigo, L.P., Atchou, G., Schena, F., Ferretti, G., 1994. Pygmy
 392 locomotion. *European journal of applied physiology and occupational physiology* 68,
 393 285–290.
- 394 Moretto, P., Bisiaux, M., Lafortune, M.A., 2007. Froude number fractions to increase walking
 395 pattern dynamic similarities: Application to plantar pressure study in healthy subjects.
 396 *Gait Posture* 25, 40–48.
- 397 Pierrynowski, M.R., Galea, V., 2001. Enhancing the ability of gait analyses to differentiate
 398 between groups: scaling gait data to body size. *Gait & Posture* 13, 193–201.
- 399 Srinivasan, M., Holmes, P., 2008. How well can spring-mass-like telescoping leg models fit
 400 multi-pedal sagittal-plane locomotion data? *Journal of Theoretical Biology* 255, 1–7.
- 401 Vaughan, C.L., Blaszczyk, M.B., 2008. Dynamic similarity predicts gait parameters for
 402 *Homo floresiensis* and the *Laetoli* hominins. *American Journal of Human Biology* 20,
 403 312–316.
- 404 Villegger, D., Delattre, N., Watier, B., Moretto, P., 2012. Froude and Strouhal dimensionless

405 numbers to study human gait: an experimental approach. *Computer methods in*
406 *biomechanics and biomedical engineering* 15 Suppl 1.
407 Wannop, J.W., Worobets, J.T., Stefanyshyn, D.J., 2012. Normalization of Ground Reaction
408 Forces, Joint Moments, and Free Moments in Human Locomotion. *J. Appl. Biomech.*
409 28, 665–676.
410 Wu, G., Siegler, S., Allard, P., Kirtley, C., Leardini, A., Rosenbaum, D., Whittle, M., D’Lima,
411 D.D., Cristofolini, L., Witte, H., Schmid, O., Stokes, H., 2002. ISB recommendation
412 on definitions of joint coordinate system of various joints for the reporting of human
413 joint motion - part 1: ankle, hip, and spine. *Journal of Biomechanics* 35, 543–548.
414 Wu, G., van der Helm, F.C.T., Veeger, H.E.J., Makhsous, M., Van Roy, P., Anglin, C., Nagels,
415 J., Karduna, A.R., McQuade, K., Wang, X.G., Werner, F.W., Buchholz, B., 2005. ISB
416 recommendation on definitions of joint coordinate systems of various joints for the
417 reporting of human joint motion - Part II: shoulder, elbow, wrist and hand. *Journal of*
418 *Biomechanics* 38, 981–992.
419

Table 1

Dimensionless numbers useful for the behavior description of the SMM determined by π theorem. The equation $f(l_0, m, v_0, k, g, \beta_0, \theta_0) = 0$ can be reduced to $\varphi(\pi_1, \pi_2, \pi_3, \pi_4) = 0$.

Dimensionless numbers (π)	Equation	Equivalent to
π_1	$l_0^2 k / m V_0^2$	Str^2
π_2	$g l_0 / V_0^2$	Nfr^{-1}
π_3	β_0	
π_4	θ_0	

With l_0 the initial spring length; k the spring stiffness; m the mass; v_0 the initial landing speed; g the gravitational acceleration; β_0 the angle of the initial landing speed; and θ_0 the initial spring angle.

420

421

Table 2

Units, dimensions and predicted scale factors of kinetic parameters

Parameters	Units (SI)	Dimension s	Predicted scale factors	Dimensionless parameters
CoM height (l)	m	L	C_L	
Body mass (m)	kg	M	C_M	
Speed (v)	$m.s^{-1}$	LT^{-1}	$C_L C_T^{-1}$	Nfr
CoM oscillation frequency (f)	s^{-1}	T^{-1}	C_T^{-1}	Str
Time (TC and TPPF)	s	T	C_T	Time ^D = Time × f
Force (VPF and BPF)	N	MLT^{-2}	$C_M C_L C_T^{-2}$	Force ^D = Force / (mlf ²)
Impulse (VI, BI and PI)	N.s	MLT^{-1}	$C_M C_L C_T^{-1}$	Impulse ^D = Impulse / (mgf)
Rate (LR)	$N.s^{-1}$	MLT^{-3}	$C_M C_L C_T^{-3}$	Rate ^D = Rate / (mlf ³)
Length (SL)	m	L	C_L	Length ^D = Length / l
Angle (Ankle, Knee and Hip)	Rad			Angle

C_L and C_M were defined by the subject's anthropometry whereas C_T was determined by the experimental conditions.

422

423

Table 3

Standard deviation of dimensionless gait parameters at each speed stage

Speed stage	EC	Ankle angle (x 10 ³)	Knee angle (x 10 ³)	Hip angle (x 10 ³)	SL ^D	TC ^D	TPPF ^D	VPF ^D	BPF ^D	VI ^D	BI ^D	PI ^D	LR ^D
1.67 m.s ⁻¹	EC _{SPEED}	2.6	2.6	2	0.10	0.10	0.08	0.46	0.03	0.11	0.01	0.01	2.53
	EC _{NFR}	2.6	2.6	2	0.09	0.07	0.08	0.41	0.03	0.15	0.01	0.01	1.65
	EC _{MOD}	2.6	2.4*#	1.8*#	0.00*#	0.05*	0.05	0.36	0.02*	0.08#	0*#	0.01	1.20*
2.22 m.s ⁻¹	EC _{SPEED}	2.8	2.9	2.2	0.13	0.08	0.06	0.40	0.03	0.10	0.01	0.01	2.47
	EC _{NFR}	2.6*	2.6*	2.2	0.12	0.07	0.05	0.36	0.03	0.14	0.01	0.01	1.59
	EC _{MOD}	3*#	2.3*#	1.6*#	0.00*#	0.05*	0.04	0.23*	0.02	0.05*#	0.01*#	0.01*#	1.32*
2.78 m.s ⁻¹	EC _{SPEED}	3	3.4	2.6	0.15	0.08	0.06	0.44	0.05	0.09	0.01	0.01	2.57
	EC _{NFR}	3	3.3	2.4*	0.14	0.07	0.05	0.38	0.04	0.12	0.01	0.01	1.90
	EC _{MOD}	2.9	2.8*#	2*#	0.00*#	0.04*#	0.03*	0.19*#	0.03*#	0.05*#	0.01*#	0.01*#	1.04*#
3.33 m.s ⁻¹	EC _{SPEED}	3.3	3.7	2.7	0.15	0.07	0.05	0.35	0.04	0.08	0.01	0.01	2.13
	EC _{NFR}	3.2	3.7	2.7	0.15	0.07	0.05	0.37	0.05	0.12	0.01	0.01	1.88
	EC _{MOD}	2.8*#	2.9*#	2.2*#	0.00*#	0.03*#	0.03*#	0.16*#	0.03*#	0.04*#	0.01*	0.01*	0.98*#
3.89 m.s ⁻¹	EC _{SPEED}	3.2	3.7	2.6	0.20	0.08	0.06	0.41	0.05	0.09	0.01	0.01	2.41
	EC _{NFR}	3.7*	3.8*	2.8*	0.16	0.05	0.04*	0.33	0.05	0.11	0.01	0.01	1.49*
	EC _{MOD}	2.9*#	3.2*#	2.1*#	0.00*#	0.03*	0.03*	0.15*#	0.03*#	0.04*#	0.01*#	0.01*#	0.96*
4.44 m.s ⁻¹	EC _{SPEED}	4	4.1	2.9	0.30	0.09	0.06	0.48	0.07	0.10	0.01	0.01	2.61
	EC _{NFR}	4.9*	4	2.7*	0.18*	0.05*	0.04	0.31	0.05	0.09	0.01	0.01	1.49*
	EC _{MOD}	3.2*#	3.3*#	2.5*#	0.00*#	0.03*#	0.02*#	0.12*#	0.02*#	0.03*#	0.01*#	0.01*#	0.72*#

The characteristic dimensions to express the gait parameters in a dimensionless form (^D) are: the mass ([M]), the CoM height ([L]) and the step frequency ([T⁻¹]).

*, #: variability significantly different from EC_{SPEED} and from EC_{NFR}. The significant lowest values of standard deviation are bolded.

424

425

426 **Figure legends**

427

428 Figure 1. The Spring Mass Model (SMM)

429

430 Figure 2. Relationship between speed, CoM oscillation frequency and CoM height under the
431 three experimental conditions for each stage of speed.

432

433 Figure 3. (A) Running vertical reaction force (F_z) over time. 1: Time of Contact (TC);
434 2: Vertical Peak Force (VPF); 3: Loading Rate from 10% to 90% of vertical peak force (LR);
435 4: Vertical Impulse (VI). (B) Running antero-posterior reaction force (F_y) over time.
436 1: Braking Peak Force (BPF); 2: Time to Propulsive Peak Force (TPPF); 3: Braking Impulse
437 (BI); 4: Propulsion Impulse (PI).

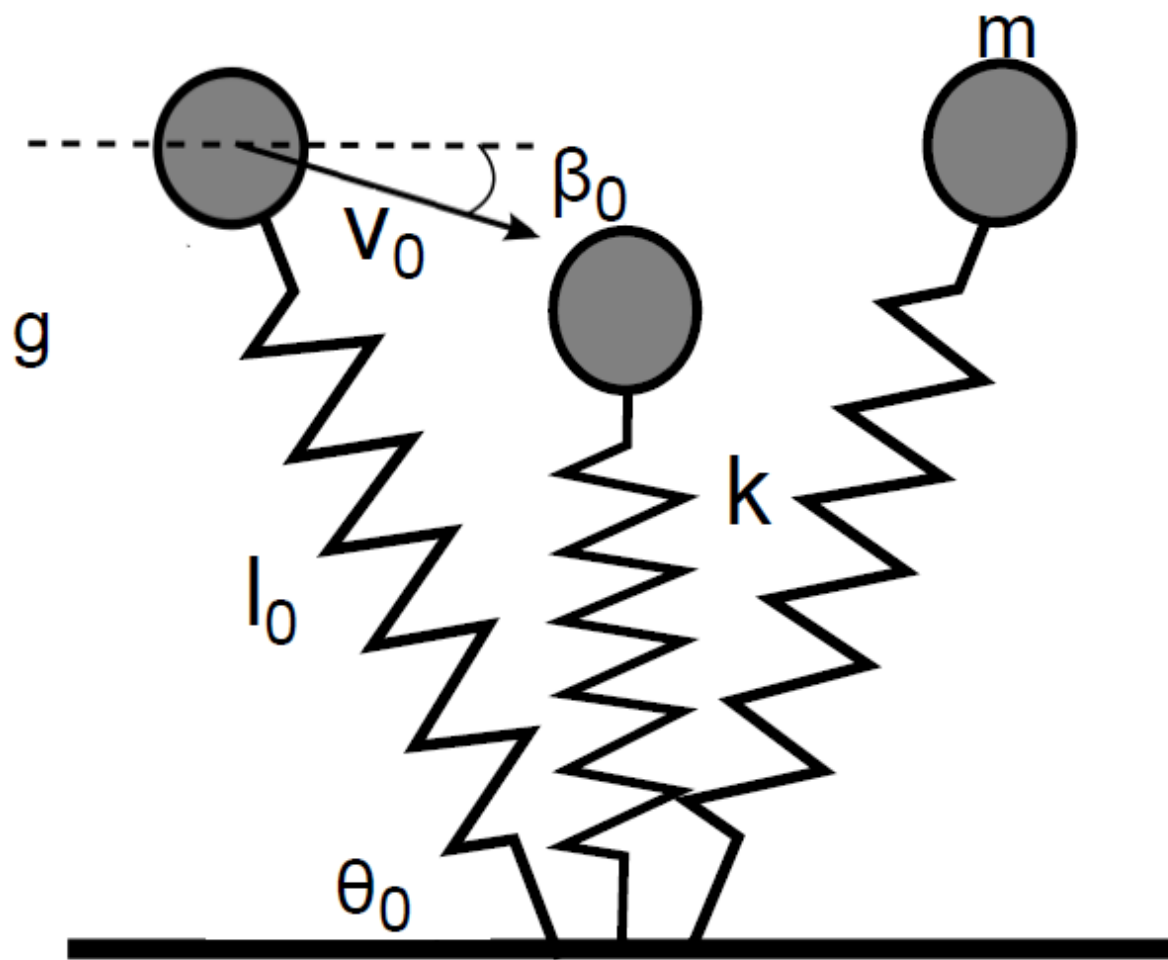
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439 Figure 4. Correlations between predicted and measured scale factors of body mass (C_M), step
440 time (C_T) and kinetic parameters (TC, time of contact; TPPF, time to propulsive peak force;
441 VPF, vertical peak force; BPF, braking peak force; VI, vertical impulse; BI, braking impulse;
442 PI, propulsive impulse; and LR, loading rate). The scale factor correlations whose the
443 Wilcoxon test revealed a difference between predicted and measured scale factors was set to
444 0. Lightest grey, dark grey and black bars represent respectively dynamic similarity for
445 EC_{SPEED} , EC_{NFR} and EC_{MOD} .

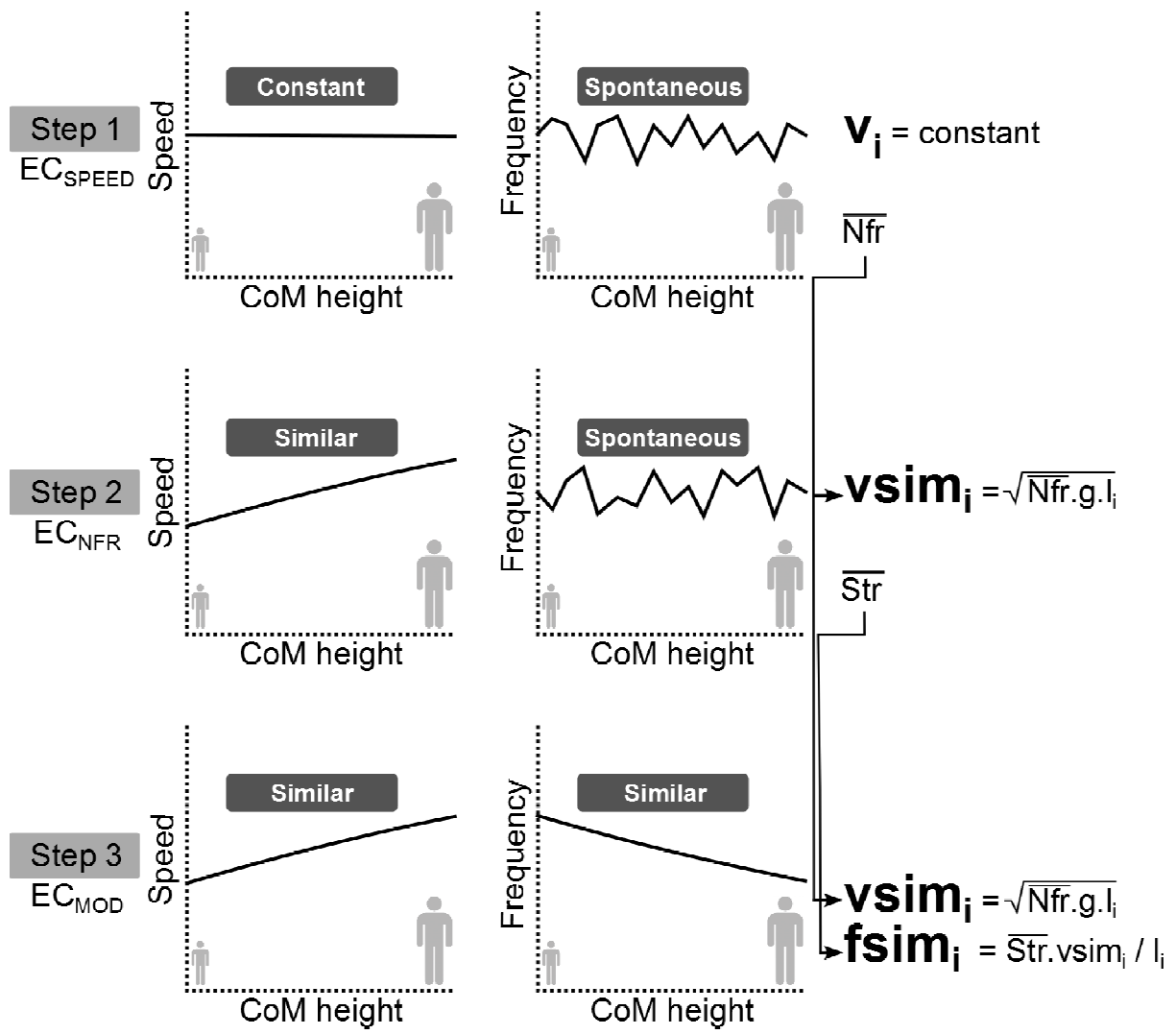
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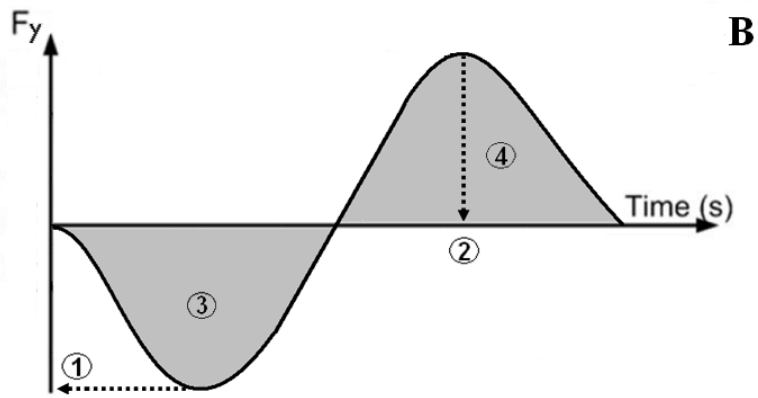
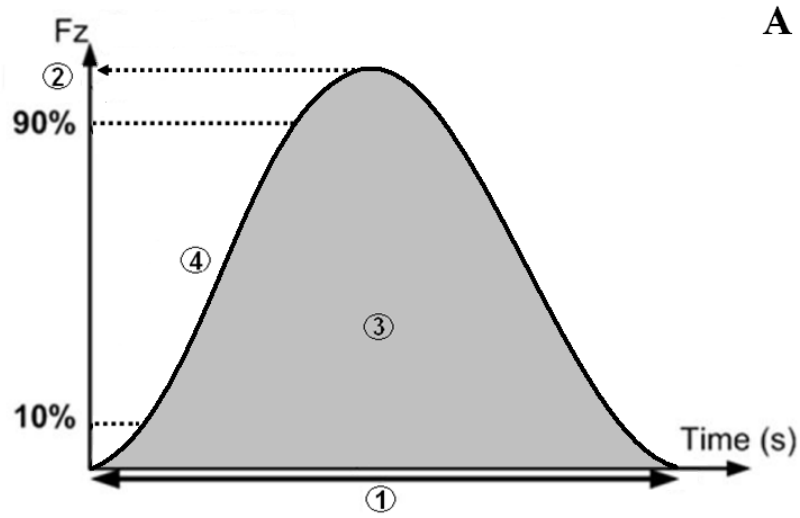
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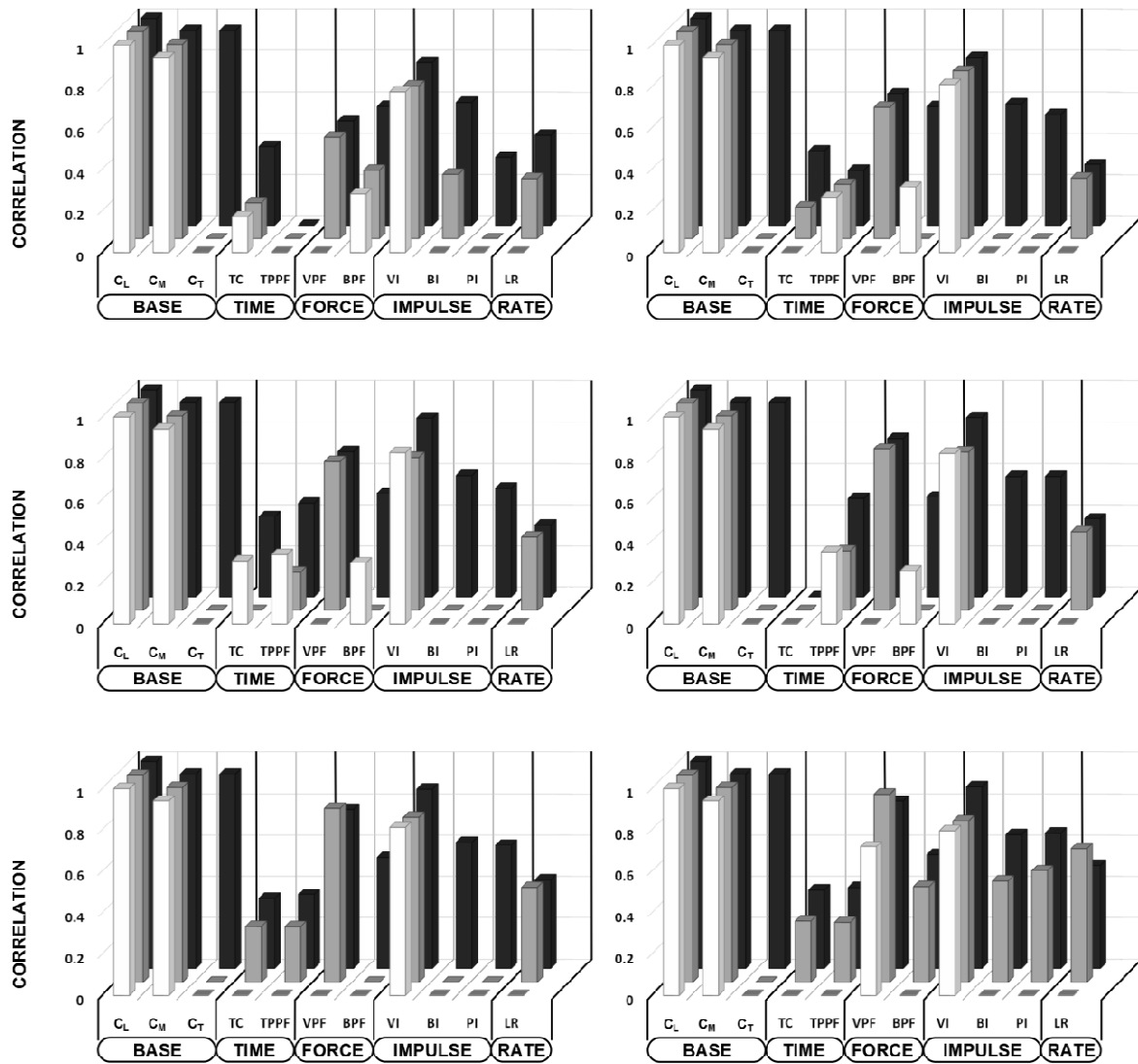
449
450 Fig. 1



451
452 Fig. 2



453
454 Fig. 3



455
456 Fig. 4