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Original Article

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Word count (abstract): 237 words

Word count (Introduction through References with appendix): 3475 words

Number of figure: 4 figures

Number of table: 3 tables

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Abstract

The aim of this study was to test the hypothesis that running at fixed fractions of Froude (Nfr) and Strouhal (Str) dimensionless numbers combinations induce dynamic similarity between humans of different sizes. Nineteen subjects ran in three experimental conditions, i) constant speed, ii) similar speed (Nfr) and iii) similar speed and similar step frequency (Nfr and Str combination). In addition to anthropometric data, temporal, kinematic and kinetic parameters were assessed at each stage to measure dynamic similarity informed by dimensional scale factors and by the decrease of dimensionless mechanical parameter variability. Over a total of 54 dynamic parameters, dynamic similarity from scale factors was met for 16 (mean r = 0.51), 32 (mean r = 0.49) and 52 (mean r = 0.60) parameters in the first, the second and the third experimental conditions, respectively. The variability of the dimensionless preceding parameters was lower in the third condition than in the others. This study shows that the combination of Nfr and Str, computed from the dimensionless energy ratio at the center of gravity (Modela-r) ensures dynamic similarity between different-sized subjects. The relevance of using similar experimental conditions to compare mechanical dimensionless parameters is also proved and will highlight the study of running techniques, or equipment, and will allow the identification of abnormal and pathogenic running patterns. Modela-r may be adapted to study other abilities requiring bounces in human or animal locomotion or to conduct investigations in comparative biomechanics.

Keywords: Spring Mass Model; Dimensionless Parameters; Center of Mass; Similar Speed; Similar Frequency

1. Introduction

Originally used in the fluid mechanics field, the concept of dynamic similarity enables two different-sized systems to be considered as scaled models by setting them in equivalent experimental conditions. It suggests that when two systems are dynamically similar, one could be identical to the other by multiplying (i) all lengths (L dimension) by one scale factor C_L, (ii) all masses (M dimension) by another scale factor C_M, and (iii) all times (T dimension) by a third scale factor C_T. Furthermore, scale factor for all other mechanical parameters depending on the three preceding dimensions, such as speed, force, and impulse, can be computed from C_L, C_M, and C_T. The concept was originally applied in fluid mechanics, and more recently in biology, ecology, and biomechanics considering that, if isometric, a small subject is a scaled model of a tall one. This concept has also been applied to compare locomotion between different species (Alexander, 1989; Minetti et al., 1994; Vaughan and Blaszczyk, 2008) and to study similarities between human of different sizes during walking and running (Moretto et al., 2007; Delattre and Moretto, 2008; Delattre et al., 2009).

A Spring Mass Model (SMM, Fig. 1) is commonly used to compare locomotion between animals and humans as it takes into account an elastic component and modelizes the rebound occurring during jumping and running (Alexander, 1989). It consists in a body mass represented at the Centre of Mass (CoM) oscillating at the end of a massless spring. This model is commonly used to represent the CoM mechanical behavior of human running (Blickhan, 1989; Mcmahon and Cheng, 1990). Its kinematic depends on seven physical variables: gravity (g), mass (m), stiffness (k), initial spring length (l_0), initial spring angle (θ_0), initial landing velocity (v_0), and the angle of the initial landing velocity (β_0).

An approach to compare similar locomotion and to ensure dynamic similarity between specimens is based on the dimensionless approach focusing on locomotion models like SMM. Part of this approach rests in the π theorem stated by Buckingham (Buckingham, 1914). It reduces the number of variables by considering dimensionless numbers computed from the characteristic variables of a specific problem. This theorem states that a physical equation using N_P physical variables, that are dependant of N_D base dimensions, necessitates N_P-N_D dimensionless numbers (π) to describe the mechanical behavior of a system. Applying the π theorem to the SMM, the seven aforementioned physical variables ($N_P = 7$) are dependent on three base dimensions ($N_D = 3$), L (m), M (kg), and T (s). Thus, four dimensionless numbers are necessary to completely describe the movement of both systems. These four dimensionless numbers given by the theorem come from the seven physical variables as presented in table 1. Each of them can be expressed in terms of Nfr or Str. Consequently, the four dimensionless numbers are Str, Nfr, β_0, and θ_0 (Tab 1). Nfr (v_0^2 / gl_0) is the Froude number representing the dimensionless speed and Str (fl_0 / v_0) is the Strouhal number corresponding to the dimensionless oscillatory frequency, i.e. the dimensionless form of the step frequency f ($f = \sqrt{k/m}$). The SMM modelizes the behavior of the CoM. To be in accordance with the fundamental physic principle, the Nfr and Str computation should take the position of the CoM into account rather than the leg length. This is why “l” refers to CoM height.

Nfr and Str dimensionless numbers have been used to determine experimental running conditions. Delattre et al. (Delattre et al., 2009) showed that neither Nfr nor Str were sufficient to characterize running mechanics or to establish inter-subject dynamic similarities, but each leads its own contributions. Indeed, Nfr contributes to observe similarities of antero-posterior kinetic events while Str contributes to the temporal organization. Very recently, a link has been highlighted between Nfr and Str during running (Villeger et al., 2012). According to Alexander (Alexander, 1989), these authors suggested a concomitant use of these dimensionless numbers for running gait. To this end, the Modela-r dimensionless number has been developed from mechanical simulation of SMM (Delattre and Moretto, 2008). It is equal to the combination of Nfr and Str, which equals the ratio of Kinetic (E_K) and Potential (E_P) Energies over Elastic Energy (E_E) with $E_K = 0.5mv^2$ (m the mass, g the gravity, and v the speed), $E_P = mgh$ (h the CoM height) and $E_E = 0.5k\Delta l^2$ (k the stiffness and Δl the variation of spring length)(Eq. 1). The ratio (EK+EP)/EE would be theoretically constant for a SMM and would correspond to a witness of the energy transfer at the CoM. As mentioned by Wannop et al. (2012), Modela-r has never been experimentally validated.

$$Modela\text{-}r = \frac{E_K+E_P}{E_E} = \frac{1}{Str^2}\left(\frac{2}{Nfr}+1\right) \qquad \text{(Eq. 1)}$$

Inspired by these recent works, our study aims to ensure dynamic similarity to different-sized subjects using a combination of Nfr and Str for running through the introduction of Modela-r as a dimensionless number issued from the energy transfer at the CoM.

2. Methods

2.1. Population

Nineteen subjects (n = 19) took part in this study after signing an informed consent document. Their characteristics were (mean ± sd [min; max]): age 23 ± 5 [18; 36] years, height 1.79 ± 0.07 [1.68; 1.94] m, and mass 80.7 ± 11 [64; 102.9] kg. They were chosen so as that the tallest was the heaviest, and vice versa. The experimentation was approved by the ethical committee of the University of Toulouse.

2.2. Experimental conditions

2.2.1. General procedure

For 3-dimensional analysis, 42 reflective markers were fixed on subject bone landmarks (Wu et al., 2002, 2005). Participants performed running tests barefoot with speed and/or step frequency determined from Nfr and Str. Experimentation was conducted on a treadmill (PF 500 CX, PRO FORM, Villepreux, FRANCE) mounted on a large forceplate sampled at 1 kHz (AMTI, Watertown, MA, USA). The positions of reflective marker were recorded by twelve optoelectronic cameras sampled at 200 Hz (VICON, Oxford's metrics, Oxford, UK). After a familiarization period, the subjects had to perform three trials per running test (Hamill and Mcniven, 1990) that were repeated in different experimental settings. The CoM height (l_i) was determined from the i^{th} subject's anatomic position ($i \in [1,n]$) with the anthropometric model of De Leva (de Leva, 1996). The center of rotation of the hip was determined using the SCoRE method (Ehrig et al., 2006).

2.2.2. Experimental steps

The experimentation was separated into the three steps detailed below and in fig. 2.

EC_{SPEED}

The subjects performed six stages of running with speeds set at 1.67, 2.22, 2.78, 3.33, 3.89, and 4.44 $m.s^{-1}$ (Eq. 2). These six speed stages were indexed as $k \in [1, 6]$. The first experimental condition consisted in setting the same constant speed for all the subjects. At speed stage k:

$$v_{ik} = 1.111 + 0.556 \times k = v_k \quad \text{(Eq. 2)}$$

EC_{NFR}

The second experimentation time consisted of imposing six stages of running with similar velocities. A mean $\overline{\mathrm{Nfr}_k}$ was computed from EC_{SPEED} for each stage of speed (Eq. 3). Then, similar velocities at speed k were determined from $\overline{\mathrm{Nfr}_k}$ (Eq. 4) for each subject.

$$\overline{Nfr_k} = (1/n)\sum_{i=1}^{n} Nfr_{ik} = (1/n)\sum_{i=1}^{n} \frac{v_k^2}{gl_i} \quad \text{(Eq. 3)}$$

$$vsim_{ik} = \sqrt{\overline{Nfr_k} gl_i} \quad \text{(Eq. 4)}$$

EC_{MOD}

The third experimentation time consisted of imposing six stages of running with similar velocities (Eq. 4) and similar frequencies. A mean $\overline{Str_k}$ was computed from EC_{NFR} for each stage of speed (Eq. 5). Then, similar frequencies at speed k for each subject were determined from $\overline{Str_k}$ (Eq. 6)

$$\overline{Str_k} = (1/n)\sum_{i=1}^{n} Str_{ik} = \frac{1}{n}\sum_{i=1}^{n} \frac{f_{ik} l_i}{vsim_{ik}} \quad \text{(Eq. 5)}$$

$$fsim_{ik} = \overline{Str_k}\frac{vsim_{ik}}{l_i} \quad \text{(Eq. 6)}$$

2.3. Parameters assessed

4^{th} order zero lag Butterworth filters were applied to kinematic and kinetic data with a cut off frequency set at 6 Hz and 10 Hz respectively (Goldberg and Stanhope, 2013). Then, 5 consecutive cycles were averaged at each stage of speed.

The ground reaction forces (GRF) were measured by a large force platform under the treadmill. A threshold of 10 N was used to detect the contact phase in running. The kinetic parameters suggested by Delattre et al. (Delattre et al., 2009) to study the GRF similarities during running were adapted. Indeed, eight parameters were studied aiming at reader comprehension of the results (Fig. 3). The different parameters are detailed in Fig. 3 legend.

The flexion extension angles at the ankle, the knee, and the hip were also considered and expressed in radian to respect the international unity system and a dimensionless form. In order to compare angle variability, the averaged cycle was normalized to 100 points wherein each corresponded to a percentage of the cycle.

The mass (m), the CoM height (l), and the CoM oscillation frequency (f), were considered to compute the dimensionless values of the kinetic parameters and to normalize them with respect to the basic dimensions [M, L, and T^{-1}] (Table 2). A "D" has been added as an exponent of the parameter acronym to differentiate the dimensionless value from the real one. Thus, the relative stride length, the relative contact time (duty factor), and the relative peak of force were noted as SL^D, TC^D, and VPF^D for running, respectively.

2.4. Analysis to consider similarity

The similarity analysis was a two step procedure. The first step was based on the correlation between the scale factors predicted from basis scale factors and measures. The second step was to verify the decrease of variance of the dimensionless parameters. Experimental setups that enable concomitantly the increase in the scale factors correlation and the decrease in the variability will be considered as successful means to induce dynamic similarity between different subjects.

A scale factor was a ratio of a mechanical parameter of one subject to another. With 19 subjects, 171 scale factors were built for each parameter. Basis scale factors (C_L, C_M and C_T) were derived from the three basis dimensions of any system (length, mass and time, respectively). C_L was calculated by subject height ratios, predicted C_M was computed from $C_M = C_L^3$ because the subjects had theoretically the same density index, and predicted C_T depended on the experimental conditions. Predicted scale factors were developed from the basis scale factors (Table 1) and represented how the individuals' parameters should be related if the conditions of dynamic similarity were met. Measured scale factors were those developed from the measurements of the mechanical parameters. For instance, the predicted scale factor between two subjects S_i and S_j for the braking peak was $C_{BPF} = C_{FORCE} = C_M C_L C_T^{-2}$ whereas the measured scale factor was $C_{BPF} = BPF_i / BPF_j$ with

BPF the measured values. When for a given parameter all predicted scale factors equaled all measured scale factors, it could be stated that the parameter was similar or proportional from one subject to another. We reiterate that C_L and C_M (= CL^3) were given by anthropometry; however, C_T was dependent on experimental conditions and is presented thereafter.

EC_{SPEED}

At constant speed k, the speed scale factor (table 2) between subjects (i and j) was $C_{SPEED} = v_{ik}/v_{jk} = C_L C_T^{-1} = 1$, thus $C_T = C_L$ with $j \in [1,n]$ and $i \neq j$.

EC_{NFR}

The speed scale factor between two similar velocities ($C_L C_T^{-1}$) was equal to $C_L^{0.5}$ (Eq. 7) that induced a $C_T = C_L^{0.5}$ time scale factor.

$$\frac{Vsim_{ik}}{Vsim_{jk}} = \frac{\sqrt{\overline{Nfr_k} g l_i}}{\sqrt{\overline{Nfr_k} g l_j}} = \sqrt{\frac{l_i}{l_j}} = C_L^{0.5}, \text{ thus } C_T = C_L^{0.5} \qquad \text{(Eq. 7)}$$

EC_{MOD}

The frequency scale factor between two similar frequencies (C_T^{-1}) was equal to $C_L^{-0.5}$ (Eq. 8) that induced the time scale factor of $C_T = C_L^{0.5}$.

$$\frac{fsim_{ik}}{fsim_{jk}} = \frac{\overline{Str_k} Vsim_{ik}/l_i}{\overline{Str_k} Vsim_{jk}/l_j} = \frac{Vsim_{ik}}{Vsim_{jk}} \frac{l_j}{l_i} = C_L^{0.5} C_L^{-1} = C_L^{-0.5}, \text{ thus } C_T = C_L^{0.5} \qquad \text{(Eq. 8)}$$

It should be noted that the decrease of variance of dimensionless parameters signifies a more similar behavior (Pierrynowski and Galea, 2001).

2.5. Statistical analysis

All statistical analyses were performed with the STATISTICA software (STATISTICA

V6, Statsoft, Maison-Alfort, FRANCE). For all statistical tests, normality was checked using the Kolmogorov-Smirnov test. For normal distributions, parametric tests were performed other than non parametric tests were used.

Statistical analysis performed on kinetic parameter scale factors was divided into two steps. First, a Spearman coefficient was computed between predicted and measured scale factors under each experimental condition. Only significant correlations ($p<0.05$) were taken into account. Then, Wilcoxon paired tests were conducted to identify if there were significant difference between the predicted and the measured scale factors. If the Spearman correlation coefficients were significant and the Wilcoxon test did not reveal significant difference between predicted and measured scale factors for a kinetic parameter, then the parameter was considered as similar from one subject to another. In addition to the kinetic parameters, the same tests were repeated on mass (C_M) and on step time (C_T).

3 repeated factors ANOVA (EC_{SPEED}, EC_{NFR}, and EC_{MOD}) was conducted for ankle, knee, and hip angles at each stage of speed ($p<0.05$) to detect the significant effect of the experimental conditions on the inter-subject variance. A Tukey post hoc comparison enabled a refinement of the analysis.

The homogeneity of variance of the dimensionless gait parameters SL^D, TC^D, $TPPF^D$, VPF^D, BPF^D, VI^D, BI^D, PI^D, and LR^D between the three experimental conditions was tested with a Levene test ($p<0.05$). Then, the Fisher and Snedecor F-test ($p<0.05$) was performed as a post hoc test to highlight which variance was significantly different from the others. It was repeated for the six speed stages.

3. Results

For kinetic parameter scale factors, two criteria were taken into account to determine if one experimental condition produced more dynamic similarities than the others: first, the numbers of parameters for which the measured and predicted scale factors were correlated and non-statistically different from each other; then, the mean of the correlation value for these parameters. The dynamic similarity results are presented below and in Fig. 4. They were met for 16, 32, and 52 parameters out-of 54 dynamic parameters in EC_{SPEED}, EC_{NFR}, and EC_{MOD}, respectively. No similarities were found on C_T (step time) in EC_{SPEED} and EC_{NFR}. The mean coefficients of correlation for all parameters were 0.51, 0.49, and 0.60 in EC_{SPEED}, EC_{NFR}, and EC_{MOD}.

The variances of ankle, knee, and hip angles are presented in table 3. The lowest variability of angles of knee and hip was met in EC_{MOD} for all speeds. In EC_{MOD}, the variability of ankle angles was the highest at the 2.22 $m.s^{-1}$ stage whereas it was the lowest at the last three speed stages. Moreover, EC_{NFR} generated more variability of ankle angles than the other conditions at the two last stages.

Referring to table 3, EC_{NFR} allowed a reduction of the variability of a total of 13 dimensionless parameters compared to EC_{SPEED}. The variability of 64 dimensionless parameters was decreased in EC_{MOD} compared to EC_{SPEED}. EC_{MOD} enabled a reduction of the variability of 52 dimensionless parameters compared to EC_{NFR}.

4. Discussion

This study aimed to ensure dynamic similarity between different-sized subjects using a new dimensionless number, Modela-r. As a combination of Nfr and Str, Nmodela-r accounts for the energy transfer at the CoM during running.

The increase of correlations between predicted and measured mechanical scale factors associated with the decrease of the dimensionless parameter variability highlights the interest of the association of Nfr and Str to induce dynamic similarity. In our study, EC_{MOD} leads to more dynamic similarity than the other conditions at each stage of speed. Thus, in order of importance, EC_{MOD} and EC_{NFR} lead to more similar gait parameters than EC_{SPEED}. Our results are in line with those of Delattre et al. (Delattre et al., 2009) and Alexander (Alexander, 1989) who suggested using a combination of Nfr and Str dimensionless numbers to obtain similarities on running patterns. Moreover, we have shown that Nfr alone brings similarities and its combination with Str leads to further similarities. As defined in 2.4. (EC_{NFR} and EC_{MOD}), the time constraint generated a theoretical relationship between C_L and C_T as $C_T = C_L^{0.5}$. Thus, the correlation between measured and predicted scale factors of time was higher (0.94) in EC_{MOD}. However, the C_T dynamic similarity was met only in EC_{MOD} with a correlation of 0.94. Thus, in EC_{NFR} the spontaneous frequency was not proportional (different from $C_L^{-0.5}$) in our study. This is in accordance with Delattre et al. (Delattre et al., 2009). Indeed, they reported correlations of -0.27 and 0.99 between predicted and measured scale factors of stride frequency (or stride time) in experimental conditions which respected the same Nfr and Str, respectively. A non-proportional spontaneous frequency in EC_{NFR} could be an explanation of the effect of the additional use of Str on dynamic similarity in EC_{MOD}.

Based on robust physic theory as π theorem, four dimensionless numbers (Nfr, Str, β_0, and θ_0) are necessary to describe the behavior of the SMM which modelizes the CoM movement in running gaits. Our model enabled the computation of Nfr and Str at the CoM and the determination of similar speeds and similar step frequencies from the CoM height and the CoM oscillation frequency. In this study, only two of the four dimensionless numbers are necessary to describe the movement of the spring mass models. As Bullimore and Donelan (Bullimore and Donelan, 2008) have shown with two unconstrained simulations of SMM, two dimensionless numbers are not sufficient to ensure dynamic similarities. Indeed, from the same values of two dimensionless numbers they have simulated different SL^D (~2.96 and ~5.52), TC^D (~0.31 and ~0.2), and VPF^D (~2.4 and ~4). Referring to our data, the variability of the dimensionless parameters (VPF^D, TC^D, and SL^D) from the same dimensionless numbers (Nfr and Str in EC_{MOD}) was very low. This discrepancy suggests that human locomotion in our case cannot be summarized as unconstrained simulations. Indeed, the organization of the movement suggests that for an association of Nfr and Str, a constrained behavior corresponds. This can be an explanation of the lower variability of our measured dimensionless parameter in EC_{MOD}. Moreover, the variability of SL^D was close to 0 in EC_{MOD}. SL^D is the inverse of Str (Alexander, 1989) and explains its zero variability in EC_{MOD} wherein Str is taken into account.

The locomotion model used in this study is constrained by the gravity and an elastic phenomenon. The gravity constraint is taken into account in Nfr and the elastic phenomenon is strongly dependent on the general stiffness (k), which is introduced in Str. The elastic phenomenon (Cavagna et al., 1964) during running is taken into account in Modela-r (Delattre and Moretto, 2008). Modela-r is a witness of the energy transfer that occurs at the CoM and can be expressed as a combination of Nfr and Str (Eq. 1). Thus, subjects, who move at the same Modela-r number, move similarly. More precisely, the use of Modela-r as a combination of Nfr and Str allows the researcher to generate similar experimental conditions that constrain energy transfer occurring at the CoM. Moreover, its development being based on the SMM behavior, Modela-r could be applied to the whole of locomotion characterized like SMM. Thus, Modela-r should be useful in comparative biomechanics between species (Alexander, 1989; Farley et al., 1993; Srinivasan and Holmes, 2008) and could be a means to construct a dimensionless database of running.

Many studies compare mechanical parameters between different populations that are not homogeneous among themselves (ex. A small specimen versus a tall one), especially normalizing the parameters by individual anthropometrical characteristics (i.e. height and mass). Besides population characteristics like heights and masses, many studies compare normalized mechanical parameters under dissimilar conditions. It means they compare parameters relative to individual characteristics under experimental conditions which themselves are not relative to individual characteristics. Indeed, small and tall subjects running at the same speed is not comparable, this is like comparing children and adults running at the same speed. In these conditions, Modela-r allows scientists to put different-sized specimens in similar conditions which makes the comparison of dimensionless parameters relevant. Indeed, if two specimens move similarly they would have the same dimensionless mechanical parameters. Then, the identification of unequal parameters could highlight abnormal running, such as expertise, lack of practice, long-distance training or fatigue.

Furthermore, a part of the inter-individual variability under similar experimental conditions is a matter of biological system variability. Indeed, two mechanical systems have to move similarly in similar conditions, or else the differences between both should come from the part of biological variability of the bio-mechanics field. Hence, similar conditions, such as EC_{MOD}, allows one to study and identify the role of significant subjects like gender (Ferber et al., 2003), stiffness (Blum et al., 2009), prostheses (Hobara et al., 2013), and ability of elastic energy storage/recoiling in running more accurately.

Finally, the movement of the CoM in running can be characterized like a SMM. Hence, the concomitant use of Nfr and Str ensures dynamic similarities between different-sized subjects. Constraining locomotion by Str and Nfr allows researchers to constrain energy transfer occurring at the CoM (Modela-r), and thus, estimate the elastic energy origin and its function more accurately. So, this study highlights the importance of using similar experimental conditions by removing the individual anthropometrical characteristics effect to compare mechanical parameters and to more accurately study serious topics in running. Modela-r has been experimentally validated and shows its usefulness in i) establishing similar experimental conditions and ii) constraining the energy transfer at the CoM. Further studies that involve application of SMM to locomotion patterns like bouncing, trotting, and running in animals would enlightened the interest of Modela-r in comparative biomechanics.

Conflict of interest statement

None.

Acknowledgements

None.

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Table 1

Dimensionless numbers useful for the behavior description of the SMM determined by π theorem. The equation $f(l_0, m, v_0, k, g, \beta_0, \theta_0) = 0$ can be reduced to $\varphi(\pi_1, \pi_2, \pi_3, \pi_4) = 0$.

Dimensionless numbers (π)	Equation	Equivalent to
π_1	l_0^2k / mV_0^2	Str^2
π_2	gl_0 / V_0^2	Nfr^{-1}
π_3	β_0	
π_4	θ_0	

With l_0 the initial spring length; k the spring stiffness; m the mass; v_0 the initial landing speed; g the gravitational acceleration; β_0 the angle of the initial landing speed; and θ_0 the initial spring angle.

Table 2

Units, dimensions and predicted scale factors of kinetic parameters

Parameters	Units (SI)	Dimensions	Predicted scale factors	Dimensionless parameters
CoM height (l)	m	L	C_L	
Body mass (m)	kg	M	C_M	
Speed (v)	$m.s^{-1}$	LT^{-1}	$C_L C_T^{-1}$	Nfr
CoM oscillation frequency (f)	s^{-1}	T^{-1}	C_T^{-1}	Str
Time (TC and TPPF)	s	T	C_T	$Time^D = Time \times f$
Force (VPF and BPF)	N	MLT^{-2}	$C_M C_L C_T^{-2}$	$Force^D = Force / (mlf^2)$
Impulse (VI, BI and PI)	N.s	MLT^{-1}	$C_M C_L C_T^{-1}$	$Impulse^D = Impulse / (mgf)$
Rate (LR)	$N.s^{-1}$	MLT^{-3}	$C_M C_L C_T^{-3}$	$Rate^D = Rate / (mlf^3)$
Length (SL)	m	L	C_L	$Length^D = Length / l$
Angle (Ankle, Knee and Hip)	Rad			Angle

C_L and C_M were defined by the subject's anthropometry whereas C_T was determined by the experimental conditions.

Table 3

Standard deviation of dimensionless gait parameters at each speed stage

Speed stage	EC	Ankle angle (x 10^3)	Knee angle (x 10^3)	Hip angle (x 10^3)	SL^D	TC^D	TPPF^D	VPF^D	BPF^D	VI^D	BI^D	PI^D	LR^D
1.67 m.s^{-1}	EC_{SPEED}	2.6	2.6	2	0.10	0.10	0.08	0.46	0.03	0.11	0.01	0.01	2.53
	EC_{NFR}	2.6	2.6	2	0.09	0.07	0.08	0.41	0.03	0.15	0.01	0.01	1.65
	EC_{MOD}	2.6	**2.4*#**	**1.8*#**	**0.00*#**	**0.05***	0.05	0.36	**0.02***	**0.08#**	**0*#**	0.01	**1.20***
2.22 m.s^{-1}	EC_{SPEED}	2.8	2.9	2.2	0.13	0.08	0.06	0.40	0.03	0.10	0.01	0.01	2.47
	EC_{NFR}	**2.6***	2.6*	2.2	0.12	0.07	0.05	0.36	0.03	0.14	0.01	0.01	1.59
	EC_{MOD}	3*#	**2.3*#**	**1.6*#**	**0.00*#**	**0.05***	0.04	**0.23***	0.02	**0.05*#**	**0.01*#**	**0.01*#**	**1.32***
2.78 m.s^{-1}	EC_{SPEED}	3	3.4	2.6	0.15	0.08	0.06	0.44	0.05	0.09	0.01	0.01	2.57
	EC_{NFR}	3	3.3	2.4*	0.14	0.07	0.05	0.38	0.04	0.12	0.01	0.01	1.90
	EC_{MOD}	2.9	**2.8*#**	**2*#**	**0.00*#**	**0.04*#**	**0.03***	**0.19*#**	**0.03*#**	**0.05*#**	**0.01*#**	**0.01*#**	**1.04*#**
3.33 m.s^{-1}	EC_{SPEED}	3.3	3.7	2.7	0.15	0.07	0.05	0.35	0.04	0.08	0.01	0.01	2.13
	EC_{NFR}	3.2	3.7	2.7	0.15	0.07	0.05	0.37	0.05	0.12	0.01	0.01	1.88
	EC_{MOD}	**2.8*#**	**2.9*#**	**2.2*#**	**0.00*#**	**0.03*#**	**0.03*#**	**0.16*#**	**0.03*#**	**0.04*#**	**0.01***	**0.01***	**0.98*#**
3.89 m.s^{-1}	EC_{SPEED}	3.2	3.7	2.6	0.20	0.08	0.06	0.41	0.05	0.09	0.01	0.01	2.41
	EC_{NFR}	3.7*	3.8*	2.8*	0.16	0.05	0.04*	0.33	0.05	0.11	0.01	0.01	1.49*
	EC_{MOD}	**2.9*#**	**3.2*#**	**2.1*#**	**0.00*#**	**0.03***	**0.03***	**0.15*#**	**0.03*#**	**0.04*#**	**0.01*#**	**0.01*#**	**0.96***
4.44 m.s^{-1}	EC_{SPEED}	4	4.1	2.9	0.30	0.09	0.06	0.48	0.07	0.10	0.01	0.01	2.61
	EC_{NFR}	4.9*	4	2.7*	0.18*	0.05*	0.04	0.31	0.05	0.09	0.01	0.01	1.49*
	EC_{MOD}	**3.2*#**	**3.3*#**	**2.5*#**	**0.00*#**	**0.03*#**	**0.02*#**	**0.12*#**	**0.02*#**	**0.03*#**	**0.01*#**	**0.01*#**	**0.72*#**

The characteristic dimensions to express the gait parameters in a dimensionless form (^{D}) are: the mass ([M]), the CoM height ([L]) and the step frequency ([T^{-1}]).

*, #: variability significantly different from EC_{SPEED} and from EC_{NFR}. The significant lowest values of standard deviation are bolded.

Figure legends

Figure 1. The Spring Mass Model (SMM)

Figure 2. Relationship between speed, CoM oscillation frequency and CoM height under the three experimental conditions for each stage of speed.

Figure 3. (A) Running vertical reaction force (Fz) over time. 1: Time of Contact (TC); 2: Vertical Peak Force (VPF); 3: Loading Rate from 10% to 90% of vertical peak force (LR); 4: Vertical Impulse (VI). (B) Running antero-posterior reaction force (Fy) over time. 1: Braking Peak Force (BPF); 2: Time to Propulsive Peak Force (TPPF); 3: Braking Impulse (BI); 4: Propulsion Impulse (PI).

Figure 4. Correlations between predicted and measured scale factors of body mass (C_M), step time (C_T) and kinetic parameters (TC, time of contact; TPPF, time to propulsive peak force; VPF, vertical peak force; BPF, braking peak force; VI, vertical impulse; BI, braking impulse; PI, propulsive impulse; and LR, loading rate). The scale factor correlations whose the Wilcoxon test revealed a difference between predicted and measured scale factors was set to 0. Lightest grey, dark grey and black bars represent respectively dynamic similarity for EC_{SPEED}, EC_{NFR} and EC_{MOD}.

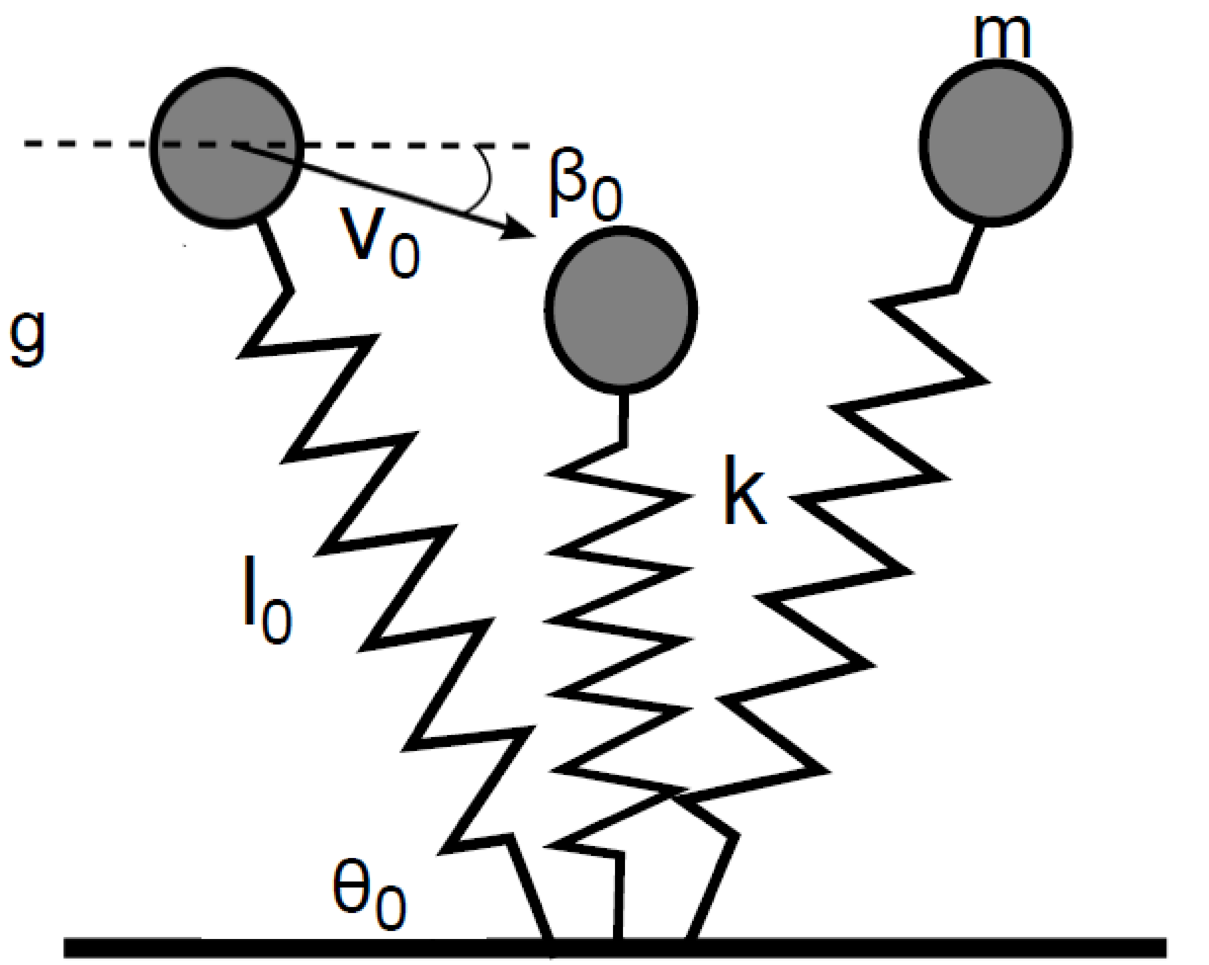
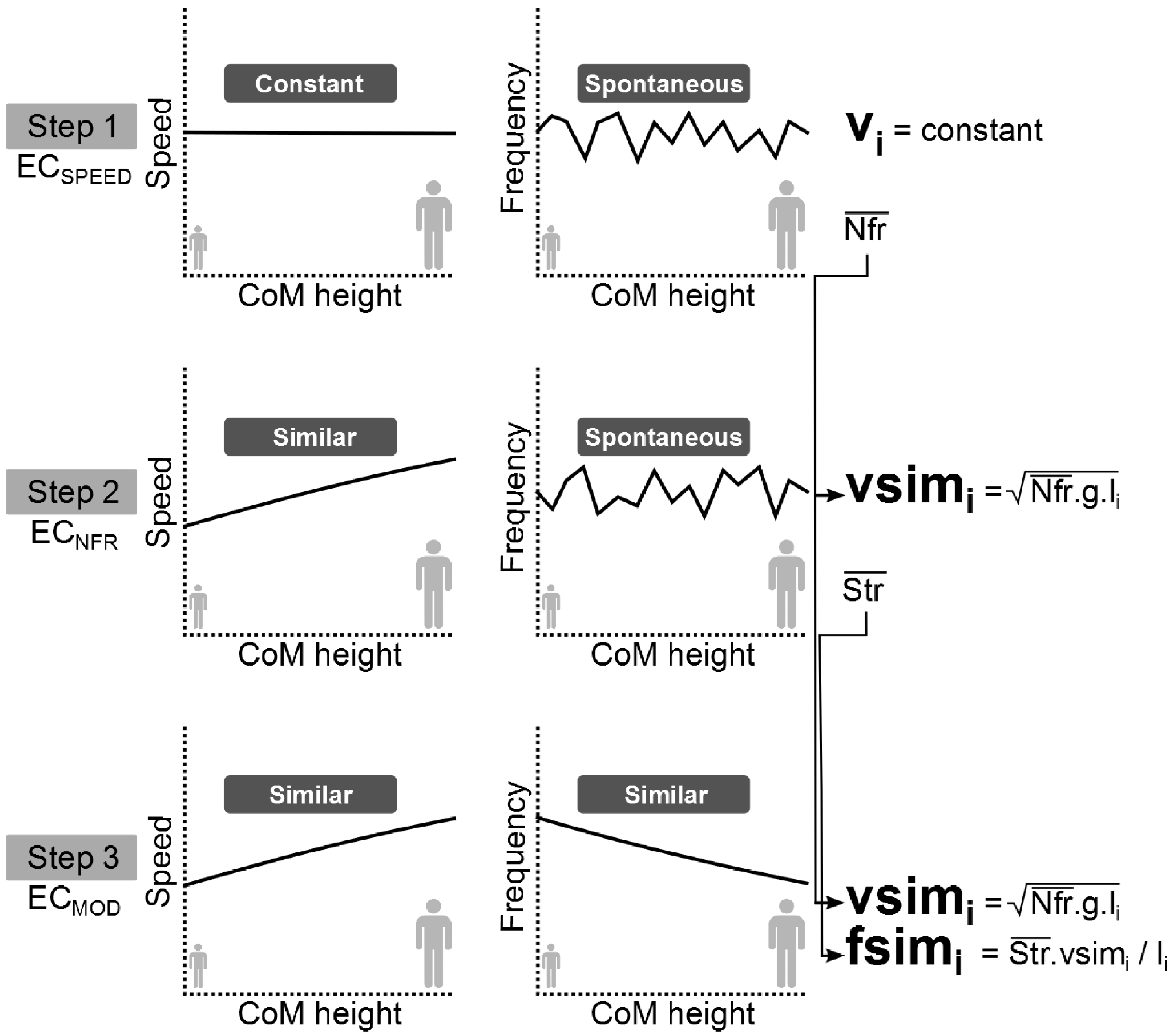


Fig. 1



Step 1
EC SPEED
Speed
Constant
CoM height
Frequency
Spontaneous
CoM height
v i = constant
Nfr
Step 2
EC NFR
Speed
Similar
CoM height
Frequency
Spontaneous
CoM height
vsim i = √Nfr.g.l i
Str
Step 3
EC MOD
Speed
Similar
CoM height
Frequency
Similar
CoM height
vsim i = √Nfr.g.l i
fsim i = Str.vsim i / l i

Fig. 2

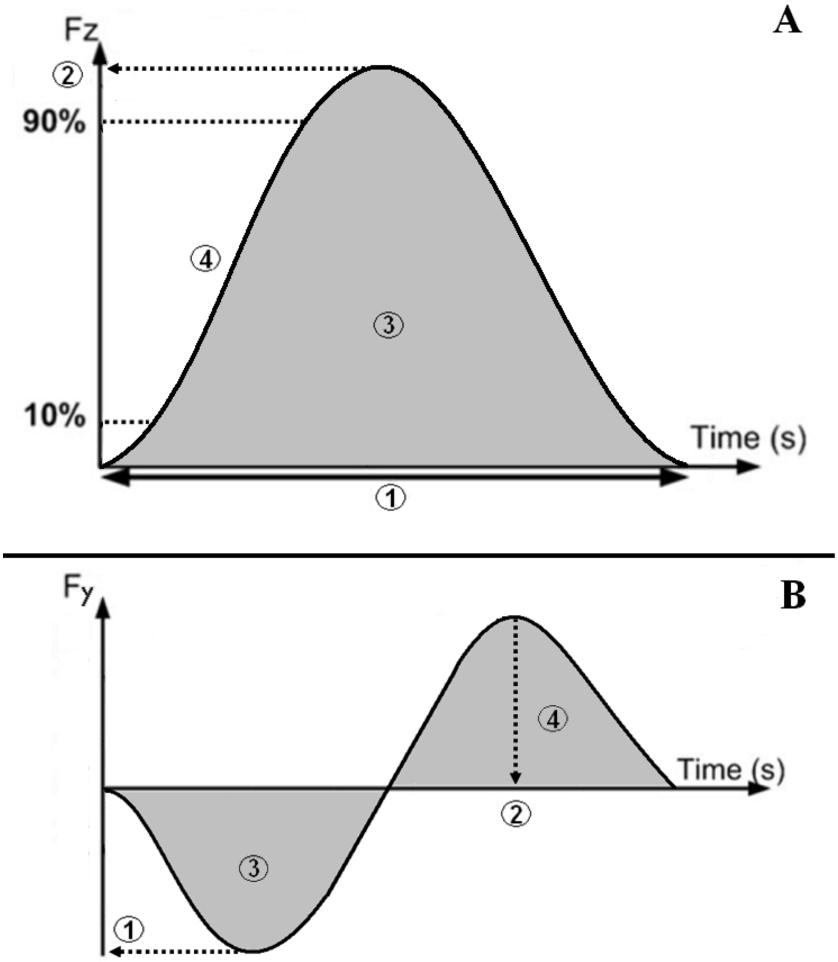


Fig. 3

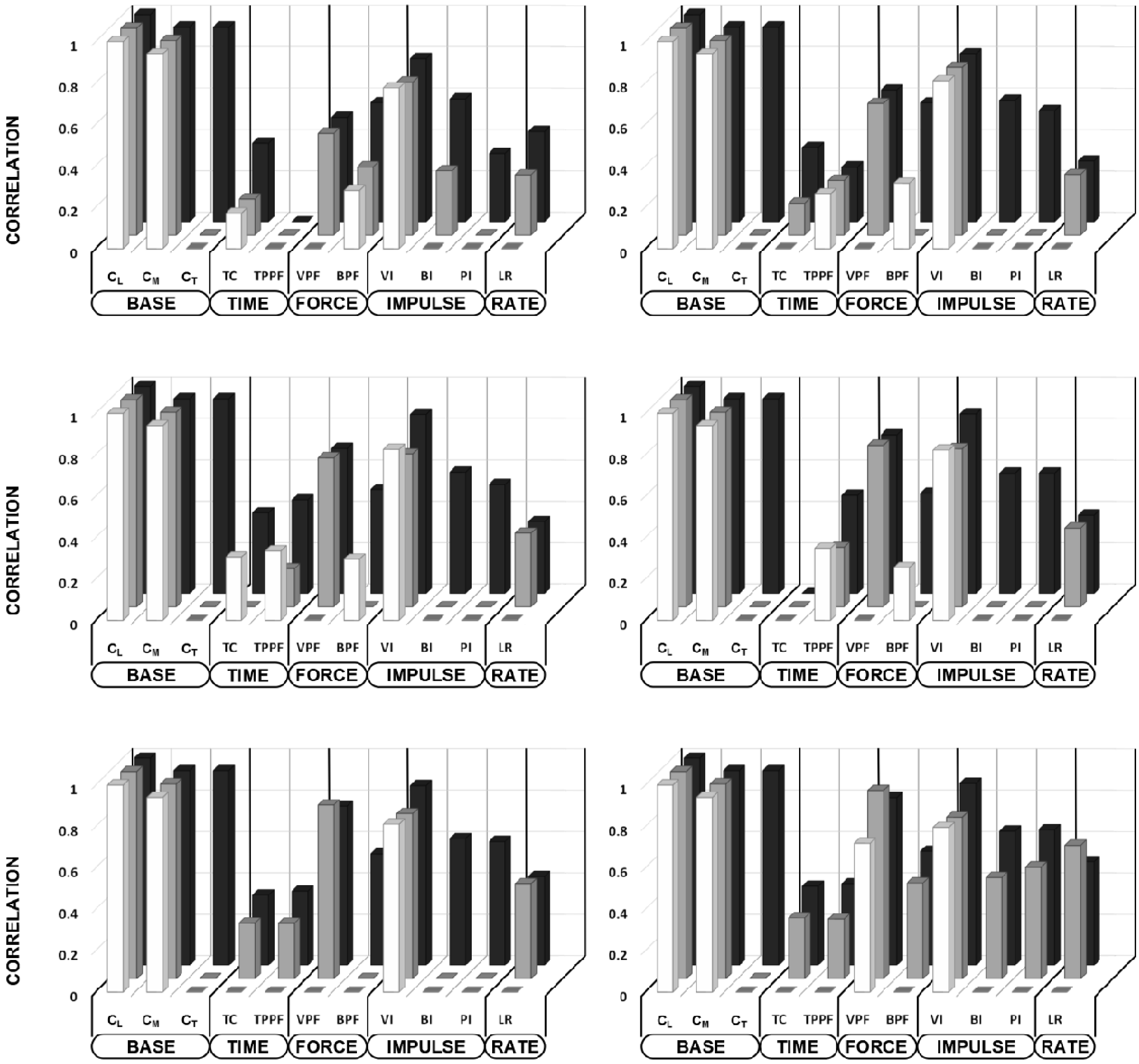

CORRELATION
0
0.2
0.4
0.6
0.8
1
C_L C_M C_T
TC TPPF
VPF BPF
VI BI PI
LR
BASE
TIME
FORCE
IMPULSE
RATE

Fig. 4