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Abstract: In the context of space rendezvous hovering phase, a event-triggered predictive controller has been designed in order to maintain the relative position of a chaser spacecraft relatively to a target for orbit servicing purposes. The main goal is to minimize the consumption and the computation burden of the controller with respect to previously developed controller.

Keywords: Event-triggered control, Predictive control, Impulsive control

1. INTRODUCTION

Mastering the spacecraft rendezvous and its automatic control will open venue of economical opportunities for space industry. For instance, for orbit servicing operations, the ability of a space tug or supply spacecraft to maintain safely and thriftily its relative position with respect to the target spacecraft is crucial for different purposes (observations, waiting for order, etc). In this study, the target spacecraft is inert and the chaser one is moved by means of chemical thrusters so that the control is modelled by an impulsive signal. Such hovering capacities can be obtained by developing efficient control algorithms. Pursuing this aim, the authors have developed an impulsive predictive controller for steering to and maintaining the chaser spacecraft in the given polytopic subset Arantes Gilz et al. (2017). The control strategy accounts for the periodic nature of the relative motion between spacecraft to steer the chaser to the set of periodic orbits that are included in a particular polytope. In fact, in Arantes Gilz et al. (2018), this predictive impulsive controller has been proved to stabilize this set even in presence of saturations: this property has been obtained by considering at least three consecutive impulsive controls for sake of controllability. That MPC scheme induces a periodic computation and execution of the multi-impulse controls. However, if this predictive controller shows efficiency in numerous cases and ability to be embedded in space-certified computation board (LEON 3), it suffers few drawbacks inherent to its nature. For instance, since computed at a given period, the obtained controls can be unnecessary or too small to be executed by the thrusters especially if the system belongs or is closed to the admissible set. Moreover, since the convergence to the admissible set is only ensured after the three impulsive controls are applied, the invariance of the admissible set can not be set before the third impulse. To overcome those drawbacks, we propose to combine the periodically triggered controller presented above with the event-triggered controller exposed in this work. The role of the first controller is to steer the system close enough to the admissible set so that the proposed event-triggered controller can take over to stabilize the admissible set. The proposed controller will compute one impulse that will systematically bring the system to the admissible set when the suitable event occurs.

To develop this event-triggered predictive controller, several challenges have to be faced. First, The relevant signals to be observed must be defined and constructed. Trigger rules have also to be set carefully to ensure the good properties of invariance and avoid bad behavior such as Zeno phenomenon. Then, the computation of the impulse is addressed by means of optimization to ensure the convergence. Note that the trigger rules should lead to the feasibility of the mathematical program. Finally the cohesion between both controller has to be carefully studied in order to obtain a global convergence and local invariance of the admissible set. Reader must keep in mind that these goals has to be achieved while taking into account the periodic nature of the relative dynamics and the non trivial description of the admissible set. If this the abstract exposes our different choice of methods, several technical aspects of this work have been omitted for sake of brevity.

2. RENDEZVOUS CONTEXT, DYNAMIC MODEL AND TECHNOLOGICAL CONSTRAINTS

The relative motion consists in the motion of a chaser spacecraft equipped with thrusters with respect to the moving local frame attached to a passive target spacecraft. Under Keplerian assumptions, the relative motion between two spacecraft in the Earth gravitational field has been expressed by means of linearized impulsive differential equations: $\dot{X}(t) = A(t)X(t) + B\Delta V_i \delta(t - t_i)$ with $X(t) \in \mathbb{R}^6$ and $A(t)$ being 2π-periodic. After a change of independent variable from time t to true anomaly ν of the target spacecraft e.g. its position on its orbit, a similar transformation is applied to obtain a second state space where the state are the parameters D exploited in several previous works Deaconu et al. (2015); Arantes Gilz et al. (2017): $D\prime(\nu) = A_D(\nu)X(\nu) + B_D(\nu)\Delta V_i \delta(\nu - \nu_i)$. This state can be viewed as the coordinates of the relative orbits (see (Deaconu, 2013, Chap 2) for full details). As stated earlier, the aim of the controller is to stabilize the set of relative periodic orbits satisfying the following relative position constraints: $\underline{x} \leq x(t) \leq \overline{x},\ \underline{y} \leq y(t) \leq \overline{y},\ \underline{z} \leq z(t) \leq \overline{z}\,, \forall t \geq t_0$. This set is described by linear but time varying conditions on state D (see details in Arantes Gilz et al. (2017)):

★ P. Arantes Gilz is supported by the ANR FASTRelax.

$$S_D^p := \left\{ D \in \mathbb{R}^6 \;\middle|\; d_0 = 0,\; \begin{array}{c} \underline{x} \leq M_x(\nu)D \leq \overline{x} \\ \underline{y} \leq M_y(\nu)D \leq \overline{y} \\ \underline{z} \leq M_z(\nu)D \leq \overline{z} \end{array}, \forall \nu \right\} \quad (1)$$

On top of that, the presence of a deadzone and saturations in the thrusters operation must be accounted for.

3. EVENT-BASED ALGORITHM

Control Law For a given true anomaly ν, $D^+(\nu)$, the state right after an impulse $\Delta V(\nu) \in \mathbb{R}^3$ is given by $D^+(\nu) = D(\nu) + B_D(\nu)\Delta V(\nu)$.

We opt for the strategy where every control impulse produce systematically an periodic orbit such that:

$$d_0^+ = d_0 + B_{D_0}(\nu)\Delta V = 0 \qquad (2)$$

Equivalently, the control that satisfy equation (2) is given by $\Delta V = B_{D_0}^{\perp}(\nu)\lambda + \Delta V^0(\nu)$, $\lambda \in \mathbb{R}$ and $B_{D_0}^{\perp}$ describes the kernel space of B_{D_0} and ΔV^0 is a particular solution of (2). Assuming the periodicity pursuit strategy, the effect of a control impulse on the cuurent state D is described by

$$D^+(\nu, \lambda) = D(\nu) + B_D\left(B_{D_0}^{\perp}(\nu)\lambda + \Delta V^0\right) \qquad (3)$$

To maintain the state D in the admissible set S_D^p, the impulse control is computed by solving the following program:

$$\begin{aligned} &\min_{\lambda \in \mathbb{R}} (B_{D_0}^{\perp}(\nu)\lambda) \\ &\text{s.t.} \begin{cases} D^+ \in S_D^p \\ \lambda \in I_{sat} \end{cases} \end{aligned} \qquad \text{(Psat)}$$

where I_{sat} describes the input saturation and deadzone condition in function of λ such that

$$I_{sat}(\nu) = \left\{\lambda \in \mathbb{R} \text{s.t. } \underline{\Delta V} \leq |B_{D_0}^{\perp}(\nu)\lambda + \Delta V^0(\nu)| \leq \overline{\Delta V}\right\} \quad (4)$$

Trigger laws The trigger law is designed to complete a threefold objective. First the problem (Psat) has to be feasible when called. Second, unnecessary controls must be avoided. Third Zeno phenomenon should be ensured to not occur. To set the trigger rules we need to define few terms. First, let the set Δ^+ be set of state D reachable with one impulse and describe by (3). Since $\lambda \in \mathbb{R}$, Δ^+ is a time-varying line. A necessary condition for the admissible set to be reachable is that the line Δ^+ intersects S_D^p. Let us denote this intersection Λ such that $\Lambda = \Delta^+ \cap S_D^p$. S_D^p being convex bounded and closed (see Arantes Gilz et al. (2017)), Λ is a segment of the line Δ^+. If $\mathrm{L}(\Lambda)$, the length of segment Λ, is different from zero, then S_D^p is reachable without input constraints. Considering these constraints, the admissible set is reachable if the set Λ_{sat}, defined by $\Lambda_{sat} = \Lambda \cap I_{sat}$, is non empty. Note that Zeno behavior is avoided by the presence of a deadzone condition in (4). Using these definitions, trigger rules can be set:

if $D(\nu) \in S_D^p$:
  Wait.
if $D(\nu) \notin S_D^p$ and $\Lambda_{sat} = \emptyset$
  Apply the three impulse strategy from Arantes Gilz et al. (2017)
if $D(\nu) \notin S_D^p$, $\Lambda_{sat} \neq \emptyset$, $\mathrm{L}(\Lambda_{sat}) < \delta$ and $\frac{d}{d\nu}\mathrm{L}(\Lambda_{sat}) < 0$
  Compute and apply $\Delta V = \arg\min(\text{(Psat)})$
else Wait.

To resume, the application of the single impulse control obtained by solving (Psat) is triggered if the set Λ_{sat} is vanishing i.e. when its length goes under a given threshold δ. Otherwise, if the admissible set is unreachable, the three impulse control can be computed as in Arantes Gilz et al. (2017) and applied.

4. FIRST RESULTS AND DISCUSSION

First results are provided on figure 1. Scenario 1 of the work Arantes Gilz et al. (2018) is rerun with the combination of the multi-impulse controller and the proposed event-triggered single impulse one. The threshold δ is 0.005 and bounds $\underline{\Delta V}$ and $\overline{\Delta V}$ are resp. equal to $0.5cm \cdot s^{-1}$ and $50cm \cdot s^{-1}$. For this scenario, the first impulses are computed by means of Arantes Gilz et al. (2018) controller. Then, the event triggered controller takes over at the end of the first period. From there, three other impulses are computed and applied each time the state lays outside of the admissible set while being reachable ($\mathrm{L}(\Lambda_{sat}) \neq 0$). Then, when the system is close enough to the admissible set ($\mathrm{L}(\Lambda_{sat}) \neq 0$), the single impulse controller takes over and stabilize the admissible set. The usage of such event-based strategy permits to save 25% of fuel consumption with respect to the periodic multi-impulse approach.

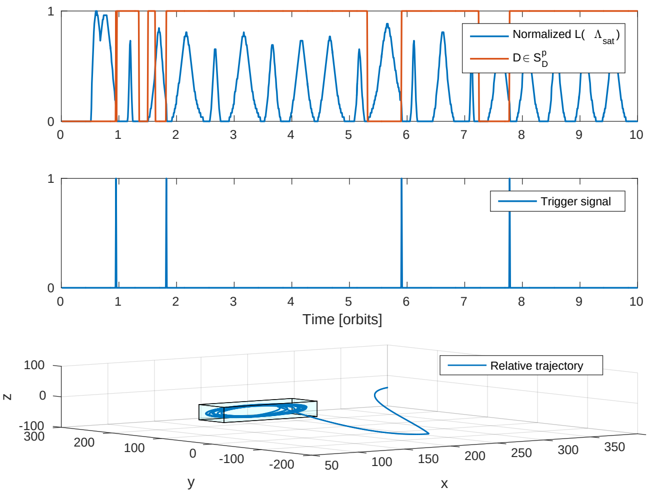

Fig. 1. Observed signals, trigger and controlled trajectories

Several technical aspects have been omitted in this extended abstract. Mainly, the evaluation of the set Λ_{sat} that is a part of the computation burden has not been exposed. In addition, if the stability of the combined controllers has not been formerly set, first hints at stability comes from the properties of the periodic multi-impulse controller.

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