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Abstract We consider a scheduling problem to which an additional decision level is added regarding the selection of the energy source used to satisfy the total power demand of tasks processed at each instant. Different energy sources are available, with different characteristics in terms of efficiency, power range and dynamics. The objective is to identify the best combination between scheduling and energy source utilization which minimizes the total duration and energy cost of the project.

Keywords energy-aware scheduling · efficiency functions · column generation · combinatorial optimization · mathematical programming

Mathematics Subject Classification (2000) MSC 49M27 · MSC 65K05 · MSC 90C27

1 Problem statement

Energy considerations are becoming paramount in the resolution of real-world applications. A rising combinatorial optimization challenge is the integration of energy constraints in deterministic scheduling models, such as job-shop scheduling or resource-constrained project scheduling. The specificity of this work is to consider multiple energy sources with constraints related to their physical, technological and performance characteristics. It is part of the project OREM (Ordonnancement de Ressources Energétiques Multiples) financed by the PGMO. The ambition of the project is to define a new and efficient methodology for the integration of the energy sources characteristics in combinatorial optimization problems.

Previous works, in collaboration with the LAPLACE electrical engineering team and the NEXTER Electronics company, have focussed on the optimization of the allocation of multiple sources of energy to predefined demand curves in hybrid electric vehicles. Non-linearities coming from energy efficiency functions make the allocation problem difficult to solve. Morevoer, flaws in existing modeling hypotheses led to significant gaps between state-of-the-art solutions and optimal ones. Global optimization-based heuristics have been designed to outperform the prior state-of-the-art [6,7]. As an alternative to non-linear modeling yielding suboptimal solutions and important computation time, a new and efficient

S. U. Ngueveu, C. Artigues, P. Lopez
CNRS, LAAS, 7 avenue du Colonel Roche, F-31400 Toulouse, France
Univ de Toulouse, INP, LAAS, F-31400 Toulouse, France
Tel.: +33-(0)5.61.33.62.90
Fax: +33-(0)5.61.33.69.36
E-mail: {ngueveu,artigues,lopez}@laas.fr

combinatorial modeling was proposed [5]. Although these studies considered the allocation of multiple energy sources and general non-linear efficiency functions, the scheduling of energy consuming activities, that would allow more flexibility for energy management was not considered.

On the other hand, the ROC team at LAAS was historically involved in resource allocation problems and production scheduling applications. Over the years the stakes of such applications have evolved towards a more responsible management of resources. In particular we considered models in production scheduling where the energy can be modulated at every instant of task processing, mainly to avoid peaks of electrical consumption. These models were solved via pure integer linear programming formulations [9] or hybrid constraint and integer programming based methods [8,1]. All our previous works on scheduling under energy constraints considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.

The OREM project aims at explicitely solving in an integrated fashion energy resource allocation problems and energy-consuming activity scheduling problems with non linear energy efficiency functions. Note that in the literature, a few promising mathematical programming-based approaches on similar problems can be found [3,4], either based on MINLP or transformations into approximate MILP. We propose an alternative approach based on piecewise linear lower and upper bounding, rather than approximating, the non linear efficiency functions. Related work in scheduling problems involving non-linear efficiency functions can also be found in the field of scheduling with continuous resources [2], mainly associated with parallel machine scheduling applications and considering theoretical complexity studies of remarkable special cases. In contrast with these studies, we aim at rather proposing mathematical decomposition methods to solve (relatively) general problems.

2 Solution procedure

The solution procedure is based on two main ideas: (i) the bounding of the nonlinear energy efficiency function, then (ii) the reformulation of the problem, which originally is a mixed-integer non-linear problem (MINLP), into two mixed integer linear problems (MILP) using the pair of bounding functions previously defined. The piecewise bounding of a function f of m variables within a tolerance value ϵ consists in identifying two piecewise linear functions denoted $\overline{f}^{\epsilon}$ and $\underline{f}^{\epsilon}$ that verify equations (1) to (3). The two MILP, denoted $\overline{\text{MILP}}$ and $\underline{\text{MILP}}$ respectively, are obtained by substituting f with $\overline{f}^{\epsilon}$ and $\underline{f}^{\epsilon}$, respectively.

$$\underline{f}^{\epsilon}(x) \leq f(x) \leq \overline{f}^{\epsilon}(x), \forall x \in \mathbb{R}^m \tag{1}$$
$$f(x) - \underline{f}^{\epsilon}(x) \leq \epsilon f(x), \forall x \in \mathbb{R}^m \tag{2}$$
$$\overline{f}^{\epsilon}(x) - f(x) \leq \epsilon f(x), \forall x \in \mathbb{R}^m \tag{3}$$

Performing the linearizations before the optimization allows not only the generation of $\overline{f}^{\epsilon}$ and $\underline{f}^{\epsilon}$ with respect to a predefined tolerance value ϵ, but also the minimization of the number of sectors of the resulting piecewise functions and therefore the minimization of number of additional integer variables in $\overline{\text{MILP}}$ and $\underline{\text{MILP}}$.

Solving a $\overline{\text{MILP}}$ generates solutions that are feasible for the original MINLP, and that have a total cost less than ϵ% higher than the optimal solution cost. Solving a $\underline{\text{MILP}}$ generates solutions that may not be feasible for the original MINLP, but whose total cost is less than ϵ% lower than the optimal solution cost and can help proving the optimality of a solution. Note that both problems share the exact same structure and only differ in terms of the numerical data of their respective piecewise functions. Therefore, a single dedicated resolution method needs to be developped and applied to solve both problems.

The underlying scheduling problem studied considers pre-emptive activities with release dates, due dates, duration and energy requirements. Because of the piecewise linear bounding-based solution method, the efficiency functions can be assumed to be piecewise linear without any loss of generality. Building on that, we proved the NP-hardness of the resulting problem. We aim at defining patterns of resource consumption via the set of activities that can be simultaneously in progress and a procedure for feasible patterns generation with regard to the predefined constraints. A column-generation-based algorithm to exploit the feasible patterns identified is being developed. It is based on a Dantzig-Wolfe decomposition which moves to the subproblem all piecewise linear cost functions. The results obtained will be presented at the conference.

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