

Supply chain transportation plans optimization Time-expanded graph enrichment heuristic

Simon Belieres¹, Nicolas Jozefowicz¹ and Frédéric Semet²

¹ CNRS, LAAS, 7 avenue du colonel Roche, F-31400 Toulouse, France

² CRISTAL, École Centrale de Lille, Cité Scientifique, B.P. 48, 59651 Villeneuve DAscq Cedex, France

Abstract. We address a problem of freight transportation in supply chains. To model flows over time in the system, we use time-expanded graphs. However, the time-expanded graphs' size increases exponentially with the number of actors and the time dimension, which makes the industrial solvers inefficient to overcome real instances. To face this difficulty, we conceived a heuristic based from Boland's and Hewitt's Dynamic Discretization Discovery. We manipulate a partially time-expanded graph, composed of a small subset of nodes and arcs, and enriches it incrementally. We produce a sparse graph with the essential elements and solve the associated problem, with much less constraints and variables.

Keywords: supply chain, time-expanded graph, heuristic

1 Introduction

This paper describes a resolution algorithm for the computation of supply chains' transportation and storage plans. We consider tri-echelon supply chain with suppliers, hubs and clients. Unfortunately, realistic instances in terms of network and time horizon quickly yield to problems too large to be sorted in a reasonable amount of time with an industrial solver. Boland and Hewitt [1] addresses to a comparable freight transportation problem over time, the Continuous Service Network Design Problem (CSNDP), of which the static version was reviewed by Crainic [2]. They conceived an ingenious method based on time-expanded graph enrichment to deal with the important data's scale. The Dynamic Discretization Discovery (DDD) first generates an initial time-expanded graph constituted with a small subset of the nodes and arcs. Successively, the CSNDP is solved and the graph is enhanced until the reach of the optimal solution. Our works were drawn from this logic and led us to the following resolution heuristic.

2 Problem description

The supply chain can be represented with a time-expanded graph $\mathcal{D}_T = \mathcal{N}_T, \mathcal{H}_T \cup \mathcal{A}_T$ derived from the "static" network $\mathcal{D} = (\mathcal{N}, \mathcal{A})$. The set of nodes \mathcal{N}_T is obtained duplicating the physical locations (suppliers $\in \mathcal{U}$, hubs $\in \mathcal{V}$, customers $\in \mathcal{W}$)

of \mathcal{N} through the time horizon. The set of arcs is decomposed into the holding arcs \mathcal{H}_T - connecting two occurrences of the same physical location - and the transportation arcs \mathcal{A}_T . Each arc $((i, t), (j, t'))$ has a travel time $t' - t$, a per-unit-of-flow cost $c_{ij} \in \mathbb{R}^{+*}$, a fixed cost $f_{ij} \in \mathbb{R}^{+*}$, and a capacity $u_{ij} \in \mathbb{N}^*$. Let \mathcal{K} denote a set of commodities, each customer has a positive demand of commodity k at time t : d_{wk}^t . So for each commodity we know the related sink w , but no predefined origin - any supplier can ship any commodity. We define SCNDP(\mathcal{D}_T) to be our Supply Chain Network Design Problem with integer variables $y_{ij}^{tt'}$ the resources required to route freight on arc $((i, t), (j, t'))$, and continuous flow variables $x_{ij}^{ktt'}$. The following is a generic valid integer linear programming formulation of this problem:

$$\begin{aligned}
\min \quad & \sum_{\mathcal{A}_T} f_{ij} y_{ij}^{tt'} + \sum_{\mathcal{K}} \sum_{\mathcal{A}_T} c_{ij} x_{ij}^{ktt'} + \sum_{\mathcal{K}} \sum_{\mathcal{H}_T} c_{ii} x_{ii}^{ktt} \\
\text{s.t.} \quad & \sum_{\mathcal{A}_T \cup \mathcal{H}_T} x_{ij}^{ktt'} - \sum_{\mathcal{A}_T \cup \mathcal{H}_T} x_{jl}^{ktt'} \begin{cases} = 0 & \forall (j, t) \in \mathcal{V}_T \\ \geq d_{wk}^t & \forall (j, t) \in \mathcal{W}_T \end{cases} \quad \forall k \in \mathcal{K} \\
& \sum_{\mathcal{K}} x_{ij}^{ktt'} \leq u_{ij} y_{ij}^{tt'} \quad \forall ((i, t), (j, t')) \in \mathcal{A}_T \\
& x_{ij}^{ktt'} \geq 0, \quad y_{ij}^{tt'} \geq 0, \quad \forall ((i, t), (j, t')) \in \mathcal{A}_T \cup \mathcal{H}_T, \forall k \in \mathcal{K}
\end{aligned}$$

We seek to minimize the total expenses, i.e. the transportation costs and holding costs. Constraint 1 is an adapted Kirchhoff constraint, and Constraint 2 ensures that sufficient resource capacity is available for the commodities.

3 Method

Boland and Hewitt DDD is an exact method that solves the CSNDP with free holding costs for the commodities. It is based on the iterative refining of a partially time-expanded graph, for which certain arcs under-estimate real travel times, and therefore allow extra unfeasible consolidations opportunities. However, in our problem storing commodities has a cost. We use the DDD to find the optimal flow in a transportation point of view, and combine it with a storing cost injection procedure, to enrich the partially-time expanded graph. The final solution is an upper bound we compared with the results obtained with Gurobi on the full graph. The comparison reveals our method efficiency, increasing with the instances size.

References

1. Boland N., Hewitt M., Marshall L., Savelsbergh M. (2017). *The Continuous Time Service Network Design Problem*. Operations Research, 65(5), 1303-1321.
2. Crainic T. G. (2000). *Service Network Design in freight transportation*. European Journal of Operational Research, 122(2), 272-288.