Impact of calibration on indoor positionning precision

Xavier Tolza, Pascal Acco, Jean-Yves Fourniols

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Xavier Tolza
Universite de Toulouse;
UPS, INSA, INP, ISAE; LAAS;
F-31077 Toulouse, France
Email: xtolza@laas.fr

Pascal Acco
Universite de Toulouse;
UPS, INSA, INP, ISAE; LAAS;
F-31077 Toulouse, France
Email: pacco@laas.fr

Jean-Yves Fourniols
Universite de Toulouse;
UPS, INSA, INP, ISAE; LAAS;
F-31077 Toulouse, France
Email: fourniol@laas.fr

***Abstract*—Internet of Things (IoT) business has raised a strong interest in assets positioning. While outdoor positioning mostly uses a Global Navigation Satellite System (GNNS) system, those are inoperable in indoor situations. Indoor positioning has been a very active field of study for the last decades and many approaches have been proposed, however those solutions are very often complex to deploy at industrial grade, due to complexity or cost of the hardware or the algorithms. Moreover, deploying those solution often requires a complex and expensive calibration setup such as fingerprinting to establish a radio map of the room. In this study, we focus on Bluetooth Low Energy (BLE) Received Signal Strength Indication (RSSI) positioning using maximum likelihood estimate and Cramer-Ro Lower Bound (CRLB) with the widely used Log-Distance Path Loss (LDPL) model to determine the impact of the calibration setup on the accuracy and precision of the position. Results are then matched with real BLE measurements made in an industrial-like environment.**

I. Introduction

Asset positioning has raised a great interest in the late year, especially with the IoT business.

Outdoor positioning is mainly achieved using GNNS solutions for global positionning, however this is not applicable for indoor position due to poor signal reception.

Indoor positioning has thus raise significant interest in the last decades and many solutions have been proposed, mainly based on RSSI, Time of Arrival (ToA) and Angle of Arrival (AoA), see [1] for a review and explanation of those terms. Their performances has been widely studied and the CRLB calculated for all those methods [2]

Some solutions provide very good performances, such as Ultra Wide Band (UWB), AoA, ultrasonic (US), or collaborative positionning but require expensive or difficult to integrate hardware, or have a high power consumption, limiting the usage for industrial deployments.

On the other hand, emergent low-power radio-frequency (RF) networks solutions can be really cheap, allowing to deploy autonomous infrastructure at low cost, but are more subject to interference and fading. The state-of-the-art solution for this problem is either to construct a radio map of the room (referred as fingerprinting) or using probabilistic methods such as maximum likelihood estimates.

Thus aiming low cost and autonomous indoor positionning solutions, we focus on RSSI solutions using low energy transmissions like BLE advertising or LORA. RSSI posistioning precision is greatly affected by the fading effects due to dense multipath environment, the APs placement and the calibration of the propagation model.

Fingerprinting performance compete those of probabilistic algorithms [3], a radio map could be inferred from the measurements and positions estimates using semi-supervised learning [4]. Each fingerprinting ask many transmissions and lower the energy autonomy of the setup, as the channel may vary in time these cost should be repeated periodically.

We chose to focus on probabilistic methods for several reasons: fingerprinting is a complex and costly procedure for large scale industrial facilities, its performance compete those of probabilistic algorithms [3], a radio map could be inferred from the measurements and positions estimates using semi-supervised learning [4].

II. Position estimation with uncertain path loss model

We consider the position estimation problem to be an estimation problem: knowing the RSSI channel model, our uncertainties on the model, we want to find $\hat{\theta}$ the best estimate of our position θ inside a given room from some measurements $r(\theta)$ by maximizing the likelihood function:

$$\widehat{\theta} = \underset{\theta}{\operatorname{argmax}} \left(\mathcal{L}(\theta|r(\theta))\right) \tag{1}$$

This section details the indoor RSSI channel model we have chosen, section III presents an analytic CRLB formula to lower-bound our estimate variance and finally we present a numerical application in section IV to draw conclusions and validate the analytic expression found in section III

A. Path loss model

The use of probabilistic algorithms require a propagation model, the LDPL model is a common choice for indoor Non Line of Sight (NLoS) propagation [5],

which defines the received power in relation with the distance as it follows:

$$P_R(d) = G - a_0 - 10\gamma\log_{10}(d) - R + \mathcal{V}(\sigma) \tag{2}$$

Where a_0 is the loss at a distance of 1m, γ is commonly known as path loss exponent, path loss factor or path loss gradient [5], R is the loss due to receiver's antenna and $\mathcal{V}(\sigma)$ the measuring noise modeled as a random Gaussian variable.

Fast fading phenomenon is removed by the preprocessing step, usually averaging and removing outliers using median or sometimes Kalman filtering.

NLoS situation is taken into account by the log-normal probality of shadowing $\mathcal{V}(\sigma)$ and does not worth be modelled seperatly as for time delay techniques.

Depending on setup constraints the Path Loss parameters are known with relative precision depending on the calibration setup when it exists.

B. Modelization of uncertainties

It is common for maximum likehood estimates to express the measurements in the form of a gaussian noise:

$$y = \mathcal{N}(f(x), \sigma) \tag{3}$$

In the following section we're developping the exact same form with a matrix representation of σ and $f(x)$ by considering the uncertainties as a sum of dependant gaussian noises.

Let's consider we make I measures in a rectangular room from N Access Points (APs) placed at coordinates $\omega_n = \begin{bmatrix} x_n & y_n \end{bmatrix}$ with $i \in [0, N-1]$

If we note $\theta = \begin{bmatrix} x & y \end{bmatrix}$ our position, $r_{n,i}$ the i^{th} RSSI measurement from AP n at θ, $\overline{r_n}$ the expected measured value from AP n, $r = \begin{bmatrix} r_{0,0} & \dots & r_{N-1,0} & r_{0,1} & \dots r_{N-1,I-1} \end{bmatrix}^T$ the vector of measurements at position θ, and $\overline{r}(\theta) = \begin{bmatrix} \overline{r_0} & \dots & \overline{r_{N-1}} \end{bmatrix}^T$ the mean expected value from all access points.

Then, the following relation between $\overline{r}(\theta)$ and r can be deduced:

$$r = \mathbb{1}_{I,1} \otimes (\overline{r}(\theta) + M(\theta)\mathcal{V}_\Gamma) + \mathcal{V}_{NI,1} \tag{4}$$

With $\mathbb{1}_{I,1}$ the I by 1 matrix full of ones, $\otimes$ the Kronecker product, $\mathcal{V}_{NI,1}$ a centered gaussian vector of shape NI by 1 and variance σ (the measuring error) and:

$$\begin{aligned}
\mathcal{V}_\Gamma &= \begin{bmatrix} \mathcal{V}_{a_0} & \mathcal{V}_R & \mathcal{V}_G^T & \mathcal{V}_\gamma \end{bmatrix}^T \text{ the model uncertainties} \\
\overline{r}(\theta) &= G + \mathbb{1}_{N,1}(a_0 + R) + \gamma\Delta(\theta) \\
M(\theta) &= \begin{bmatrix} \mathbb{1}_{N,1} & \mathbb{1}_{N,1} & I_N & \Delta(\theta) \end{bmatrix} \\
G &= \begin{bmatrix} G_0 \, \dots \, G_{N-1} \end{bmatrix}^T \\
\Delta(\theta) &= \begin{bmatrix} 5\,\log_{10}(d_0(\theta)^2) \, \dots \, 5\,\log_{10}(d_{N-1}(\theta)^2) \end{bmatrix}^T
\end{aligned}$$

If we do not calibrate G (resp. R, a_0 and γ), then we can consider it as a random variable normaly distributed around an average $\overline{G}$ value:

$$\begin{aligned}
G &= \overline{G} + \mathcal{V}(0, \sigma_G) = \overline{G} + \mathcal{V}_G \\
a_0 &= \overline{a_0} + \mathcal{V}(0, \sigma_{a_0}) = \overline{a_0} + \mathcal{V}_{a_0} \\
R &= \overline{R} + \mathcal{V}(0, \sigma_R) = \overline{R} + \mathcal{V}_R \\
\gamma &= \overline{\gamma} + \mathcal{V}(0, \sigma_\gamma) = \overline{\gamma} + \mathcal{V}_\gamma
\end{aligned}$$

If we consider we are calibrating one of those, we can set σ_G (resp. σ_R, σ_{a_0}, σ_γ) to zero.

We can exprim r as a correlated random variable, which simplifies (4) to:

$$r = \mathbb{1}_{I,1} \otimes \overline{r}(\theta) + \mathcal{W} \quad \text{where} \quad \mathcal{W} \sim \mathcal{G}(0, \Phi) \tag{5}$$

With Φ the covariance matrix of $r(\theta)$:

$$\begin{aligned}
\Phi(\theta) =& \mathbb{1}_{I,1} \otimes \left((\sigma_{a_0}^2 + \sigma_R^2)\mathbb{1}_{N,N} + \sigma_G^2 I_N + \Delta(\theta)\Delta(\theta)^T \sigma_\gamma^2 \right) \\
&+ \sigma^2 I_{NI} \qquad (6) \\
=& \mathbb{1}_{I,1} \otimes \Phi_N(\theta) + \sigma^2 I_{NI} \qquad (7)
\end{aligned}$$

Hense, Φ correspond to the matrix of uncertainties and noise from which we can calculate the likelihood.

III. Expression of CRLB

Althrough the CRLB for RSSI has already been calculated in the past [2], this section aims to infer the CRLB as a function of not only the channel model but also the model uncertainties (Φ). In the next calculus we simplified the problem to differentiate the likelihood, by assuming $\sigma_\gamma = 0$ so that Φ is constant with respect to θ, simplifying the diferentiation of Φ, which is equivalent to say we are calibrating γ. Calculations without this simplification is a subject for future research.

A. Generic room and APs disposition

First, we will consider a generic AP disposition. From (5), the Fisher information matrix is the following:

$$\mathcal{I}(\theta) = E\left[J_{\mathcal{L}}^T(\theta|r) J_{\mathcal{L}}(\theta|r)\right] = \frac{\partial \overline{r}(\theta)}{\partial \theta}^T \Phi^{-1} \frac{\partial \overline{r}(\theta)}{\partial \theta} \tag{8}$$

With $J_{\mathcal{L}}^T(\theta|r)$ the jacobian of the likelihood function. Using the Moore-Penrose [6] pseudo inverse of $\frac{\partial \overline{r}(\theta)}{\partial \theta}$ (noted $\left(\frac{\partial \overline{r}(\theta)}{\partial \theta}\right)^+$) and (7) comes the CRLB:

$$\begin{aligned}
\mathcal{I}(\theta)^{-1} =& \left(\sigma_G^2 + \frac{\sigma^2}{I}\right) \left(\frac{\partial \overline{r}(\theta)}{\partial \theta}^T \frac{\partial \overline{r}(\theta)}{\partial \theta} \right)^{-1} \\
&+ (\sigma_{a_0}^2 + \sigma_R^2) \left(\frac{\partial \overline{r}(\theta)}{\partial \theta}\right)^+ \mathbb{1}_{N,N} \left(\frac{\partial \overline{r}(\theta)}{\partial \theta}\right)^{+T}
\end{aligned}$$

As we are interested in ϵ the distance error, we will approach it using the trace of the CRLB:

$\epsilon^2 \geq \mathrm{Tr}\left(\mathcal{I}(\theta)^{-1}\right)$. This can be calculated using the expression of $\frac{\partial \overline{r}(\theta)}{\partial \theta}$:

$$\begin{aligned}
\epsilon^2 \geq \left(\frac{\ln(10)}{10\gamma}\right)^2 \Bigg[& \left(\sigma_G^2 + \frac{\sigma^2}{I}\right) \frac{\mathrm{Tr}(\alpha)}{\det(\alpha)} \\
&+ (\sigma_{a_0}^2 + \sigma_R^2) \frac{\mathrm{Tr}(\alpha\beta^T\beta\alpha)}{\det(\alpha)^2} \Bigg]
\end{aligned} \tag{9}$$

With:

$$\alpha = \left(\frac{\partial \overline{r}(\theta)}{\partial \theta}^T \frac{\partial \overline{r}(\theta)}{\partial \theta}\right) = \sum_{n=0}^{N-1} \frac{1}{d_n^4(\theta)} \begin{bmatrix} \Delta x_n^2 & \Delta x_n \Delta y_n \\ \Delta x_n \Delta y_n & \Delta y_n^2 \end{bmatrix}$$
$$\beta = \sum_{n=0}^{N-1} \frac{\begin{bmatrix} \Delta x_n & \Delta y_n \end{bmatrix}}{d_n^2(x,y)}$$
$$\begin{bmatrix} \Delta x_n & \Delta y_n \end{bmatrix} = \begin{bmatrix} (x - x_n) & (y - y_n) \end{bmatrix}$$

Here, α and β only depend on our position in the room and on the APs placement.

B. Application to an infinite grid

It may be useful to see how the CRLB is related to the density of APs. Therefore, we will here introduce a "density" parameter to the previously calculated expression (9). Let's consider an infinite room with a grid of APs spaced of Δ meters. As the problem becomes symetric and repeat every Δ, we consider the receiver's coordinates relative inside a square formed by it's closest access points: δx and $\delta y \in [0, 1]$, N_x and N_y are the indexes of the bottom left access-point of the square.

Hense the distance to each access point Δx_n can be rewritten in function of Δ:

$$\Delta x_n = \Delta \times (\delta x + N_x + n) \tag{10}$$
$$\Delta y_n = \Delta \times (\delta y + N_y + n) \tag{11}$$

Which makes the following changes in the previous equations:

$$\alpha = \tfrac{1}{\Delta^2}\alpha' \quad \beta = \tfrac{1}{\Delta}\beta' \quad \det(\alpha) = \tfrac{1}{\Delta^4} det(\alpha') \tag{12}$$

The final form of the CRLB becomes the following:

$$\epsilon^2 \geq \frac{\Delta^2}{\gamma^2} \frac{\ln(10)^2}{100} \left[\left(\sigma_G^2 + \frac{\sigma^2}{I}\right) \frac{\mathrm{Tr}(\alpha')}{\det(\alpha')} + (\sigma_{a_0}^2 + \sigma_R^2) \frac{\mathrm{Tr}(\alpha' \beta'^T \beta' \alpha')}{\det(\alpha')^2} \right] \tag{13}$$

IV. Numerical application

A. Experimental setup

The measuring device we used are some fflyTrack [7]: BLE beacons with 2.4GHz and an additional 868 MHz transceiver (used to forward measured BLE RSSI values to a computer).

We made our experiments in three places: a mechanical workshop (36 meters length squared building with industrial grade machines lined every 7 meters, full of metal frames which has been chosen because of its likeliness with industrial warehouses, Fig. 2), a 40 by 20 meters empty gym (chosen to limit the multipath and obstacles fading) and a meeting office of 11 by 6 meters.

Eight APs were placed around the room and one in the middle. A total of 100 measurements for each AP were made approximately each 2 meters (depending on the floor marking) (see Fig. 2) and the median value of those measurements was retained to obtain a vector of 9 RSSI for each measuring point.

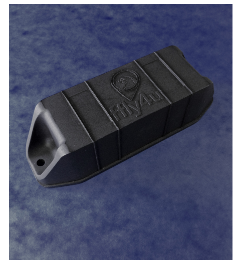

Fig. 1: Photography of the device we used

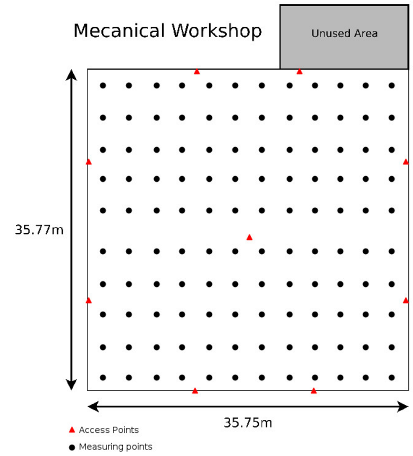

Fig. 2: Schematic of the workshop and measuring points

Using 16 devices, we made multiple measurements in 3 different rooms in various conditions: device mounted on a gaz container, on a wooden stick on laying on a table.

We measured each time the distance to each AP and the RSSI. We found the following results:

$$G = R \approx \mathcal{N}(0, 3.54) \quad a_0 \approx \mathcal{N}(51.76, 2.27)$$
$$\gamma \approx \mathcal{N}(1.39, 0.12) \quad \sigma \approx 4.04$$

A numerical application gives the results detailled in Table I

Delta	Rel. Coord.	I	ϵ_1	ϵ_2	ϵ_3
5	(0.5, 0.5)	1	2.13	0.48	0.48
5	(0.5, 0.5)	20	2.08	0.11	0.11
5	(0.1, 0.5)	1	3.14	1.95	1.16
5	(0.1, 0.5)	20	3.09	1.87	1.01
10	(0.5, 0.5)	1	4.26	0.96	0.96
10	(0.5, 0.5)	20	4.15	0.21	0.21
10	(0.1, 0.5)	1	6.27	3.89	2.31
10	(0.1, 0.5)	20	6.17	3.73	2.03
20	(0.5, 0.5)	1	8.51	1.92	1.92
20	(0.5, 0.5)	20	8.30	0.43	0.43
20	(0.1, 0.5)	1	12.54	7.79	4.62
20	(0.1, 0.5)	20	12.35	7.47	4.05

TABLE I: CRLB distance error where ϵ_1 is without any calibration, ϵ_2 is calibrating APs ($\sigma_G = 0$) and ϵ_2 is calibrating APs and receiver ($\sigma_G = \sigma_R = 0$). Coordinates are δx and δy: relatives to the meshing

We can see that if we do not calibrate the APs, the error is of the same magnitude order than the meshing Δ, meaning the positionning is only cellular. If we want a smaller precision than a cellular setup, we necessarily have to calibrate APs.

On the other hand, calibrating the receiver has a moderated impact on the error, depending on the position in the room (impact is null at the center of the room, more generally when distances to each AP are equal), thus depending on the cost of this operation and the precision desired, it may be avoided by industrials.

We verified our calculus using a simulation to get a hundred likelihood estimates for different Δ and I values, comparing the simulated covariance matrix and the analytical values from (9) in several points of the room.

The 2D covariance matrix was plotted using a 2-sigma ellipsis, the result for $\Delta = 10m$ is shown on figure Fig. 3.

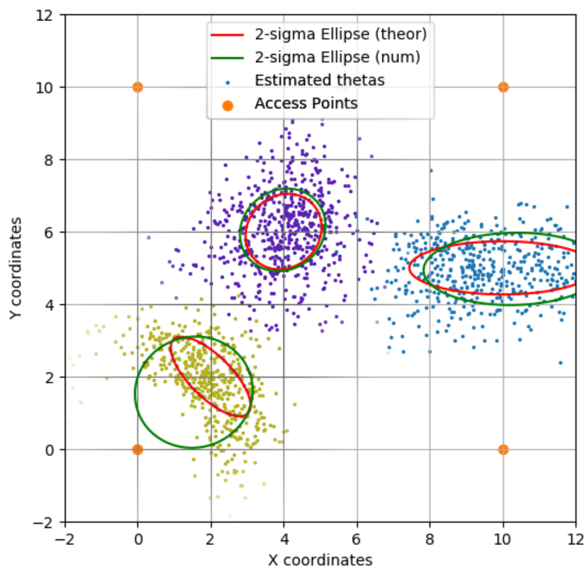

Fig. 3: A map showing the CRLB on three points with the equivalent numerically computed covariance matrix plotted in 2-sigma

We can see the CRLB is matching the numerically computed covariance matrix as it is always smaller, but close to the APs where the path loss model is less linear, it looses meaning and coherence relatively to the simulates values.

V. Conclusion

In this paper we give an analytical expression (cf (9)) of the CRLB in function of the uncertainties on a LDPL. We verified numerically our expression using a simulation and real RSSI measurements from three configurations in three rooms. This shown several points:

- It is needed to calibrate APs gains to achieve a positionning more precise than a cellular (also called geofencing) positionning.
- Calibration of the receiver gain has a moderated impact on the global precision
- Increasing the number of measurements only minimizes the error relative to measurements, thus the error won't converge to zero by measuring more RSSIs

Those results still need to be extended by adding the path loss exponent uncertainty (σ_γ), but can already lead to more applications such as an amelioration of existing APs optimal position finding algorithms [8] to find the best configuration minimizing the average CRLB.

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