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Article

# Shading Ratio Impact on Photovoltaic Modules and Correlation with Shading Patterns

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**Abstract:** This paper presents the study of a simplified approach to model and analyze the performance of partially shaded photovoltaic modules using the shading ratio. This approach integrates the characteristics of shaded area and shadow opacity into the photovoltaic cell model. The studied methodology is intended to improve the description of shaded photovoltaic systems by specifying an experimental procedure to quantify the shadow impact. Furthermore, with the help of image processing, the analysis of the shading ratio provides a set of rules useful for predicting the current–voltage behavior and the maximum power points of shaded photovoltaic modules. This correlation of the shading ratio and shading patterns can contribute to the supervision of actual photovoltaic installations. The experimental results validate the proposed approach in monocrystalline and polycrystalline technologies of solar panels.

**Keywords:** partial shading; photo-generated current; photovoltaic performance; maximum power point; image processing.

## 1. Introduction

Nowadays, the integration of photovoltaic (PV) systems into electrical grids is becoming increasingly widespread as a promising alternative distributed-energy resource [1]. Their ease of installation and adaptability have encouraged their integration into urban-area and rural-area energy grids. However, shadows from surrounding structures affect the PV installations, which causes power losses and structural failures [2][3]. Several authors have therefore developed modeling approaches to better understand the impact of shadows on PV systems [4][5][6]. Despite these important contributions, the observed behavior and harmful conditions suggest the need for improving shadow impact quantification [7][8]. Indeed, innovative modeling and supervision approaches are required to better understand and prevent the production losses in PV systems [9][10]. Also, innovative approaches can improve the design of power converters and control strategies to reduce the shadow impact [11][12]. As a result, the development of novel methods to quantify and supervise the shadow impact is currently an important issue for improving PV system performance [13][14].

The previously mentioned research area relies on reverse-bias behavior of shaded PV-cells. A widely accepted model was presented by Bishop for describing the shaded PV-cell behavior in reverse-bias [15]. Quaschnig *et al.* extended the model proposed by Bishop to the two-diodes model [16]. Kawamura *et al.* simulated the previous Bishop model while considering shadow transmittance in order to study the corresponding I–V characteristics [17]. In order to obtain a more dynamic model, Guo *et al.* investigated the influence of moving shadows on the PV-power characteristics [18]. Afterwards, Olalla *et al.* simulated large PV systems with high granularity using diffuse irradiance to model the partial shaded effects [19]. In addition, Díaz *et al.* proposed a generalized and simplified model while

34 considering the shadow geometry [20].  
35 For the study of the shading ratio, Silvestre *et al.* extend the Bishop model to analyze the performance  
36 of PV modules [21]. Jung *et al.* proposed a mathematical model for the output characteristics of a  
37 photovoltaic module including three key factors and the photo-current for a different shading ratios  
38 [22]. In Reference [23], Yong *et al.* presents a non-disruptive cell-level characterization of a photovoltaic  
39 module extracting the shunt resistances and the short-circuit currents of individual cells by using a  
40 partial shading technique with two different shading ratios. He *et al.* study the hot-spot issues in  
41 a PV module in different numbers of PV-cells using several shading ratio scenarios [24]. The work  
42 presented in Reference [25] develops a simulation and modeling of PV modules performance under  
43 partial shading for several shadow rates testing single cells in PV modules to analyze the influence of  
44 the shadow rate on the most important PV module parameters.  
45 As shown through this brief historical background, the researchers have progressively developed more  
46 detailed and extensive approaches to describe the shaded PV system behavior and the influence of  
47 the shading ratio. However, research on evolutionary PV installations currently requires accurate but  
48 simplified analysis given the variable nature of shadows in real-world applications [26][27][28].  
49 In this context, our work proposes a more accurate definition of the shading ratio and an innovative  
50 experimental set-up to integrate the shadow properties into the shaded PV model. This work includes  
51 the analysis of the shading ratio to quantify the shadow impact on PV installations. This shading  
52 ratio associates the shadow characteristics of the shaded area and the shading factor. Furthermore,  
53 with the help of image processing methods, the proposed approach adds a novel experimental  
54 set-up to analyze and supervise the shadow impact using the shading ratio. This analysis provides  
55 a set of rules useful for predicting the current–voltage behavior of shaded photovoltaic modules.  
56 Additionally, the correlation between the shading ratio and the shadow image patterns allowed for  
57 developing a simplified expression to localize the maximum power points (MPPs) in actual shaded  
58 conditions. Finally, these correlations were experimentally validated, which provides fundamentals  
59 for the applications of image processing methods to quantify and supervise the shadow impact on PV  
60 installations. Figure 1 outlines the methodology that uses the shading ratio and image processing.  
61 This paper is organized as follows. Section II presents the modeling background. Section III describes  
62 the proposed approach. In section IV, simulations of shaded PV modules are analyzed. Section V  
63 explains in detail the experimental setup for validating and correlating the proposed approach with  
64 shadow image patterns. Finally, experimental results are discussed.

## 65 2. Photovoltaic Model for Shaded Conditions

66 Shaded PV modules have a high risk of structural failures and a high risk of losing power  
67 production. Several authors have studied this behavior at level of PV-cells [15][28]. From proposed  
68 models, the approach presented by bishop has allowed for a suitable agreement with controlled  
69 tests [15]. However, the complex nature of the shading phenomenon has demonstrated the need for  
70 complementing these approaches [29]. This section describes the shaded PV behavior and current  
71 modeling methods.

### 72 2.1. Shaded PV Modules and Modeling Background

73 A typical partially shaded setup is used for the test in this study, which is shown in Figure 2.  
74 This experimental shading test was performed on 14 February, 2017 in sunny weather. The ambient  
75 temperature was 15°C and the global solar irradiation in the horizontal plane was 910 W/m<sup>2</sup> at 1:00pm.  
76 The experimental results in Figure 2 illustrate the drastic impact on the I–V and P–V curves. The partial  
77 shadows can produce multiple maximum power points (MPPs). In addition, studies have shown that  
78 these partial shadows can lead to overheating and hot-spot issues [3][30].

79 Several authors have studied this shaded behavior. Bishop presents a model for the reverse-bias  
80 characteristics of shaded solar cells based on previous works regarding the avalanche breakdown  
81 theory [15]. The authors propose a numerical simulation [16] and then the authors investigated the

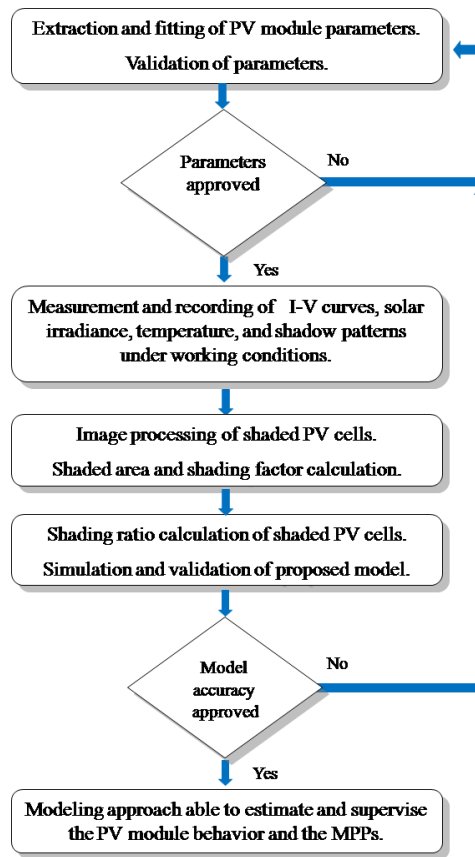


Figure 1. Modeling methodology using the shading ratio and image processing.

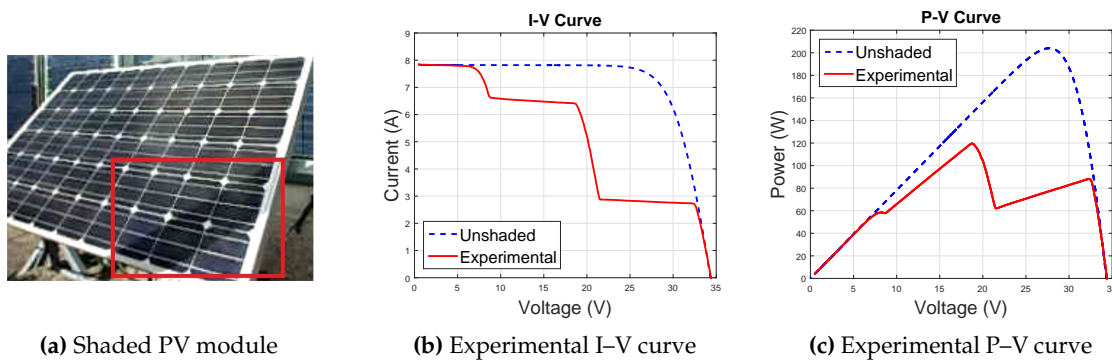


Figure 2. PV module under partially shaded conditions.

82 I-V characteristic under shadow conditions [17]. The work presented an alternative model for various  
 83 types of PV-cells [31]. The study in Reference [21] describes the PV performance in relation with the  
 84 shadow rate. Thermal stability and hot-spot risks are studied in Reference [32]. The work in Reference  
 85 [18] outlines a study of the shadow movement influence. For shaded PV installations, a discrete I-V  
 86 model is presented [20]. Other studies have correlated the shaded impact with PV power production  
 87 [33][34]. In Reference [35], authors deal with shaded PV installations in urban environments using  
 88 3D modeling. A simplified method is presented in Reference [9] for simulating the output power  
 89 of shaded PV systems. However, the complex nature of the shading phenomenon suggests that

90 proposed approaches can be extended to improving the PV module performance [26]. As a first step,  
91 the following section describes the model proposed by Bishop at the level of shaded PV-cells.

## 92 2.2. Shaded PV-Cell Model

93 This section describes the approach proposed by Bishop to model shaded PV-cells in PV modules  
94 because of the granularity and the scalability of PV systems [15]. Under shaded conditions, the  
95 PV-cells can be forced to carry current in reverse bias. As such, a negative voltage appears at the  
96 PV-cell terminals and causes dangerous reverse current to increase [32]. Bishop explains this current  
97 multiplication effect through Equation (1) by modeling shaded PV-cells using a non-linear multiplier  
98 factor.

$$I = I_{ph} - I_0 \left[ e^{\left(\frac{V_c + IR_s}{V_t}\right)} - 1 \right] - \frac{V_c + IR_s}{R_p} \left[ 1 + k \left( 1 - \frac{V_c + IR_s}{V_b} \right)^{-n} \right] \quad (1)$$

99 Equation (1) represents the relation between the PV-cell current  $I$  and the PV-cell voltage  $V_c$  [15].  
100 Where  $R_s$  is the series resistance associated with conductive losses and  $R_p$  is the shunt resistance  
101 associated with distributed losses inside of the p-n material.  $I_0$  is defined as the inverse saturation  
102 current and  $V_t$  is the thermal voltage [5]. In the non-linear multiplier factor,  $k$  is the fraction of current  
103 involved in avalanche breakdown,  $V_b$  the breakdown voltage, and  $n$  is the avalanche breakdown  
104 exponent.  $I_{ph}$  is the photo-generated current given by Equation (2).

$$I_{ph} = [I_{sc\_STC} + (C_{Ti} (T_c - T_{STC}))] \frac{G_i}{G_{STC}} \quad (2)$$

105 where  $G_i$  is the incident irradiance,  $C_{Ti}$  is the thermal current coefficient, and  $T_c$  is the cell  
106 temperature.  $I_{sc\_STC}$ ,  $T_{STC}$ ,  $G_{STC}$  are the short-circuit current, the cell temperature, and the incident  
107 irradiance for Standard Test Conditions (25°C, 1000W/m<sup>2</sup>), respectively. Equation (2) becomes the  
108 expression for the totally illuminated photo-generated current  $I_{ph_{Ti}}$  when  $G_i$  is the incident irradiance  
109 on the totally unshaded cells.

110 The model proposed by Bishop is able to describe the PV-cell behavior for completely unshaded and  
111 shaded conditions [15]. However, this model disregards the geometric and the optical properties of  
112 partial shadows, which can lead to significant loss of accuracy. Indeed, the photo-generated current  
113 in Equation (1) depends on a uniform irradiance and partial shading is not discussed by Bishop in  
114 Reference [15]. Some authors have extended the scope of this model to consider shadow properties  
115 [20][36]. However, experimental methods to quantify these shadow properties are less widespread in  
116 the literature because of shadow complexity [29]. The next section describes the proposed approach  
117 for calculating partially shaded PV modules when considering quantifiable shadow characteristics.

## 118 3. Proposed Approach for Partially Shaded PV Modules

119 The previous section described a widespread approach to model shaded PV Modules. However,  
120 experimental results have shown that this approach can lose accuracy under actual partially shaded  
121 conditions [5]. Given the complex nature of the shading phenomenon, the shadow analysis requires the  
122 inclusion of the shadow properties without increasing the computational effort due to the scalability  
123 of PV systems. These concerns have encouraged the development of the proposed approach through  
124 this section.

### 125 3.1. Partially Shaded PV-Cell Model

126 Figure 3 shows that in a PV module the partially shaded cells have two main shadow features.  
127 The first feature is the shadow geometry represented by  $a_s + a_i = 1$  where  $a_s$  is the fraction of shaded  
128 cell area and  $a_i$  is the fraction of illuminated cell area. The second shadow feature includes the optical  
129 properties of the solar irradiance on the PV module represented by the shadow transmittance  $\tau$  and the

130 shading factor  $S_f$ . The shadow transmittance  $\tau$  is defined by the ratio between the scattered irradiance  
 131  $G_s$  on the shadow and the incident irradiance  $G_i$  where  $\tau = G_s/G_i$  [18].  $\tau = 0$  means that all the  
 132 available irradiance is blocked in the interest region. In contrast,  $\tau = 1$  means that all the available  
 133 irradiance shines on the interest region because the scattered irradiance becomes  $G_s = G_i$ . The shading  
 134 factor  $S_f$  is defined in Equation (3) to describe the shadow opacity [37],

$$S_f = 1 - \frac{G_s}{G_i} \quad (3)$$

135 where  $0 \leq S_f \leq 1$ .  $S_f = 0$  means that the available irradiance shines on the interest region. In  
 136 contrast,  $S_f = 1$  means that all available irradiance is blocked in the interest region. Then, the relation  
 137 between  $\tau$  and  $S_f$  is given by Equation (4),

$$S_f + \tau = 1 \quad (4)$$

138 Physical meaning of Equation (4) shows that the shadow parameters of shading factor  $S_f$  and  
 139 shadow transmittance  $\tau$  are complementary. For instance, a totally shaded PV-cell ( $a_s = 1$ ) with a  
 140 shading factor  $S_f = 0.8$  means that only the 20% of the available irradiance achieves the PV-cell surface  
 141 which represents a shadow transmittance  $\tau = 0.2$ .

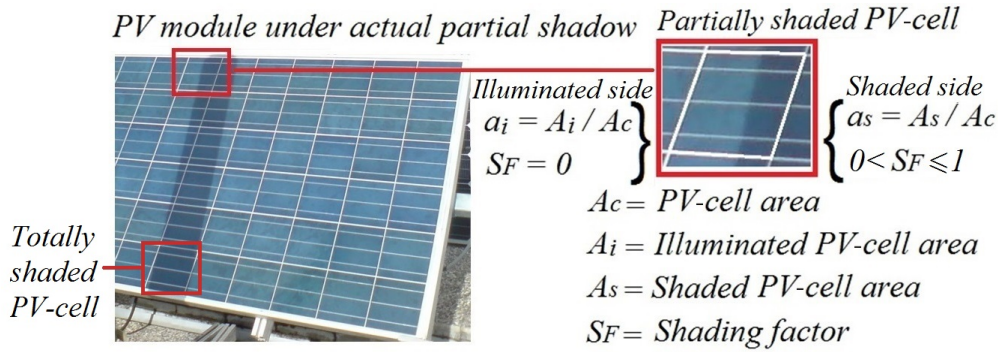


Figure 3. Partially shaded PV-cell.

142 Figure 4a shows a 3D schematic section of a partially shaded PV-cell. In Figure 4,  $I_{phi_i}$  and  $I_{phi_s}$   
 143 represent the photo-generated currents in the illuminated and shaded areas.  $I_{ph_T}$  defined as the total  
 144 photo-generated current. As shown in Figure 4a, electron-hole pairs are generated when photons  
 145 arrive at the p-n junction in the illuminated area. As a result, a photo-generated current  $I_{phi_i}$  is produced  
 146 in the illuminated area. In contrast, fewer photons can arrive to the p-n junction in the shaded area,  
 147 which produces lower photo-generated current  $I_{phi_s}$  in the shaded area. Therefore, using a simplified  
 148 approach, the total photo-generated current  $I_{ph_T}$  depends on contributions of shaded and unshaded  
 149 areas, which is defined in Equation (5). Figure 4b shows the equivalent circuit for the photo-generated  
 150 currents [29].

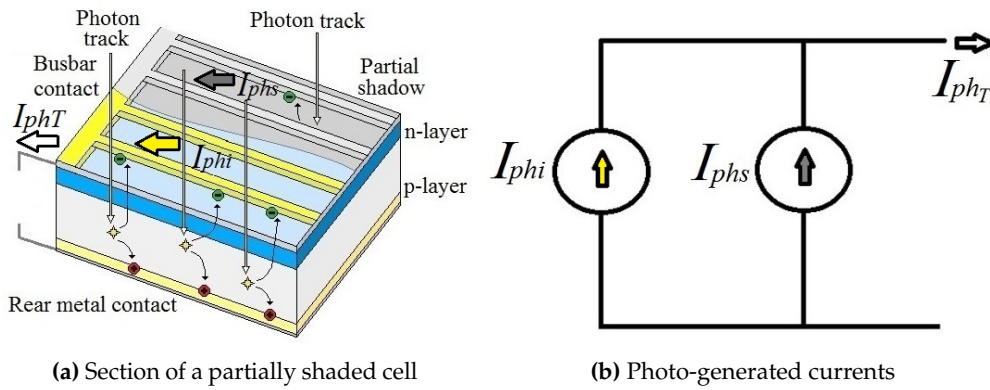
$$I_{ph_T} = I_{phi_i} + I_{phi_s} \quad (5)$$

151 Using the current density definition  $J = I/A$  for linking the electrical characteristics and the  
 152 shadow geometric, we obtain Equation (6).

$$I_{ph_T} = J_{phi_i} A_i + J_{phi_s} A_s = J_{phi_i} a_i A_c + J_{phi_s} a_s A_c \quad (6)$$

153 Considering the relation between the illuminated and shaded current densities given by the  
 154 shadow transmittance,  $J_{phi_s} = \tau J_{phi_i}$ ,

$$I_{ph_T} = J_{phi_i} A_c (a_i + \tau a_s) \quad (7)$$



**Figure 4.** Photo-generated currents in a partially shaded PV-cell.

155 as described previously  $S_f + \tau = 1$  and  $a_s + a_i = 1$ . Thus,

$$I_{ph_T} = J_{phi} A_c (1 - a_s S_f) \quad (8)$$

156 Given that  $J_{phi}$  represents the photo-generated current produced per unit cell area in the  
 157 illuminated side and  $A_c$  defined as the total PV-cell area, the factor  $J_{phi} A_c$  can be interpreted as the  
 158 photo-generated current  $I_{ph_{Ti}}$  that should be provided by the PV-cell in totally illuminated conditions .  
 159 Therefore, Equation (7) can be rewritten as seen below.

$$I_{ph_T} = I_{ph_{Ti}} (1 - a_s S_f) \quad (9)$$

160 The physical meaning of Equation (9) represents that the total photo-generated current  $I_{ph_T}$  is  
 161 proportional to the totally illuminated photo-generated current  $I_{ph_{Ti}}$  given a ratio that depends on  
 162 the shadow properties [29]. Equation (9) shows that the total photo-generated current depends on  
 163 the shaded area percentage  $a_s$  and the shadow opacity  $S_f$  but is independent of the shadow shape.  
 164 Defining this relation by the shading ratio  $\delta$ , Equation (10) is obtained.

$$\delta = 1 - a_s S_f \quad (10)$$

165 Thus, the total photo-generated current  $I_{ph_T}$  is given through Equation (11). In the  $I_{ph_T}$  expression,  
 166 the totally illuminated photo-generated current  $I_{ph_{Ti}}$  is evaluated using Equation (12) and considering  
 167  $G_i$  as the incident irradiance in totally unshaded conditions, which was clarified previously in Equation  
 168 (2).

$$I_{ph_T} = I_{ph_{Ti}} \delta \quad (11)$$

$$I_{ph_{Ti}} = [I_{sc\_STC} + (C_{Ti} (T_c - T_{STC}))] \frac{G_i}{G_{STC}} \quad (12)$$

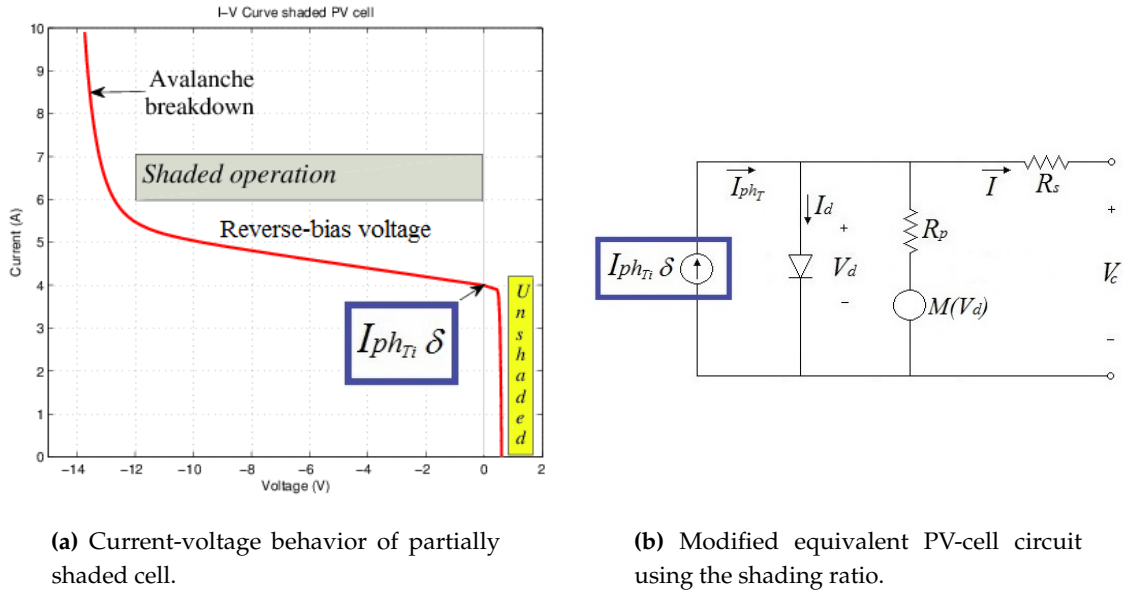
169 In addition, Equation (13) is defined by considering  $I_{sc_{Ti}}$  as the totally illuminated short-circuit  
 170 current for unshaded cell conditions.

$$I_{ph_{Ti}} \approx I_{sc_{Ti}} \quad (13)$$

171 Thus,

$$I_{ph_T} \approx I_{sc_{Ti}} \delta \quad (14)$$

172 We propose extending the model presented by Bishop [15] while using  $I_{ph_T}$  for reformulating  
 173 Equation (1) and Equation (15). At this point, it is important to highlight that the shading ratio  $\delta$   
 174 depends on the quantifiable parameters  $a_s$  and  $S_f$ . Therefore,  $\delta$  is also quantifiable. Figure 5a outlines  
 175 the current-voltage behavior of a shaded PV-cell according to Equation (15). The equivalent PV-cell  
 176 circuit is shown in Figure 5b. This simplified  $\delta$  factor improves the description scope of shaded PV  
 177 systems including measurable shadow features without needing to increase the computational effort.



**Figure 5.** Equivalent circuit and I-V curve of partially shaded PV-cell.

$$I = \underbrace{I_{ph_T} \delta}_{I_{ph}} - I_0 \left[ e^{\left( \frac{V_c + IR_s}{V_t} \right)} - 1 \right] - \frac{V_c + IR_s}{R_p} \left[ 1 + k \left( 1 - \frac{V_c + IR_s}{V_b} \right)^{-n} \right] \quad (15)$$

178 The Equation (15) is a non-linear equation which can be solved using numerical methods. The  
 179 numerical method usually employs to solve this type of equations is the Newton-Raphson method [4].  
 180 The method starts with a function  $f(V_c)$  defined as  $f(V_c) = 0$  as rewritten below in Equation (16),

$$f(V_c) = I_{ph_T} \delta - I - I_0 \left[ e^{\left( \frac{V_c + IR_s}{V_t} \right)} - 1 \right] - \frac{V_c + IR_s}{R_p} \left[ 1 + k \left( 1 - \frac{V_c + IR_s}{V_b} \right)^{-n} \right] \quad (16)$$

181 Given that the function satisfies the condition  $f'(V_c) \neq 0$ , the following iterative process is  
 182 repeated until a sufficiently accurate value is reached.

$$V_{c_{n+1}} = V_{c_n} - \frac{f(V_{c_n})}{f'(V_{c_n})} \quad (17)$$

183 The solution of the iterative process in Equation (17) describes the PV-cell voltage given the  
 184 influence of the shading ratio  $\delta$  and a known  $I$  current. The solution of this iterative process is  
 185 performed for a range of  $I$  currents from 0 to  $I_{sc}$  and for the respective shading ratios  $\delta$  of shaded PV  
 186 cells. This method allows calculating the voltage at the PV-cell level under several working conditions.  
 187 Nevertheless, series connections of PV-cells form PV modules and it is required to go in depth about  
 188 this aspect. The following section presents a systematic perspective to analyze the influence of the  
 189 shading ratio on PV modules.

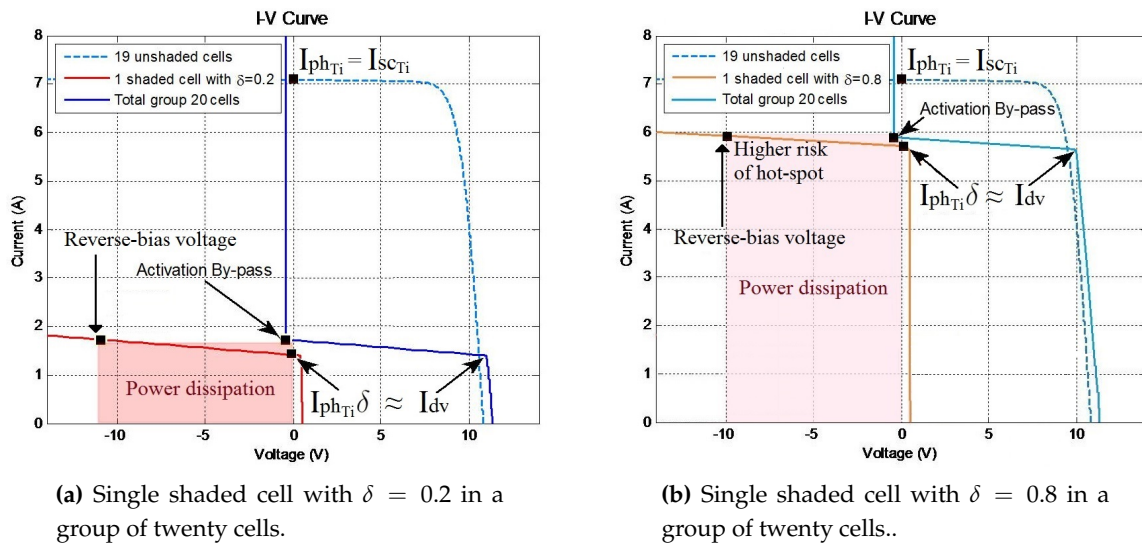


### 3.2. Influence of the Shading Ratio $\delta$ on the PV Module Behavior

This section relates the previous proposed approach with shadow patterns to extend the shadow impact analysis at the level of PV modules. Series connections of PV-cells form PV modules, and PV module manufacturing usually connect by-pass diodes to groups of PV-cells for decreasing the damage risk [30]. Thus, the voltage in a PV module  $V_p$  with  $m$  groups of  $q$  PV-cells and by-pass diode voltage  $V_{BD}$  is given by Equation (18).

$$V_p = \sum_{j=1}^m V_{G_j} \text{ where } V_{G_j} = \begin{cases} \sum_{i=1}^q V_{c_i} & \text{if } \sum_{i=1}^q V_{c_i} \geq 0 \\ V_{BD} & \text{if } \sum_{i=1}^q V_{c_i} < 0 \end{cases} \quad (18)$$

The PV-cell voltages  $V_{c_i}$  come from the solution of the non-linear Equation (16) by applying the numerical Newton-Raphson method of Equation (17). In addition, the parameters  $I_0$ ,  $R_s$ ,  $V_t$ , and  $R_p$  of Equation (16) have been extracted according to the iterative methods presented in Reference [5]. The parameters  $k$  and  $n$  of the multiplier factor proposed by Bishop in Reference [15] have been extracted using non-linear curve fitting methods from experimental results in shaded conditions with unconnected by-pass diodes. The parameter  $V_b$  depends on the PV module technology and it has been fitted according to operation regions proposed in Reference [30].



**Figure 6.** Influence of  $\delta$  in a group of twenty cells with a single shaded cell.

Solutions of Equation (15) and Equation (18) for a group of twenty cells with a single shaded cell provides the results in Figure 6. As shown in the Figure 6, a partial shadow in a single cell can change drastically the normal behavior of the group. Denoting  $I_{dv}$  as the divergence current where the I-V curve diverges of normal operation in shaded conditions given by  $\delta < 1$ , the comparison of results in Figure 6a and Figure 6b allows deducing that the behavior of  $I_{dv}$  described by Equation (19).

$$I_{dv} \approx I_{ph_{T_i}} \delta \quad \text{for } \delta < 1 \quad (19)$$

In addition, the totally illuminated short-circuit current was considered in Equation (13) as  $I_{sc_{T_i}} \approx I_{ph_{T_i}}$ . Then,

$$I_{dv} \approx I_{sc_{T_i}} \delta \quad \text{for } \delta < 1 \quad (20)$$

The Equation (19) is deduced because the voltage in the shaded PV-cell begins to be negative when the PV-cell current is higher than  $I_{ph_{T_i}} \delta$  which leads to a prominent change of the I-V curve. If

212 the PV-cell current follows increasing, the PV-cell voltage is each time more negative until achieve  
 213 the activation of the by-pass voltage. In this operation condition, the shaded PV-cell dissipates power  
 214 due to the negative voltage and risk of damage can arise. Figure 6 shows that the situation can get  
 215 worse if the shading ratio is higher because the dissipate power increases. This situation can induce to  
 216 hot-spots if the partial shadows are small and permanent. Failures of this type have been reported in  
 217 literature and require preventive actions to avoid the deterioration of the PV system performance [38].

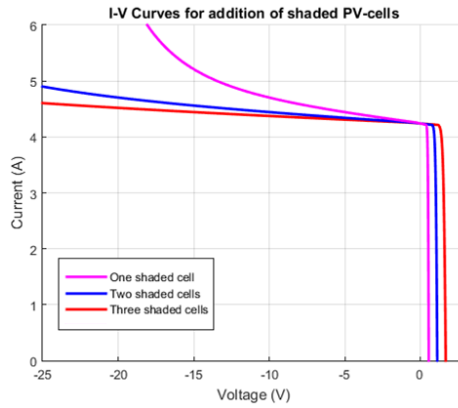


Figure 7. I-V Curves for addition of shaded PV-cells.

218 Figure 6 also allows deducing the relation between the shaded PV-cells in a group with the  
 219 lowest shading ratio. Assuming a case in which the shaded PV-cells of Figure 6a and Figure 6b are  
 220 in the same group, the lowest shading ratio of Figure 6a would lead the group toward the by-pass  
 221 activation condition. Therefore, the shading ratio of the Figure 6b would have a minimum impact in  
 222 the divergence current because the by-pass diode is already active. This operation principle can be  
 223 extended to several shaded cells with different shading ratios because the lowest shading ratio is the  
 224 first in activate the by-pass diode.

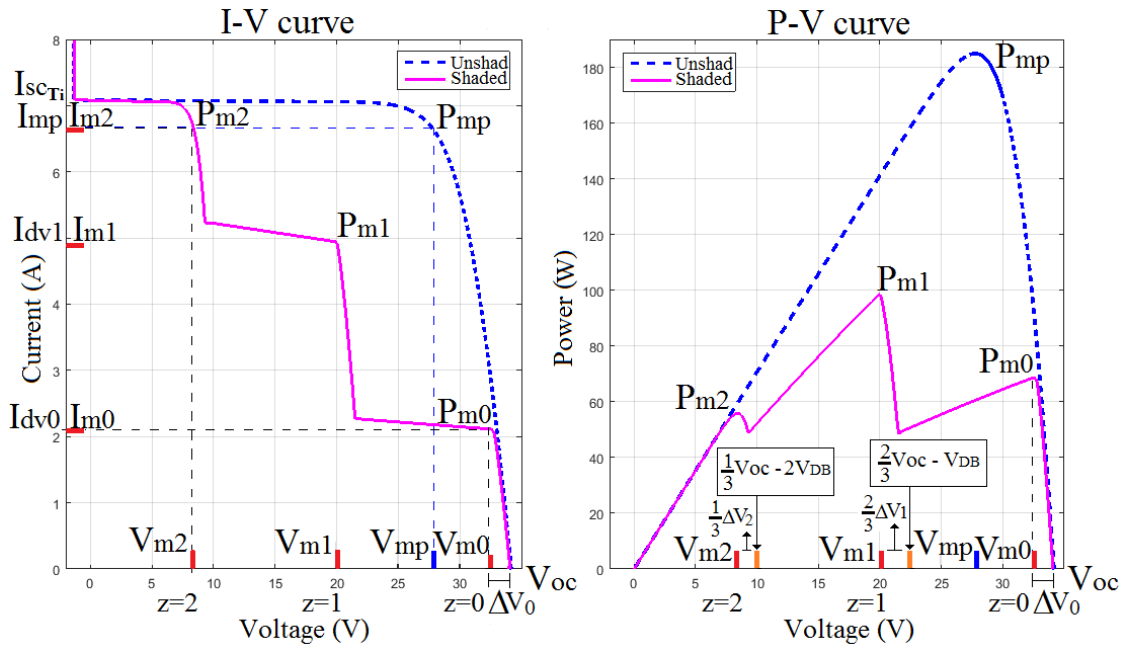


Figure 8. I-V and P-V curves for a partially shaded PV module.

225 Figure 6 shows that the reverse-bias voltage is critical for the structural healthy of the PV-module.  
 226 For that reason, in Figure 7 is depicted the I-V behavior in reverse-bias condition for several shaded

227 cells. In this case, one PV-cell has higher slope because the proximity of the breakdown voltage. In  
 228 contrast, the illustrative example of Figure 7 shows that increasing  $N$  shaded PV-cell multiplies the  
 229 negative voltage  $N$  times because the PV-cell are connected in series. Therefore, the slope in the  
 230 negative region decrease and for a given current interval  $Slope = \Delta I / (N * \Delta V)$ .

231 Figure 8 extends the analysis to several shaded PV cells in a PV module. These figures show  
 232 the interrelation between the divergence currents and the maximum power points (MPPs). As  
 233 shown in Figure 8,  $z$  represents the index for the lowest shading ratios  $\delta_z$  in each group where  
 234  $z = \{0, 1, 2, \dots, g - 1\}$  and  $g$  is the total number of groups connected in series.  $V_{mz}$  and  $I_{mz}$  are the  
 235 voltages and currents at the MPPs. The relation between the MPPs and the lowest shading ratios  $\delta_z$  is  
 236 given by the behavior of the divergence currents  $I_{divz}$  and the local MPPs. Figure 8 allows for deducing  
 237 that  $I_{divz} \approx I_{mz}$  because the MPPs arise around the current divergence. Nevertheless, an exception to  
 238 this pattern is presented in unshaded groups where  $I_{mz} \approx I_{mp}$ .  
 239  $\Delta V_z$  is defined in Equation (21) as a proportional relation between the voltage difference  $V_{oc} - V_{mp}$   
 240 and the corresponding shading ratio  $\delta_z$  for the shading ratios arranged from the lower to the higher  
 241  $\delta_z < \delta_{z+1}$ . Physical meaning of Equation (21) represents that the voltage displacement of  $V_{mz}$  in  
 242 relation to the local MPPs in unshaded condition is associated to the shading ratio  $\delta_z$ .

$$\Delta V_z \approx (V_{oc} - V_{mp}) \delta_z \quad \text{for } z = \{0, 1, \dots, g - 1\}, g = \text{number of groups, and } \delta_z < \delta_{z+1} \quad (21)$$

243 All groups are connected in series and each group proportionally contributes to the open circuit  
 244 voltage  $V_{oc}$ . For this reason, the voltage  $V_{mz}$  at the local MPPs is expressed as a fraction of  $V_{oc}$  and  $\Delta V_z$ .  
 245 For the illustrative example of the Figure 8, the expressions for the voltages  $V_{mz}$  at the local MPPs  
 246 are given from Equation (22) to Equation (24) where  $V_{BD}$  is the forward by-pass diode voltage which  
 247 displaces the proportion of  $V_{oc}$ .

$$V_{m0} = V_{oc} - \Delta V_0 \quad (22)$$

$$V_{m1} = \left(\frac{2}{3}V_{oc}\right) - \left(\frac{2}{3}\Delta V_1\right) - V_{BD} \quad (23)$$

$$V_{m2} = \left(\frac{1}{3}V_{oc}\right) - \left(\frac{1}{3}\Delta V_2\right) - 2V_{BD} \quad (24)$$

248 A general expression of  $V_{mz}$  is deduced in Equation (25) for  $g$  groups and  $z = \{0, 1, \dots, g - 1\}$ ,

$$V_{mz} = \left(\frac{g-z}{g}\right) V_{oc} - \left(\frac{g-z}{g}\right) \Delta V_z - (zV_{BD}) \quad (25)$$

249 For  $\delta_z < 1$  and  $\delta_z < \delta_{z+1}$ ,

$$P_{mz} = V_{mz} I_{mz} \approx V_{mz} I_{divz} \approx V_{mz} I_{scTi} \delta_z \quad (26)$$

$$P_{mz} \approx \left[ \left( \left( \frac{g-z}{g} \right) (V_{oc} - \Delta V_z) \right) - (zV_{BD}) \right] I_{scTi} \delta_z \quad (27)$$

250 For unshaded groups  $\delta_z = 1$  and  $P_{mz}$  is given by Equation (28),

$$P_{mz} = V_{mz} I_{mp} \quad (28)$$

$$P_{mz} \approx \left[ \left( \left( \frac{g-z}{g} \right) V_{mp} \right) - (zV_{BD}) \right] I_{mp} \quad (29)$$

Equation (27) and Equation (29) allow a fast approximation to the MPPs for known shadow patterns and unshaded operation parameters. The procedure to evaluate the MPPs is described as follows:

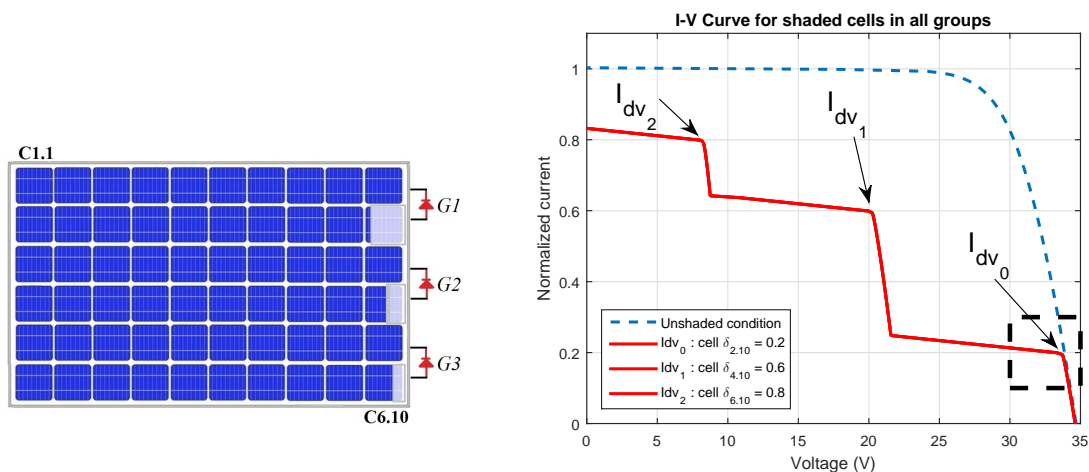
- Step 1:** Determination of the lowest shading ratios  $\delta_z$  in each group. Arrangement of shading ratios from the lower to the higher  $\delta_z < \delta_{z+1}$ .
- Step 2:** Evaluation of  $V_{mp}$ ,  $I_{mp}$ ,  $I_{scTi}$ , and  $V_{oc}$  from unshaded condition. Considering  $V_{BD} \approx 0.7V$ .
- Step 3:** Calculation of  $P_{mz}$  for  $z = \{0, 1, \dots, g - 1\}$  using Equation (27) if  $\delta_z < 1$  or Equation (29) if  $\delta_z = 1$ .
- Step 4:** \* In the special case of  $\delta_z = \delta_{z+1}$ , the sequence of values for  $P_{mz}$  and  $P_{mz+1}$  are evaluated normally. However, only the highest value of power defines the region for the local MPP.

Given the proposed modeling approach through this section, the next stage will analyze the simulation of shaded PV modules.

#### 4. Simulation Analysis of Shaded PV Modules

The cases of shadow patterns in this section have been selected to illustrate the potential features of proposed approaches in simulation. First, two cases describe the impact of single shaded cells scattered in several groups and the impact of shaded cells grouped in a single group. Then, two cases are intended to show the shadow movement impact. The final simulation targets a shaded PV string. The simulations have been performed in a conventional computational platform by solving Equation (16), Equation (17), and Equation (18) according to the lineaments presented in section 3. In addition, the simulated shading ratios  $\delta$  are set for analysis and further correlation with experimental patterns. The shaded PV-module images in this section have only a character illustrative and do not represent any software in particular.

The nominal parameters of the simulated PV modules are  $I_{sc} = 8.3A$  and  $V_{oc} = 37.3V$  with simulation conditions of incident irradiance  $G_i = 850W/m^2$  and cell temperature  $T_c = 45^{\circ}C$ . The cases consider a uniform shading factor  $S_f = 0.8$ . We also consider a conventional PV module with sixty cells distributed in groups of twenty cells connected to by-pass diodes [3]. The analysis uses a matrix notation  $ij$  where  $\delta_{ij}$  represents the shading ratio of a PV-cell in the relative position  $ij$  in a PV module.



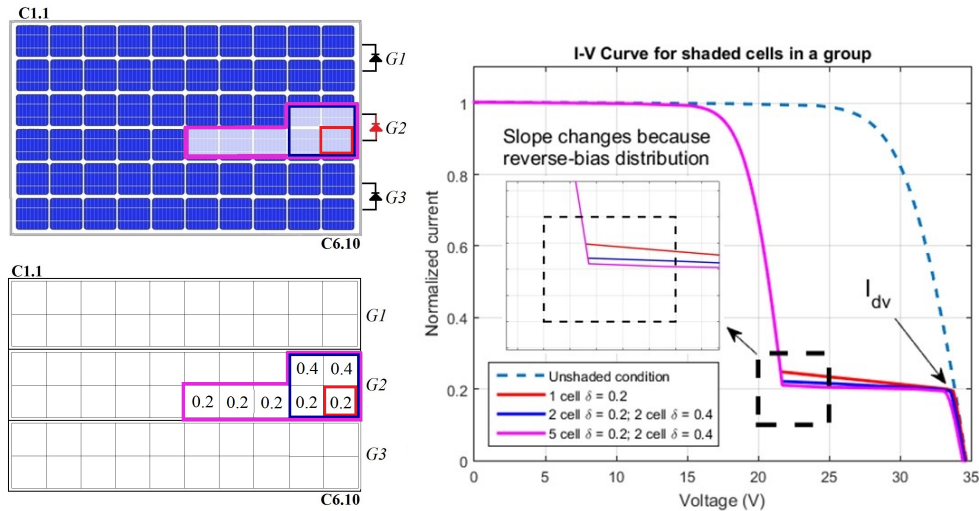
(a) Single shaded cells in all groups.  
Case:  $G1 - \delta_{2,10} = 0.2$ ,  $G2 - \delta_{4,10} = 0.6$ ,  $G3 - \delta_{6,10} = 0.8$

(b) I-V curve for single shaded cells in all groups.

**Figure 9.** Study case of shadow pattern for single shaded cells in all groups.

The first case depicted in Figure 9 shows all groups with a single shaded cell. This simulation is intended to study the impact of single shaded cells in the normal current-voltage behavior. In

279 Figure 9, the PV module current  $I_{PV}$  is normalized in ratio to  $I_{scTi} = 7.1A$ . Therefore, in the Y-axis, the  
 280  $I_{Norm} = I_{PV} / I_{scTi}$ . This simulation case shows that the lowest divergence current  $I_{dv}$  is proportional to  
 281 the shaded cells with the lowest value of  $\delta$ . For instance, the first divergence current  $I_{dv_0}$  in Figure 9b  
 282 is caused by the PV-cell with  $\delta_{2,10} = 0.20$  of group one. Figure 9 confirms that the divergence current  
 283  $I_{dvz}$  due to each group is close to  $I_{dvz} = \delta_z I_{scTi}$  where  $\delta_z$  depends on the shaded cell  $C_{ij}$  with the lowest  
 284 value of  $\delta$  in the group.



(a) Shaded PV-cells in a single group. Sub-case1 :red. Sub-case2 :blue. Sub-case3 :magenta.

(b) Simulation results. Sub-case1 :red. Sub-case2 :blue. Sub-case3 :magenta.

**Figure 10.** Representation of three sub-study cases with shadow patterns distributed in a single group.

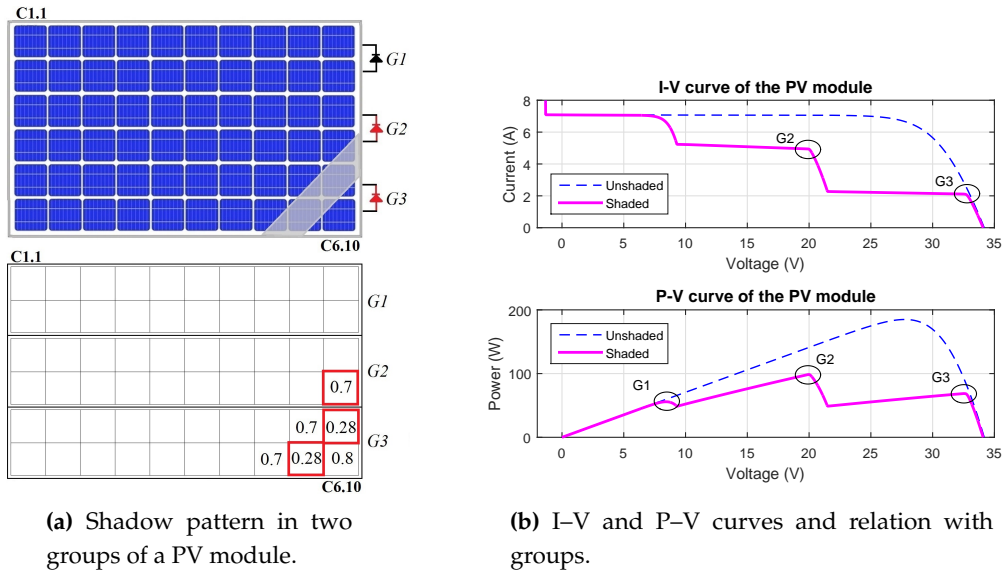
285 Figure 10 depicts three sub-cases of distributed shaded cells in a single group. As shown in Figure  
 286 10a, the first sub-case has one shaded cell with  $\delta = 0.2$ . The second sub-case has two shaded cells with  
 287  $\delta = 0.2$  and the other two cells with  $\delta = 0.4$ . The third sub-case has five cell with  $\delta = 0.2$  and the other  
 288 two cells with  $\delta = 0.4$ . The simulation results show that the lowest value of  $\delta$  in a group with several  
 289 shaded cells causes the divergence current  $I_{dv}$ . In addition, Figure 10b illustrates that a greater number  
 290 of shaded cells in a group causes a decrease in the I–V curve slope. This phenomenon is due to the  
 291 behavior of the shaded cells in the reverse-bias as described previously. As a result, a single shaded  
 292 cell in a group has higher I–V curve slope and more risk of hot-spots than a group with several shaded  
 293 cells because the reverse-bias voltage and power dissipation distribution [2][3].

294 Figure 9 and Figure 10 allowed for the analysis of single shaded PV cells and single shaded  
 295 groups. However, the shadow displacement in daily conditions can generate several irregular shadow  
 296 patterns. To describe this more realistic aspect, Figure 11 and Figure 12 illustrate two irregular shadow  
 297 patterns from a hypothetical pole, antenna, or chimney.

**Table 1.** Maximum power points - Case: diagonal shadow.

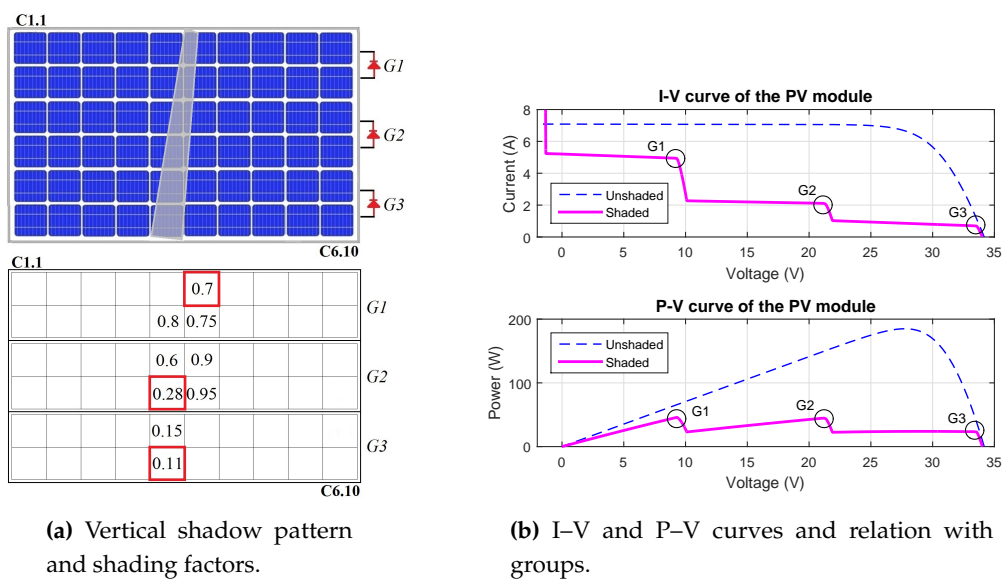
$z$	0	1	2
$\delta_z$	0.28	0.7	1.0
Group	G3	G2	G1
$P_{mz}$ approx.(W)	64.09	<b>95.02</b>	53.65
$P_{mz}$ simul.(W)	68.06	<b>98.39</b>	55.27
Rel. error	0.06	0.03	0.03

298 Figure 11a shows a diagonal shadow pattern and the associated shading ratios. In this case,  
 299 Figure 11b shows that the group three with  $\delta_{6,9} = \delta_{5,10} = 0.28$  produces the lowest divergence



**Figure 11.** I-V and P-V behavior for diagonal shadow pattern in a PV module.

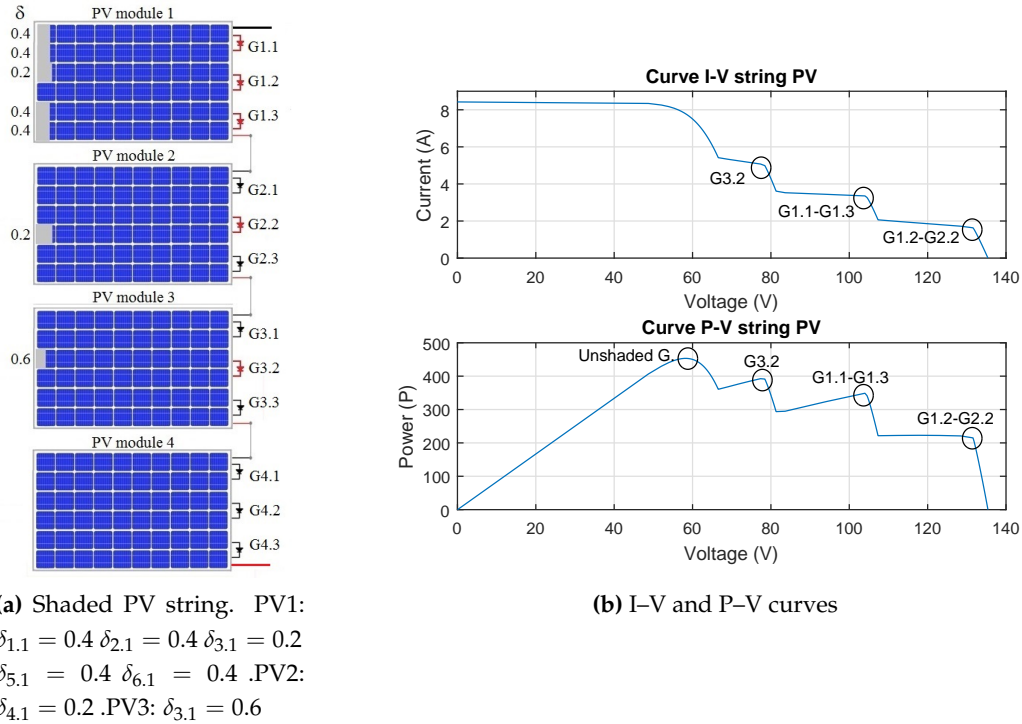
300 current  $I_{dv_0} = 0.28I_{scT_i} \approx 2A$ , and the group two with  $\delta_{4.10} = 0.7$  produces the divergence current  
 301  $I_{dv_1} = 0.7I_{scT_i} \approx 5A$ . After finding  $I_{dv_0}$  and  $I_{dv_1}$ , the maximum power points (MPPs) are calculated  
 302 using Equations (27) and Equations (29) described in section 3.2. Table 1 lists these approximate and  
 303 simulated MPPs. The results in Table 1 show a suitable agreement between actual and estimated MPPs.  
 304 This simplified method allows for quickly identifying the global MPP and its source.  
 305 Figure 12a describes a pattern in all vertical groups. This figure shows that the divergence currents  
 306 depend on the lowest  $\delta$  in each group. In addition, the other shaded cells impact the slope of the  
 307 I-V curve without relevant contribution to the  $I_{dv}$ . Table 2 shows the simplified calculation of the  
 308 approximate MPPs using Equation (27) and Equation (29) of section 3.2. These results show that the  
 309 group G1 provides the global MPPs which agrees with the simulation result. Therefore, Table 1 and  
 310 Table 2 confirm that the MPPs can be localized from the lowest  $\delta$  in each group and the parameters of  
 311 unshaded operation without an exhaustive calculation from all the shaded PV cells.



**Figure 12.** I-V and P-V behavior of vertical shadow pattern in a PV module.

**Table 2.** Maximum power points - Case: vertical shadow.

$z$	0	1	2
$\delta_z$	0.11	0.28	0.7
Group	G3	G2	G1
$P_{mz}$ approx.(W)	26.02	41.54	<b>43.05</b>
$P_{mz}$ simul.(W)	24.07	44.53	<b>45.72</b>
Rel. error	0.081	0.067	0.058

**Figure 13.** I-V and P-V curves of a shaded PV string.

312 Figure 13 depicts the final studied case at level of PV string. To facilitate understanding, this  
 313 figure highlights the most significant shaded PV cells in an irregular shadow pattern. The simulations  
 314 results allow identifying four regions. The region  $R_1$  depends on G1.2 and G2.2. In this region, PV1  
 315  $\delta_{3,1}$  and PV2  $\delta_{4,1}$  cause the lowest  $I_{dvo}$  in the PV string.  $R_1$  is extended by around 20V because the  
 316 bypass activation of two groups. G1.1 and G1.3 produce region  $R_2$ . The divergence current  $I_{dvo}$  in  $R_2$   
 317 is proportional to the 40% of  $I_{scTi}$  which is caused by PV1  $\delta_{1,1} = \delta_{2,1} = \delta_{5,1} = \delta_{6,1} = 0.4$ . Region  $R_3$   
 318 is produced by G3.2 with the single PV-cell PV3  $\delta_{3,1} = 0.6$ . The bypass activation point and the I-V  
 319 curve slope are higher in region  $R_3$ ; therefore, this single cell is more vulnerable to dissipate power  
 320 and generate hot-spots (see Figure 6b). Finally,  $R_4$  depends on the unshaded PV groups and provides  
 321 the highest MPP of all regions.

322 Table 3 lists the MPPs for the studied PV string. These results illustrate a special case of Equation (27)  
 323 and Equation (29) to evaluate the approximate MPPs where equal  $\delta_z$  appear in different groups. For  
 324 this case, the sequence of values in Equation (27) and Equation (29) are evaluated normally; however,  
 325 only the highest MPP of equal  $\delta_z$  is taking into account to define the MPP region and the global  
 326 MPP. Finally, the results in Figure 13 and Table 3 confirm that this simplified methodology provides  
 327 a suitable approximation to the MPPs at level of PV strings. The next section summarizes the main  
 328 identified findings.

**Table 3.** Maximum power points - Case: shaded string

$z$	0	1	2	3	4	5
$\delta_z$	0.2	0.2	0.4	0.4	0.6	1.0
Group	G1.2	G2.2	G1.1	G1.3	G3.2	Unsh.G.
$P_{mz}$ approx.(W)	216.1	197.1	340.5	304.2	382.5	<b>464.2</b>
$P_{mz}$ simul.(W)	220.9	-	348.8	-	391.30	<b>453.2</b>
Rel. error	0.022	-	0.024	-	0.023	0.024

#### 329 4.1. Identified Patterns between the Shading Ratio and the PV Module Behavior

330 The following findings highlight the patterns identified from the interaction between the shading  
331 ratio and the partial shadows.

- 332 • The divergence currents  $I_{dVz}$  are proportional to the lowest shading ratio  $\delta_z$  in each shaded PV  
333 group. Thus,  $I_{dVz} \approx \delta_z I_{scTi}$  for  $\delta_z < 1$ .
- 334 • Shaded cells have a minimal impact on the I–V curve if their shading ratio is greater than the  
335 lowest shading ratio in the same group.
- 336 • In a group, shaded cells with shading ratios close to the lowest shading ratio have a lower  
337 overheating risk because the reverse bias voltage is distributed between them.
- 338 • A single shaded cell in a group with higher shading ratio has a greater probability of being a  
339 hot-spot because of the power dissipation despite the by-pass diodes.
- 340 • The MPPs can be quickly identified from the lowest shading ratio in each group and the  
341 parameters for unshaded conditions.
- 342 • The above-mentioned patterns can be extended at the level of PV strings.

343 The next section presents the experimental tests to validate the proposed approach correlating the  
344 current voltage-behavior with shadow image patterns.

### 345 5. Experimental Validation and Discussion

346 This section describes the experimental setup for validating the analysis proposed in section 3. In  
347 addition, this section outlines an experimental procedure to quantify the shading ratio using image  
348 processing methods. Experimental results are discussed.

#### 349 5.1. Test for Partially Shaded PV Modules

350 The developed experiments consider two shadow cases as depicted in Figure 15 and Figure 17.  
351 Furthermore, the tests use monocrystalline and polycrystalline PV modules to compare these common  
352 commercial technologies. The PV module characteristics are listed in Table 4.

**Table 4.** PV modules under testing.

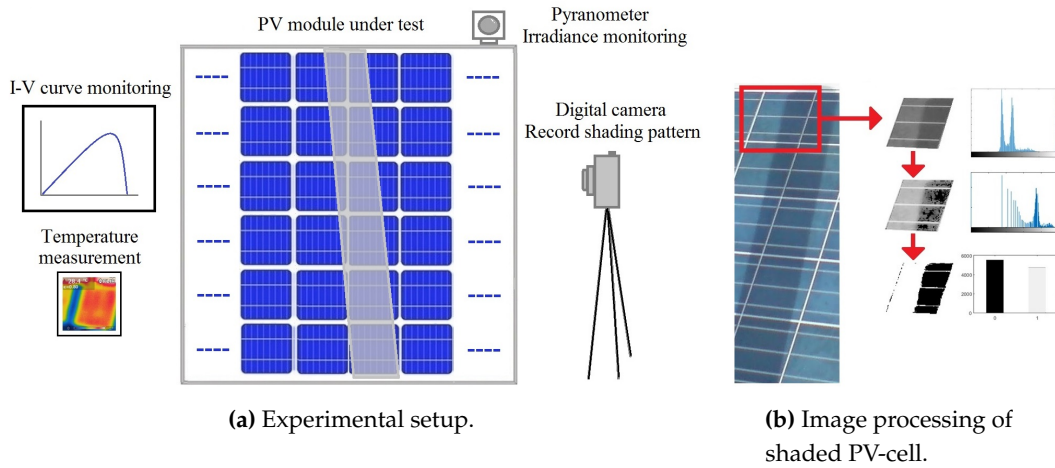
Type Reference	Monocrystalline Tenesol TE 2200	Polycrystalline Yingli solar YL290p-35b
<i>Electrical parameters at STC</i>		
Maximum Power ( $P_{mp}$ )	250 Wp	290 Wp
Voltage at $P_{mp}$ ( $V_{mp}$ )	30.3 V	35.8 V
Current at $P_{mp}$ ( $I_{mp}$ )	8.3 A	8.1 A
Open circuit voltage ( $V_{oc}$ )	37.3 V	45.3 V
Short-circuit current ( $I_{sc}$ )	8.6 A	8.62 A

353 Figure 14a depicts the experimental setup performed in the platform ADREAM of the LAAS  
354 in Toulouse, France (43°33'44.3"N 1°28'38.3"E). In this setup, an I–V curve tracer (model MP-160,  
355 EKO Instruments, Japan) is used to detect the current–voltage signals. Furthermore, a pyrometer  
356 (model SP-Lite, Kipp&Zonen, Netherland) monitors the solar irradiance and a thermographic camera



357 periodically measures the PV module temperature.  
 358 Simultaneously, a digital camera records the shadow pattern, which is shown in Figure 14b. The  
 359 analysis of the shaded PV-cell areas is performed using image processing methods after contour  
 360 selection. The selected image is converted from gray-scale image to binary image through digital  
 361 processing based on the histogram and Otsu's method [39]. Finally, the shaded area is calculated using  
 362 Equation (30) where  $p_b$  is the total number of black pixels and  $p_w$  is the total number of white pixels.

$$a_s = \frac{p_b}{p_b + p_w} \quad (30)$$



**Figure 14.** Experimental setup for model validation and image processing.

363 The experimental test is described as follows:

364 **Step 1:** Simultaneous measurements and recording of I-V curves, solar irradiance, PV module  
 365 temperature, and shadow patterns.

366 **Step 2:** Selection of synchronized I-V curves and image shadow patterns for analysis.

367 **Step 3:** Image processing for measurement of shaded PV-cell areas in selected shadow pattern.

368 **Step 4:** Shading ratio calculation for the PV-cell with the largest shaded area  $a_{sL}$  using Equation (31).  $I_{phT_i}$   
 369 is calculated using Equation (12).  $I_{dvL}$  is the first divergence-current point in the experimental  
 370 I-V curve.

371 **Step 5:** Calculate the shading factor  $S_f$  for the PV-cell with the largest shaded area  $\delta_L$  using Equation (32).  
 372 In this experimental setup, the shading factor is considered uniform on the shaded cell because  
 373 the I-V curve measurements and the shaded PV module image recording are synchronized.

374 **Step 6:** Evaluate the shading ratio  $\delta_{ij}$  for each shaded PV-cell.

375 **Step 7:** The calculated shading ratios are used to evaluate the I-V and P-V characteristics of the PV  
 376 modules.

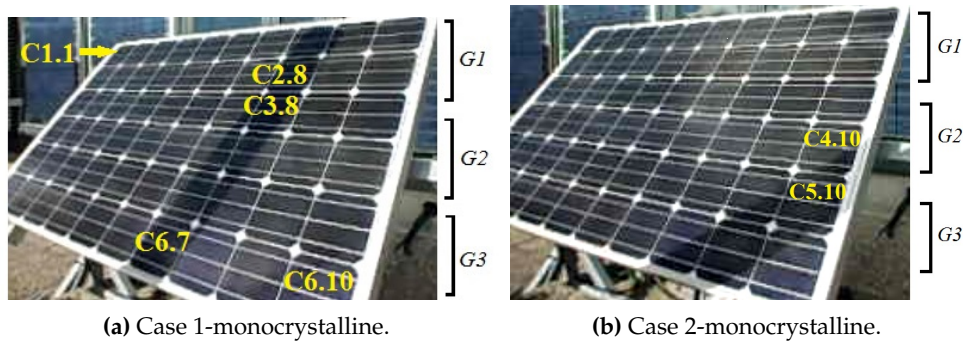
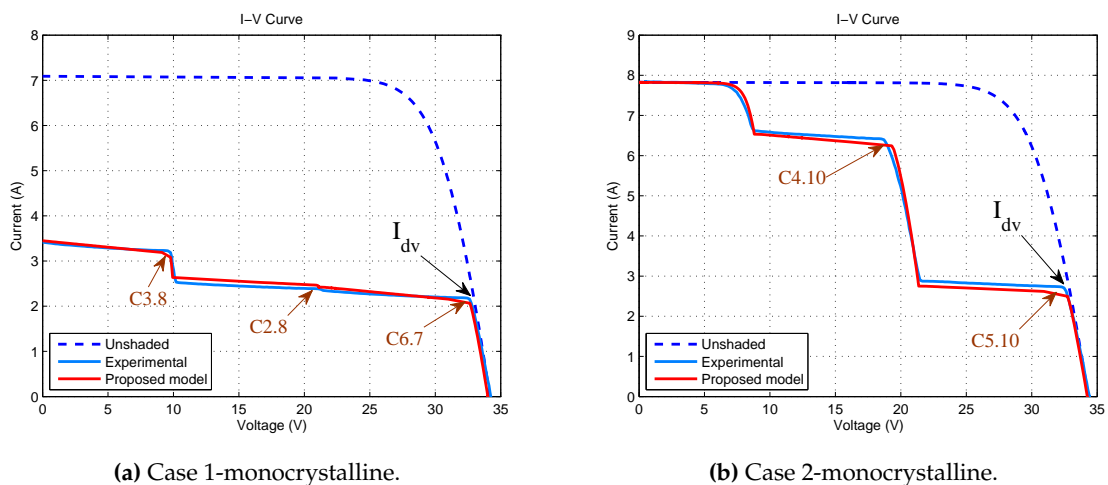
$$\delta_L = \frac{I_{dvL}}{I_{phT_i}} \quad (31)$$

$$S_f = \frac{1 - \delta_L}{a_{sL}} \quad (32)$$

377 Table 5 summarizes the parameters and values for calculating the shading factor. The shading ratios  
 378  $\delta_{ij}$  are calculated for each shaded PV-cell while considering their shaded area  $a_{ij}$  and the same shading  
 379 factor  $S_f$ . Tables 6 and 8 list the shaded cell areas  $a_{ij}$  obtained after image processing and the calculated  
 380 shading ratios  $\delta_{ij}$ . These shading ratios are used to evaluate the I-V and P-V characteristics for the PV  
 381 modules. Table 7 and Table 9 summarize the MPPs. Lastly, the experimental and calculated I-V curves  
 382 are depicted in Figure 16 and Figure 18.

**Table 5.** Shading factor results for the PV modules under testing.

Type	Monocrystalline		Polycrystalline	
	Case 1	Case 2	Case 1	Case 2
$G_i$	820 W/m <sup>2</sup>	910 W/m <sup>2</sup>	710 W/m <sup>2</sup>	540 W/m <sup>2</sup>
$T_c$	31°C	31°C	31°C	30°C
$I_{phT_i}$	7.07 A	7.85 A	6.14 A	4.67 A
$I_{dvL}$	2.16 A	2.7 A	1.33 A	1.66 A
$a_{sL}$	0.98	0.91	0.97	0.90
$\delta_L$	0.31	0.34	0.22	0.36
$S_f$	0.70	0.72	0.8	0.71

**Figure 15.** Experimental tests for monocrystalline PV module.**Figure 16.** I-V curves for test with monocrystalline PV module.

## 383 5.2. Discussion of Results

384 The experimental results confirm the correlation between the shading ratio  $\delta$  and the I-V curve.  
 385 For instance, Table 6 and Figure 16 experimentally show that the first divergence points in the I-V  
 386 curves are caused by the lowest shading ratios of PV-cells C6.7 and C5.10. Additionally, Table 8 and  
 387 Figure 18 allow for validating this interpretation.

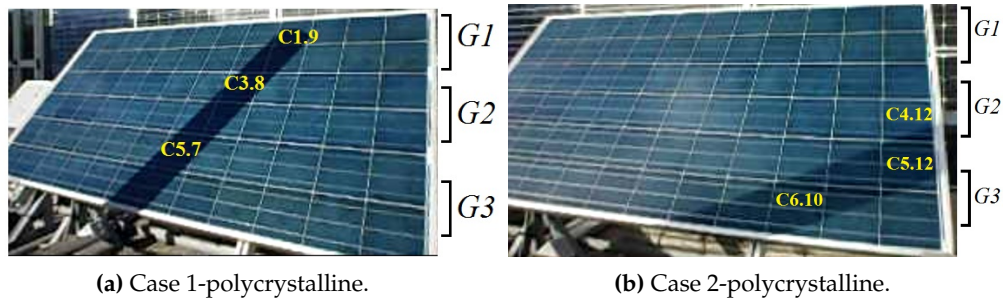
388 Considering a uniform  $S_f$ , the results also demonstrate that the smaller shaded cell areas in comparison  
 389 with the larger shaded cell areas in the same group provide a minimal contribution to the I-V curve.  
 390 For instance, the PV-cells C3.7 and C5.8 of case 1-monocrystalline (Table 6) have a minimal impact  
 391 on the I-V characteristics of Figure 16a. In contrast, shaded cells with small shaded areas are able to

**Table 6.** Shaded area and shading ratio for monocrystalline PV module TE 2200.

Group	Case 1			Case 2		
	$C_{ij}$	$a_{ij}$	$\delta_{ij}$	$C_{ij}$	$a_{ij}$	$\delta_{ij}$
1	C1.8	0.94	0.34			
	<b>C2.8</b>	<b>0.96</b>	<b>0.33</b>			
2	C3.7	0.17	0.88	<b>C4.10</b>	<b>0.25</b>	<b>0.82</b>
	<b>C3.8</b>	<b>0.80</b>	<b>0.44</b>			
	C4.7	0.40	0.72			
	C4.8	0.50	0.65			
3	C5.7	0.80	0.44	C5.9	0.35	0.75
	C5.8	0.20	0.86	<b>C5.10</b>	<b>0.91</b>	<b>0.34</b>
	<b>C6.7</b>	<b>0.98</b>	<b>0.31</b>	C6.8	0.40	0.72
				C6.9	0.89	0.37
				C6.10	0.15	0.89

**Table 7.** Maximum power points - Monocrystalline.

z	Case 1			Case 2		
	0	1	2	0	1	2
$\delta_z$	0.31	0.33	0.44	0.34	0.82	1.0
Group	G3	G1	G2	G3	G2	G1
$P_{mz}$ approx.(W)	<b>70.3</b>	48.3	28.7	85.5	<b>120.3</b>	59.2
$P_{mz}$ exper.(W)	<b>70.2</b>	51.6	31.1	88.2	<b>119.4</b>	60.2
Rel. error	0.001	0.064	0.078	0.031	0.008	0.017

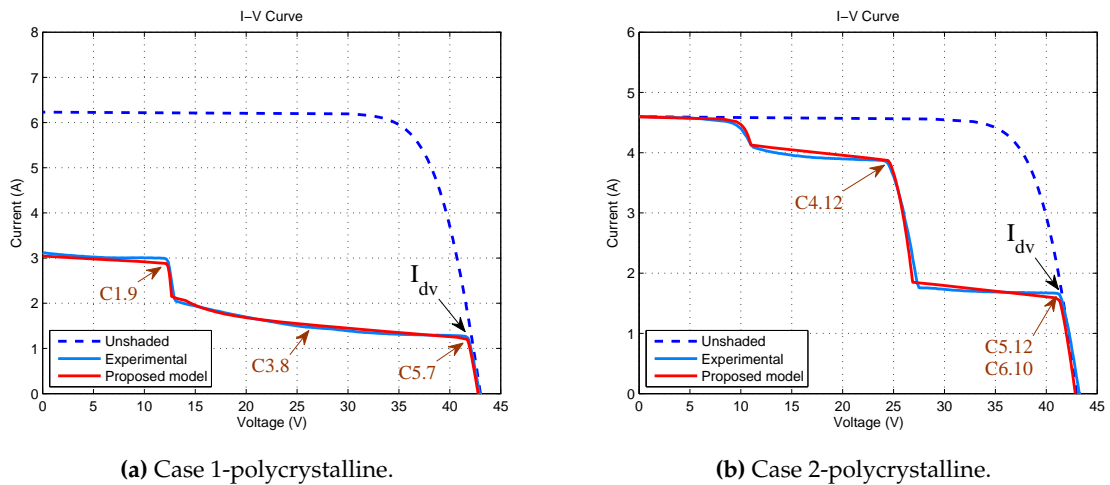
**Figure 17.** Experimental tests for polycrystalline PV module.

392 modify the I–V curve if they have the lowest shaded area in the group. For example, the PV-cell C4.12  
 393 in Case 2-polycrystalline is able to cause changes in the I–V curve of Figure 18b.

394 Figure 18a and Figure 18b show that shaded PV-cells in a group, with shading ratios near to the  
 395 lowest shading coefficient in the group, produce I–V curves with lower slopes because the behavior  
 396 of the reverse-bias voltage. For instance, PV-cells C5.12 and C6.10 of Case 2-polycrystalline cause  
 397 a lower slope than the caused by the PV-cell C4.12 in Case 2-polycrystalline. Therefore, PV-cell  
 398 C4.12 has more risk of dissipating power. Table 10 lists the slopes for the case 2-monocrystalline  
 399 and the case 2-polycrystalline which have single shaded cell in a group. Results in Table 10 show a  
 400 slightly difference in the case monocrystalline but more significant difference in case polycrystalline.  
 401 Authors also have addressed a detailed experimental study about the partial shading and the slope  
 402 identification which has been reported in Reference [3].

403

$$MSE = \frac{1}{n} \sum_{i=1}^n (e_i)^2 \quad (33)$$



**Figure 18.** I–V curves for tests with polycrystalline PV module.

**Table 8.** Shaded area and shading ratio for polycrystalline PV module YL290p-35b.

Group	Case 1			Case 2		
	$C_{ij}$	$a_{ij}$	$\delta_{ij}$	$C_{ij}$	$a_{ij}$	$\delta_{ij}$
1	<b>C1.9</b>	<b>0.68</b>	<b>0.46</b>			
	C2.8	0.64	0.49			
	C2.9	0.66	0.47			
2	<b>C3.8</b>	<b>0.96</b>	<b>0.23</b>	<b>C4.12</b>	<b>0.19</b>	<b>0.86</b>
	C3.9	0.11	0.91			
	C4.7	0.65	0.48			
	C4.8	0.65	0.48			
	C5.6	0.10	0.92	C5.10	0.21	0.85
3	<b>C5.7</b>	<b>0.97</b>	<b>0.22</b>	C5.11	0.80	0.43
	C5.8	0.11	0.91	<b>C5.12</b>	<b>0.90</b>	<b>0.36</b>
	C6.6	0.65	0.48	C6.8	0.22	0.84
	C6.7	0.65	0.48	C6.9	0.78	0.45
				<b>C6.10</b>	<b>0.90</b>	<b>0.36</b>
				C6.11	0.37	0.74

**Table 9.** Maximum power points - Polycrystalline.

	Case 1			Case 2		
	0	1	2	0	1	2
$\delta_z$	0.22	0.23	0.46	0.36	0.86	1.0
Group	G3	G2	G1	G3	G2	G1
$P_{mz}$ approx.(W)	<b>55.5</b>	37.8	33.4	68.4	<b>93.8</b>	44.9
$P_{mz}$ exper.(W)	<b>55.2</b>	42.1	36.9	67.8	<b>94.9</b>	45.7
Rel. error	0.006	0.103	0.094	0.01	0.01	0.02

**Table 10.** Slopes of I–V curves.

Region	Monocryst. Case2	Region	Polycryst. Case2
8V-18V	37mA/V	12V-24V	20.37mA/V
22V-32V	33.8mA/V	28V-42V	11.11mA/V

404 We use the mean square error (MSE) to assess the modeling accuracy based on the shading ratio.  
 405 The MSE values listed in Table 11 illustrate the model accuracy for the experimental and simulated I–V  
 406 curves depicted in Figures 16 and Figures 18. Table 11 shows that the proposed approach is suitable

**Table 11.** Mean squared error (MSE) from simulated and experimental I–V curves for model validation.

	Monocrystalline		Polycrystalline	
	Case 1	Case 2	Case 1	Case 2
MSE	0.92	1.07	0.89	1.01

for describing the current–voltage behavior of partially shaded PV modules in both monocrystalline and polycrystalline technologies. However, the behavior of polycrystalline modules slightly varies from monocrystalline modules because of the lower breakdown voltage in polycrystalline technology [30]. This phenomenon is more appreciable in the region from 13V to 40V of Figure 18a, which can lead to higher risk of hot-spots [30].

The MSE of model validation in Table 11 also shows a slightly difference for case 1 and case 2 in both technologies. This difference can be produced by several factors such as changes in the internal parameters, current path, or leakage currents. Indeed, some authors have shown that series and shunt resistances are affected by the irradiance conditions [21]. Soto *et al.* suggest that the series resistance depends on the irradiance level because its value decreases for lower irradiance and even can suffer negative values [40]. Earlier works also indicate negative values for the series resistance under low irradiance [41]. In Reference [21], the series resistance increases at the same time does the shadow rate which increases the amount of power dissipated by the series resistance. Nevertheless, most authors consider these variations less relevant by treating the serie resistance independent of the incident irradiance and temperature and obtaining sufficient accuracy [42][43]. In contrast, the study of the low irradiance conditions on the shunt resistances have been more widespread in literature because the strongly impact of the reverse-bias conditions [44][45].

For the local maximum power points (MPPs), the results from Table 6 to Table 9 show the integration of image processing methods with the proposed modeling for fast localization of the global MPP. The approximate MPPs for monocrystalline and polycrystalline cases are calculated using Equations (27) and Equations (29), and results are registered in Table 7 and Table 9. Indeed, these results highlight the correlation between the lowest shading ratios and the MPP calculation. These characteristics of simplified and fast localization of MPPs are potentially applicable to current supervision methods of power production based on image recognition [13][14].

Finally, the proposed methodology through this section can contribute to the supervision strategies based on image processing by considering the following findings in terms of shaded areas:

- Shaded cells with the highest shaded area in each group cause the divergence currents.
- Several shaded cells in a single PV group negligibly modify the operation point imposed by the PV-cell with the highest shaded area.
- Localized shadows on single shaded cells in a group are more harmful because overheating can arise.
- Uniform shadows on several cells of the same group cause less structural risks.
- The MPPs can be quickly localized considering the shaded PV-cells with the highest shaded areas in each group.

### 5.3. Comparison with Other Approaches

In this section, the contributions presented through this paper are compared with the existing schemes in literature. Methodologies in Table 12 address the PV modeling concerns using different perspectives. Reference [15] describes the reverse-bias behavior using a non-linear multiplication factor associated to the shunt resistance current. However, the impact of partial shadows is not discussed. The second approach proposes a discrete method to ensure convergence [20]. This paper presents a generalized method mainly based in the Bishop modeling to simulate the electrical behavior of PV installations by discretizing currents and voltages in PV systems. In contrast, quantification methods of shadow parameters are out of this paper’s scope [20]. The authors of Reference [36] integrate tools

450 to forecast PV energy production. The PV installation is described at a high-granularity single-cell  
 451 level and the non-intuitive influence of small-area shadows is predicted. The authors highlight the  
 452 high impact of small shadows in power production. However, the structural healthy is not covered  
 453 [36].

454 The approach in [46] develops a fast computing method to emulate shaded PV modules. In this paper,  
 455 the PV module performance is analyzed for parallel and series connections of PV-cells exposed to  
 456 equivalent external conditions by using the Brune method. However, this approach overlooks the  
 457 influence of the reverse-bias behavior. The authors of Reference [47] describe the shaded PV behavior  
 458 using the two-diodes model. The accurateness of the modeling technique is validated by real time  
 459 simulator data and compared with the neural network approach and the single-diode model. However,  
 460 this approach disregards the impact of partially shaded PV cells. The methodology in Reference  
 461 [34] presents an accurate and simplified expression for MPPs at a multi-string level. The PV array is  
 462 simulated by employing an enhanced version of the single-diode model and reformulated in an explicit  
 463 manner with the Lambert W function. However, the irradiance on shaded PV groups is considered  
 464 uniform.

465 In comparison with these approaches, the distinctive aspect of our work is to develop and study  
 466 a methodology for quantifying a ratio able to describe the shaded behavior without increasing the  
 467 computational complexity. Additionally, the proposed methodology provides a useful expression to  
 468 fast determination of MPPs using image processing methods and unshaded parameters. Nevertheless,  
 469 the proposed approach can be improved by studying other PV-cell parameters and applying image  
 470 recognition methods for estimating non-uniform shading factors.

**Table 12.** Comparison with existing schemes in literature.

Ref.	Accur.	Characteristic	Advantage	Comment
[15]	Med.	Non-linear factor	Reverse-bias behavior model	Not Partial shading
[20]	High	Discrete method	Convergence and processing time	Partial shading <sup>1</sup>
[36]	High	Integration tools	Energy prod. with shadow model	Impact structural healthy
[46]	Med.	Matrix equations	Fast computing - array emulation	Not reverse-bias <sup>1</sup>
[47]	Med.	Two-diode model	Fast computing	Not reverse-bias <sup>1</sup>
[34]	High	MPPs Multistring	Simplified MPPs expression	Uniform irradi. in groups <sup>1</sup>
Prop.	High	shading ratio	Simplified. Correlation I-V. MPPs Quantification shadow parameters	Other PV parameters Non-uniform shading factor

1. No method quantifying shadow.

## 471 6. Conclusions

472 This paper presented a complementary approach to describe the behavior of partially shaded PV  
 473 modules. The proposed approach presented a more accurate definition of the shading ratio  $\delta$  that is  
 474 suitable for describing the relation between the shaded area and the shading factor with the partial  
 475 shading behavior. The studied approach specified a methodology able to quantify experimentally the  
 476 shadow characteristics and the shading ratio  $\delta$ . Furthermore, the analysis of the results allowed us to  
 477 establish the interrelation between the shadow patterns and changes in I-V and P-V characteristics.  
 478 A simplified expression was developed to quickly calculate MPPs using the lowest shading ratio in  
 479 each group and the normal operation parameters. The experimental results validated the proposed  
 480 approach in monocrystalline and polycrystalline technologies. Further analysis should consider  
 481 non-uniform shading factors and other PV-cell parameters such as the series and the shunt resistances.  
 482 In future work, a supervision method should be developed by integrating image-processing methods  
 483 to the output power monitoring in PV installations.

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#### 490 **Nomenclature**

$\delta$	shading ratio
$\tau$	Shadow transmittance
$a_i$	Percentage of illuminated area
$a_s$	Percentage of shaded area
$A_c$	Cell area
$A_i$	Illuminated area
$A_s$	Shaded area
$C_{Ti}$	Thermal current coefficient
$C_{Tv}$	Thermal voltage coefficient
$G_i$	Incident irradiance
$G_s$	Irradiance in shaded area
$G_{STC}$	Irradiance for <i>STC</i>
$I$	Cell current
$I_{dv}$	Divergence current
$I_{mp}$	Current at MPP
$I_{ph}$	Photo-generated current
$I_{phT}$	Total $I_{ph}$
$I_{phTi}$	Completely illuminated $I_{ph}$
$I_{phTs}$	Completely shaded $I_{ph}$
$I_0$	Inverse saturation current
491 $I_{sc\_STC}$	Short-circuit current for <i>STC</i>
$J$	Current density
$J_{ph}$	Photo-current density
$k$	Fraction of current in avalanche
$n$	Avalanche breakdown exponent
<i>MPP</i>	Maximum Power Point
$P_{mp}$	Power at MPP
<i>PV</i>	Photovoltaic
$R_p$	Shunt resistance
$R_s$	Series resistance
$S_f$	Shading factor
<i>STC</i>	Standard Test Condition
$T_c$	Cell temperature
$T_{STC}$	Temperature for <i>STC</i>
$V_b$	Breakdown voltage
$V_{BD}$	By-pass diode voltage
$V_c$	Cell voltage
$V_G$	Group voltage
$V_{mp}$	Voltage at MPP
$V_p$	Module voltage
$V_t$	Thermal voltage

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