Circular formation control for cooperative target tracking with limited information

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Abstract

This paper addresses the problem of encircling and tracking a moving target with a fleet of unicycle-like vehicles. A new control law is developed to steer the vehicles to an evenly spaced formation along a circumference, the center of which tracks the motion of the target. The strategy proposed relies only on the relative positions of the agents with respect to the target, expressed in the local frame of each vehicle. The absolute position, velocity and acceleration of the target are unknown. Additionally, the robustness of the proposed control law in the presence of external disturbances is analyzed. Communication among agents is used to maintain the vehicles equally spaced in the circular formation. Simulation results illustrate the effectiveness of the proposed strategies.

Key words: Target tracking, autonomous mobile robots, motion control, cooperative control

1 Introduction

Formation control, which has steadily emerged as an important problem in the general area of motion coordination of multi-agent systems, has received widespread attention in the literature; see for example [1–5] and the references therein. Among the multitude of formation control strategies proposed, circular formation control has been the subject of intensive research efforts because of its manifold applications that include target tracking and source-seeking missions [6,7]. In its simplest form, circular formation control refers to the problem of making a group of vehicles converge to and move along a circumference centered at a desired point, while distributing themselves uniformly along the circumference. A natural extension of the problem is to study the related problem of steering the vehicles to a desired formation along a circumference, while requesting that the center of the latter track a moving target.

Circular collective motion of a network of unicycle-like agents under several communication constraints was studied in [8]. Based on potential functions and oscillator models, the authors proposed Lyapunov-based feedback control laws to stabilize the agents to a fixed circular formation. The problem of circular formation of nonholonomic vehicles was studied in [9] and [10] using cyclic pursuit strategies. Circumnavigation around a fixed target using only bearing measurements was studied in [11]. In all of these references, the center of the desired circular formation was taken as fixed.

An extremely relevant and challenging problem is that of circular formation control when the center of the formation is no longer fixed but is required to vary with time, instead. This problem is easily motivated in the context of applications where a number of vehicles must perform collaborative tasks that require the formation to move in directions that are determined on-line and to adopt a particular desired geometric configuration. For instance, in source seeking applications a formation is driven to follow the gradient of a scalar field generated by a source of interest [7,12]. Target tracking problems also require the consideration of time-varying formations. Cooperative approaches to meet the above challenges using fleets of vehicles have been studied in the literature, see for example [10,13], where formation control design builds

[1] This work was supported in part by the H2020 EU Marine Robotics Research Infrastructure Network (Project ID 731103) and the Portuguese FCT Project UID/EEA/5009/2013.

upon hybrid control techniques and the theory of oscillator models, respectively. Based on cyclic pursuit strategies, the authors in [14] derived a cooperative control law to capture a moving target with a fleet of vehicles modeled as simple integrators. In the work reported in [15,16], a gradient-based control approach using vector fields was presented to enforce a single integrator vehicle model to converge to time-varying target curves. Circular formation control with a moving center for vehicles modeled as single or double integrators was studied in [17].

The problem of time-varying circular formation control is considerably more challenging when a nonholonomic dynamic model is adopted for the vehicles, instead of a single or double integrator. Using cyclic pursuit techniques and local information only, the problem of circular formation control around a common fixed center was studied in [18]. In the latter reference, a distributed control was done by considering that only one anonymous vehicle knows the parameters describing the desired circular formation. In [19], a cyclic pursuit based strategy was proposed to stabilize a group of unicycle vehicles to a circular formation around a target, using only bearing angle information of the target and the neighbours. In the two references above, the theoretical results only hold for a fixed target.

Based on previous circular formation control results described in the literature [8], a new controller designed to make nonholonomic vehicles converge to a circumference, the center of which tracks a time-varying reference, was derived in [20]. The main idea is to generate circular trajectories for a properly chosen stable autonomous exosystem (also called virtual system) and to enforce the multi-agent system to track them. In this setup, the absolute position of the agents in a global reference frame and the desired velocity and acceleration of the time-varying center are needed to design the tracking controller that will ensure global asymptotic stabilization of the circular formation. A distributed reconfigurable control law was proposed in [10] to enforce a group of vehicles to follow and encircle a moving target while adopting an evenly spaced formation. In this study, only the target velocity and local information (distances and bearing angles) in addition to communication among neighbors agents are required. However, the proposed strategy ensures only local stability. Furthermore, the tracking position errors with respect to the target are locally bounded but do not converge to zero. The recent paper [21] addressed the problem of moving-target circular formation control for nonholonomic vehicles using only measurements in the local Frenet-Serret frame of each vehicle. The cooperative controller proposed is based on cyclic pursuit ideas and requires both the velocity and the acceleration of the time-varying target trajectory.

Motivated by the above consideration, this paper addresses the problem of encircling a moving target with a fleet of unicycle-like vehicles using limited motion information. Firstly, a new control law is derived to steer the vehicles to a circular formation whose center tracks a time-varying target. The control design proposed relies only on local information: the relative positions of the pursuing vehicles with respect to the target expressed in the local frame of each vehicle. Unlike previous results reported in the literature, this strategy dispenses with the need to measure the absolute positions of the vehicles and the velocity and acceleration of the target. The methodology developed builds upon the ideas advanced in [20], yielding an autonomous stable exosystem that generates circular trajectories and a tracking control law that stabilizes the vehicles to the circular motion around the moving target. The new proposed controller further decreases the amount of information required, when compared to our previous work [22], since it does not need the velocity of the target to be known. Secondly, the robustness of the proposed control law in the presence of external disturbances is theoretically analyzed. Communication among the vehicles is used to maintain the vehicles equally spaced along the circular formation. Simulation results illustrate the effectiveness of the proposed strategy.

The paper is organized as follows. Section 2 starts by introducing the model adopted for each of the vehicles and some basic concepts related to the underlying communication network topology. This is followed by a description of the main control objectives and the solution proposed, which relies on the construction of an autonomous exosystem and a tracking controller. Section 3 derives a new control law for target tracking with a circular formation of vehicles that does not require explicit knowledge of the target velocity and uses only the relative positions of the pursuing vehicles with respect to it. The robustness of the proposed strategy is studied in Section 4. Finally, Section 5 contains the main conclusions and describes problems that warrant further research.

2 Problem formulation and control strategy

2.1 Model of the agents

In what follows, a group of N vehicles are modeled as unicycles, subject to simple nonholonomic constraints, that is

$$\begin{aligned} \dot{\mathbf{r}}_i &= \mathbf{R}(\theta_i)\,[v_i,\,0]^T \\ \dot{\theta}_i &= u_i \end{aligned} \qquad i = 1, \ldots, N \tag{1}$$

where $\mathbf{r}_i \in \mathbb{R}^2$ is the position vector of vehicle i in a given inertial frame, θ_i is the heading angle, $\mathbf{R}(\theta)$ denotes the rotation matrix from body to inertial reference frame, defined by $\mathbf{R}(\theta) = \left[\begin{smallmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{smallmatrix}\right]$, and (v_i, u_i) are the control inputs, consisting of linear and rotational speed, respectively.

2.2 Communication topology

In the set-up adopted in this paper, each vehicle can transmit and receive information from a subset of the other vehicles in the group. Following a by now standard approach, we assume that the multiple vehicle communication network can be described in terms of a graph, see for example [23] for a fast paced presentation of the subject. We assume that the communication network of the multi-agent system is represented by an undirected graph. The set of vertices of the graph is denoted by $V = \{1, \ldots, N\}$ and E represents the set of edges such that $(i, j) \in E$ if agents i and j communicate with each other. Using standard notation, $\mathcal{G} = (V, E)$ denotes the corresponding undirected graph. We let $\mathcal{N}_i = \{j \in V : (i, j) \in E\}$ be the set of neighbors of agent i and $|\mathcal{N}_i|$ denote its number of neighbors. In the sequel, $\otimes$ denotes the Kronecker product and $\mathbf{I}_n \in \mathbb{R}^{n\times n}$ is the identity matrix.

2.3 Main objectives and contribution

Our main objective is to derive a cooperative control law for a group of vehicles to converge to a formation that encircles and tracks a moving target describing a trajectory $\mathbf{c}(t)$ that is a continuously differentiable function of time, that is, $\mathbf{c} \in \mathcal{C}_1(\mathbb{R} \to \mathbb{R}^2)$. To accomplish this task, the vehicles will be deployed in a uniform distributed pattern along a circular formation whose center is the moving target. In a previous paper [20], we extended the circular formation control law presented in [8] to time-varying formations, not necessarily circular, using an exosystem and a tracking controller. A translation control was proposed in which the vehicles are stabilized to a circular formation whose center tracks a time-varying reference $\mathbf{c}(t)$. It was assumed that the vehicles can compute their own absolute position $\mathbf{r}_i$ and moreover, in order to track the reference $\mathbf{c}(t)$, both the first and second order derivatives, $\dot{\mathbf{c}}(t)$ and $\ddot{\mathbf{c}}(t)$, are known. In other words, $\mathbf{c} \in \mathcal{C}_2(\mathbb{R} \to \mathbb{R}^2)$, i.e., $\mathbf{c}(t)$ was a continuous and twice differentiable function of time.

In the present paper, the goal is to design a new formation control law which relaxes the above assumptions on the problem. We improve substantially the above result by developing a target tracking control law that only requires information on the relative positions between the agents and the target, expressed in the agents' body frame, i.e., $\mathbf{R}(\theta_i)^T(\mathbf{r}_i - \mathbf{c})$, rather than in the global inertial frame. Compared to [20], the control design developed in our previous work [22] does not require the knowledge of $\ddot{\mathbf{c}}(t)$. Furthermore, no global information such as the absolute positions of the agents and target are needed. This was made possible at the price of introducing a position error which can be made arbitrarily small. The first contribution of the present paper is a new control law that does not require the computation of the target velocity. Consequently, the amount of required information decreases considerably. The second contribution of this paper is to analyze the robustness of the proposed target tracking control law with respect to external disturbances.

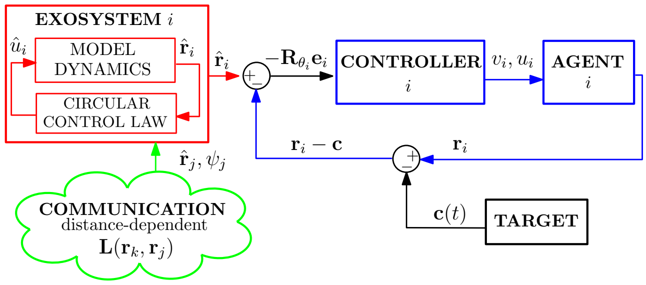

Fig. 1. *Structure for the feedback design including a decentralized exosystem. Each agent i computes its own exosystem which communicates with other exosystems by means of a communication protocol depending on the distance between the agents (the Laplacian matrix $\boldsymbol{L}$ depends on the agents' states $\boldsymbol{r}_i$).*

2.4 Control strategy

Consider that each vehicle can compute the relative position vector $\mathbf{r}_i - \mathbf{c}$, where $\mathbf{c}$ represents the position of the target. The objective for each agent is to encircle and track the target, i.e., to describe a circular motion around $\mathbf{c}(t)$. This objective can be also expressed as follows: design control laws (v_i, u_i) for the multi-agent system (1) such that the relative vector $\mathbf{r}_i - \mathbf{c}$ describes a circular motion around the origin.

In order to exploit the circular control law from [8], we introduce an exosystem represented by the multi-agent dynamics (3). The main idea is thus to make the exosystem converge to a circular motion centered at the origin and to use the exosystem trajectories $\hat{\mathbf{r}}_i$ as references for each relative vector $\mathbf{r}_i - \mathbf{c}$, see Fig 1. Each vehicle can measure its relative vector expressed in its body frame, i.e., $\mathbf{R}(\theta_i)^T(\mathbf{r}_i - \mathbf{c})$. It follows that the error between the reference and the relative position vector expressed in the body coordinates of vehicle i is given by

$$\mathbf{e}_i = \mathbf{R}(\theta_i)^T((\mathbf{r}_i - \mathbf{c}) - \hat{\mathbf{r}}_i) = \mathbf{R}(\theta_i)^T(\mathbf{r}_i - (\hat{\mathbf{r}}_i + \mathbf{c})). \quad (2)$$

The problem is to design control laws (v_i, u_i) for the multi-agent system (1) such that the error $\mathbf{e}_i$ converges to zero.

2.5 Autonomous stable exosystem

The collective motion of a group of non-holonomic agents has been extensively studied in literature. Of particular interest to our work are the results described [6,8] where the authors proposed a circular formation control law for a group of nonlinear agents modeled by (1) with unit linear velocity, i.e. $v_i = 1$, $\forall i$. In order to exploit these results, we consider an autonomous exosystem described by the unicycle dynamics:

$$\begin{aligned} \dot{\hat{\mathbf{r}}}_i &= \mathbf{R}(\phi_i) \left[|\omega_0| R_0,\, 0 \right]^T \\ \dot{\phi}_i &= \hat{u}_i \end{aligned} \qquad i = 1, \ldots, N \tag{3}$$

where R_0 denotes the desired radius of the formation, $\omega_0 \neq 0$ is the angular velocity, ϕ_i represents the angular orientation of the velocity vector $\dot{\hat{\mathbf{r}}}_i$, and $\hat{u}_i$ is the control input. Note that the linear velocity of the virtual agents of the exosystem is constant and equal to the positive value $|\omega_0| R_0$.

Communication between agents is considered in order to achieve a uniform distribution around the desired circular formation. In the setp-up adopted in this paper, each vehicle computes its own virtual exosystem (3) and communicates the virtual quantity ϕ_i to its neighbors through a undirected communication graph $\mathcal{G}$. Inspired by the synchronization problem of coupled oscillators, [8] introduced a potential function $U(\phi)$ that depends on the relative headings of the agents in the formation and induces a repulsion force to enlarge the angular distance between two connected agents of the exosystem. The potential function is designed such that the evenly spaced state corresponding to the uniform distribution of the agents along the circle is an equilibrium point of $U(\phi)$. This potential function satisfies $U(\phi) \geq 0$ and $\mathbf{1}^T \nabla U(\phi) = 0$, where $\mathbf{1}$ denotes a vector of ones and ∇ is the gradient operator, so the potential function is invariant to rigid rotations of all the agent headings. As presented in [8], the uniform distribution is only locally stable for fixed circulant graphs, and others configurations could be stabilized depending on the initial conditions. The following lemma summarizes the results presented in [8].

Lemma 1 *Let $\omega_0 \neq 0$, $\kappa > 0$, and $\kappa_u > 0$ be three control parameters, and let $R_0 > 0$ be the radius of the desired circular motion. Then, the control law*

$$\begin{aligned} \hat{u}_i &= \omega_0 (1 + \kappa \dot{\hat{\boldsymbol{r}}}_i^T \hat{\boldsymbol{r}}_i) - \frac{\partial U}{\partial \phi_i} \\ \frac{\partial U}{\partial \phi_i} &= \frac{\kappa_u}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \sum_{m=1}^{\lfloor |\mathcal{N}_i|/2 \rfloor} \frac{\sin(m\phi_i - m\phi_j)}{m}, \end{aligned} \tag{4}$$

where $\lfloor |\mathcal{N}_i|/2 \rfloor$ is the largest integer less than or equal to $|\mathcal{N}_i|/2$, ensures that the multi-agent system (3) converges to a circular motion centered at the origin of the coordinate frame with radius R_0 and the direction of rotation is determined by the sign of ω_0. Moreover, the agents are distributed along the circular formation following a particular pattern defined by an equilibrium point of $U(\phi)$.

The proof of the above result relies on Lyapunov techniques and LaSalle's Invariance Principle and can be found in [8]. For the sake of clarity, we present here a sketch of the proof. Define $\hat{\mathbf{r}} = (\hat{\mathbf{r}}_1^T, \ldots, \hat{\mathbf{r}}_N^T)^T$, $\phi = (\phi_1, \ldots, \phi_N)^T$, and consider the Lyapunov function

$$S(\hat{\mathbf{r}}, \phi) = \frac{\kappa}{2} \sum_{i=1}^{N} \left\| \dot{\hat{\mathbf{r}}}_i - \omega_0 \mathbf{R}_{\frac{\pi}{2}} \hat{\mathbf{r}}_i \right\|^2 + U(\phi) \geq 0, \tag{5}$$

where $\mathbf{R}_{\frac{\pi}{2}} = \left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right]$ denotes a rotation matrix through an angle $\frac{\pi}{2}$ counterclockwise about the origin. At the equilibrium points of the previous Lyapunov function, i.e., $S(\hat{\mathbf{r}}, \phi) = 0$, the dynamics of the exosystem (3) satisfy $\dot{\hat{\mathbf{r}}}_i - \omega_0 \mathbf{R}_{\frac{\pi}{2}} \hat{\mathbf{r}}_i = 0$, $\forall i$ and $U(\phi) = 0$. Thus, the position vector and the velocity vector are perpendicular, i.e., $\dot{\hat{\mathbf{r}}}_i^T \hat{\mathbf{r}}_i = 0$. This condition leads to the kinematic relation for the rotation of a rigid body. In other words, the vectors $\hat{\mathbf{r}}_i$ are turning around the frame origin at the equilibrium. Considering the proposed control law (4) and evaluating the derivative of $S(\hat{\mathbf{r}}, \phi)$ along the solutions of the resulting closed-loop system (3) yields:

$$\begin{aligned} \dot{S}(\hat{\mathbf{r}}, \phi) =& \kappa \sum_{i=1}^{N} \left(\dot{\hat{\mathbf{r}}}_i - \omega_0 \mathbf{R}_{\frac{\pi}{2}} \hat{\mathbf{r}}_i \right)^T \left(\ddot{\hat{\mathbf{r}}}_i - \omega_0 \mathbf{R}_{\frac{\pi}{2}} \dot{\hat{\mathbf{r}}}_i \right) \\ &+ \sum_{i=1}^{N} \frac{\partial U}{\partial \phi_i} \dot{\phi}_i \\ =& \sum_{i=1}^{N} \left(\kappa \omega_0 \hat{\mathbf{r}}_i^T \dot{\hat{\mathbf{r}}}_i (\omega_0 - \hat{u}_i) + \frac{\partial U}{\partial \phi_i} \hat{u}_i \right) \\ =& - \sum_{i=1}^{N} \left(\kappa \omega_0 \hat{\mathbf{r}}_i^T \dot{\hat{\mathbf{r}}}_i + \frac{\partial U}{\partial \phi_i} \right)^2 \leq 0. \end{aligned}$$

In conclusion, from LaSalle's Invariance Principle the solutions converge to the largest invariant set contained in the set of points where $\dot{S} = 0$. Consequently, the dynamics of the exosystem satisfy $\dot{\hat{\mathbf{r}}}_i = \omega_0 \mathbf{R}_{\frac{\pi}{2}} \hat{\mathbf{r}}_i$ which correspond to a circular motion around the origin. Additionally, the gradient term $\frac{\partial U}{\partial \phi_i}$ enforces each agent to move away from its neighbours till the equilibrium point $\frac{\partial U}{\partial \phi_i} = 0$, $\forall i$ is reached. The equilibrium point corresponding to the evenly spaced state is locally asymptotically stable for fixed connected circulant graphs.

In our previous work [20] we extended this result to deal with distance dependent communication graphs, i.e, the agents only communicate when the distance between them is smaller than the communication range. We proved that in this situation, for a large enough communication range that depends on the number of agents and the desired formation radius, the uniform distribution is the only critical point of $U(\phi)$ and therefore evenly spaced distribution of the agents along the formation is ensured.

3 Target tracking control with unknown target velocity

In this section we present a new control strategy to solve the target tracking problem with a group of unicycle-like vehicles. The objective is to enforce convergence of each $\mathbf{r}_i(t)$ to a desired trajectory $\hat{\mathbf{r}}_i(t)+\mathbf{c}(t)$ determined by adding, at each instant of time, the circular trajectory generated by the exosystem to the position $\mathbf{c}(t)$ of the target moving with velocity $\dot{\mathbf{c}}(t)$. The main idea is to consider the circular trajectories of the autonomous exosystem $\hat{\mathbf{r}}_i$ as references to be tracked by the relative vectors $\mathbf{r}_i - \mathbf{c}$ for all vehicles $i = 1, \ldots, N$. In other words, the aim is to enforce convergence of each $\mathbf{r}_i(t)$ to the desired position $\hat{\mathbf{r}}_i(t)+\mathbf{c}(t)$, where the target velocity $\dot{\mathbf{c}}(t)$ is unknown. The stability and robustness of the proposed control strategy are studied in the sequel.

3.1 Stability result

Consider the tracking error defined by (2). Then, the error dynamics are described by

$$\begin{aligned}\dot{\mathbf{e}}_i =& \mathbf{R}(\theta_i)^T(\dot{\mathbf{r}}_i - (\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}})) - u_i \mathbf{R}_{\frac{\pi}{2}} \mathbf{R}(\theta_i)^T(\mathbf{r}_i - (\hat{\mathbf{r}}_i + \mathbf{c}))\\ =& [v_i,\, 0]^T - \mathbf{R}(\theta_i)^T(\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}}) - u_i \mathbf{R}_{\frac{\pi}{2}} \mathbf{e}_i.\end{aligned}$$

We introduce the vector $\boldsymbol{\delta} = [-\delta,\, 0]^T$, where δ is an arbitrary small positive constant, to obtain:

$$\begin{aligned}\dot{\mathbf{e}}_i &= [v_i,\, 0]^T - \mathbf{R}(\theta_i)^T(\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}}) - u_i \mathbf{R}_{\frac{\pi}{2}}(\mathbf{e}_i - \boldsymbol{\delta}) - u_i \mathbf{R}_{\frac{\pi}{2}} \boldsymbol{\delta}\\ &= \Delta[v_i,\, u_i]^T - \mathbf{R}(\theta_i)^T(\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}}) - u_i \mathbf{R}_{\frac{\pi}{2}}(\mathbf{e}_i - \boldsymbol{\delta}),\end{aligned} \tag{6}$$

where $\Delta = \left[\begin{smallmatrix}1 & 0\\ 0 & \delta\end{smallmatrix}\right]$. Note that the new error vector $(\mathbf{e}_i - \boldsymbol{\delta})$ is the distance between a point located at a distance δ from the center of mass of the agent i along the x-axis of the local reference frame and the desired position $(\hat{\mathbf{r}}_i+\mathbf{c})$. The addition of $\boldsymbol{\delta}$ is similar to a feedback linearization technique that makes the control variable u_i appear directly in the position error dynamics. Consequently, in order to design simultaneously both control inputs of the unicycle-like agents, the error will converge to a neighborhood of the origin instead of to the origin itself.

In practice, the agents may only be able to measure the relative vector $\mathbf{r}_i - \mathbf{c}$, but not the target velocity $\dot{\mathbf{c}}$. In this situation, based on the control strategy presented in [22], we propose a new control law which relies only on local information. As explained, only the measurement of the relative distance between each agent and the target, expressed in its local coordinates frame, will be required.

Due to the unknown target velocity, which can be seen as an external perturbation, the error dynamics cannot be driven to zero. If the target velocity is bounded, we can use the notion of ε-stability (Sections 4.8 and 9.2 in [24]) to prove the stability and robustness of the proposed target tracking control.

Theorem 1 *Consider a differentiable function $\boldsymbol{c}(t) : \mathbb{R} \to \mathbb{R}^2$ with bounded derivative, such that*

$$\|\dot{\boldsymbol{c}}(t)\| \leq \gamma_{\boldsymbol{c}}, \quad \forall t, \tag{7}$$

for $\gamma_{\boldsymbol{c}} > 0$. Let $R_0 > 0$ be the radius of the desired circular motion, $\omega_0 \neq 0$, $\kappa > 0$, and $\kappa_u > 0$ three control parameters, and $\boldsymbol{K} \in \mathbb{R}^{2\times 2}$ a symmetric positive definite matrix. Consider the control law

$$[v_i,\, u_i]^T = \Delta^{-1}\left(\boldsymbol{R}(\theta_i)^T \dot{\hat{\boldsymbol{r}}}_i - \boldsymbol{K}(\boldsymbol{e}_i - \boldsymbol{\delta})\right), \tag{8}$$

where $\dot{\hat{\boldsymbol{r}}}_i$ is defined by the exosystem (3) driven by the control law $\hat{u}_i$ in (4). With the control law thus defined, the agents defined by (1) converge to a circular motion of radius R whose direction of rotation is determined by the sign of ω_0 and whose center converges to a ball centered at the time-varying target position $\boldsymbol{c}(t)$ with radius

$$\varepsilon_{\boldsymbol{c}} = \gamma_{\boldsymbol{c}}/\lambda_{min}(\boldsymbol{K}), \tag{9}$$

where $\lambda_{min}(\boldsymbol{K})$ denotes the minimum eigenvalue of matrix $\boldsymbol{K}$. Furthermore, the relative distance between each agent i and the moving target, i.e., $\|\boldsymbol{r}_i(t) - \boldsymbol{c}(t)\|$, converges to the set $\mathcal{R}_{\boldsymbol{c}} = [R_0 - \delta - \varepsilon_{\boldsymbol{c}}, R_0 + \delta + \varepsilon_{\boldsymbol{c}}]$.

Proof 1 *The proof is divided in two steps. Firstly, the autonomous exosystem is stabilized to a fixed circular motion, as presented in Section 2. This is done by using the circular control law (4) which guarantees that in the limit, as t tends to infinity, the trajectories of the autonomous exosystem satisfy $\dot{\hat{\boldsymbol{r}}}_i(t) = \omega_0 \boldsymbol{R}_{\frac{\pi}{2}} \hat{\boldsymbol{r}}_i(t)$, $\forall i$, as proved in Lemma 1. The exosystem is then used as an autonomous reference generator and the circular trajectories become references to be tracked by the agents with the dynamics in (1).*

The second step consists of designing a tracking controller that will make each unicycle track the desired time-varying trajectory $\hat{\boldsymbol{r}}_i(t) + \boldsymbol{c}(t)$. To this end, consider the error dynamics defined in (6) as

$$\dot{\boldsymbol{e}}_i = \Delta[v_i,\, u_i]^T - \boldsymbol{R}(\theta_i)^T(\dot{\hat{\boldsymbol{r}}}_i + \dot{\boldsymbol{c}}) - u_i \boldsymbol{R}_{\frac{\pi}{2}}(\boldsymbol{e}_i - \boldsymbol{\delta}).$$

Using the control law (8), the error dynamics become

$$\dot{\boldsymbol{e}}_i = -u_i \boldsymbol{R}_{\frac{\pi}{2}}(\boldsymbol{e}_i - \boldsymbol{\delta}) - \boldsymbol{K}(\boldsymbol{e}_i - \boldsymbol{\delta}) - \boldsymbol{R}(\theta_i)^T \dot{\boldsymbol{c}}. \tag{10}$$

To show the stability of the error system (2), consider the Lyapunov function

$$V(\boldsymbol{e}_i) = \frac{1}{2}\|\boldsymbol{e}_i - \boldsymbol{\delta}\|^2. \tag{11}$$

Differentiating $V(\boldsymbol{e}_i)$ along the solutions of the closed-loop dynamics (10) *yields*

$$\begin{aligned}\dot{V}(\boldsymbol{e}_i) \leq & -\lambda_{min}(\boldsymbol{K})\|\boldsymbol{e}_i-\boldsymbol{\delta}\|^2+\|\boldsymbol{e}_i-\boldsymbol{\delta}\|\|\dot{\boldsymbol{c}}\| \\ \leq & -\|\boldsymbol{e}_i-\boldsymbol{\delta}\|\left(\lambda_{min}(\boldsymbol{K})\|\boldsymbol{e}_i-\boldsymbol{\delta}\|-\gamma_{\boldsymbol{c}}\right).\end{aligned}$$

To prove ε-stability of the error dynamics, consider a closed ball $B_{\boldsymbol{c}}$ centered at $\boldsymbol{e}_i = \boldsymbol{\delta}$ with radius $\varepsilon_{\boldsymbol{c}} = \gamma_{\boldsymbol{c}}/\lambda_{min}(\boldsymbol{K})$. Let $\Omega_{\boldsymbol{c}} = \{\boldsymbol{e}_i - \boldsymbol{\delta} \,|\, V(\boldsymbol{e}_i) \leq \frac{1}{2}\varepsilon_{\boldsymbol{c}}^2\}$. Then $B_{\boldsymbol{c}}$ is contained in $\Omega_{\boldsymbol{c}}$ because

$$\|\boldsymbol{e}_i-\boldsymbol{\delta}\| \leq \varepsilon_{\boldsymbol{c}} \Rightarrow V(\boldsymbol{e}_i)=\frac{1}{2}\|\boldsymbol{e}_i-\boldsymbol{\delta}\|^2 \leq \frac{1}{2}\varepsilon_{\boldsymbol{c}}^2.$$

In conclusion, any solution starting in $\mathbb{R}^2 \setminus \Omega_{\boldsymbol{c}}$ satisfies $\dot{V}(\boldsymbol{e}_i) < 0$ (because $\|\boldsymbol{e}_i - \boldsymbol{\delta}\| \geq \gamma_{\boldsymbol{c}}/\lambda_{min}(\boldsymbol{K})$) and thus enters $\Omega_{\boldsymbol{c}}$ in finite time and remains there. Thus, the error dynamics converges asymptotically to a region centered at $\boldsymbol{e}_i = \boldsymbol{\delta}$ and radius $\varepsilon_{\boldsymbol{c}} = \gamma_{\boldsymbol{c}}/\lambda_{min}(\boldsymbol{K})$. In other words,

$$\|\boldsymbol{e}_i(t)-\boldsymbol{\delta}\| \leq \varepsilon_{\boldsymbol{c}}, \quad t \rightarrow \infty.$$

Considering the definition of the error in (2) *and using the previous inequality, it can be shown that at steady state the following expression holds:*

$$\left|\|\boldsymbol{R}(\theta_i)^T(\boldsymbol{r}_i(t)-\boldsymbol{c}(t))\|-\|\boldsymbol{R}(\theta_i)^T\hat{\boldsymbol{r}}_i(t)+\boldsymbol{\delta}\|\right| \leq \varepsilon_{\boldsymbol{c}}.$$

We know that the trajectories of the exosystem converge to circular motions centered at the origin and with radius R_0, i.e., $\|\hat{\boldsymbol{r}}_i\| = R_0$, for all i. Thus, the distance between the position of each agent i and the target satisfies

$$R_0-\delta-\varepsilon_{\boldsymbol{c}} \leq \|\boldsymbol{r}_i(t)-\boldsymbol{c}(t)\| \leq R_0+\delta-\varepsilon_{\boldsymbol{c}}.$$

This last inequality implies that the relative distance between each agent i and the target converges to a the set $\mathcal{R}_{\boldsymbol{c}} = [R_0-\delta-\varepsilon_{\boldsymbol{c}}, R_0+\delta+\varepsilon_{\boldsymbol{c}}]$, thus concluding the proof.

Theorem 1 proves that the tracking control law (8) enforces the agents to track and encircle the target even if the target velocity is unknown. Intuitively, if the bound of the unknown target velocity, $\gamma_{\mathbf{c}}$, is significantly large the agents may not be able to track the time-varying target trajectory. This implication is confirmed in Eq. (9): the radius of the ε-stability grows proportionally to the bound on the target's velocity. However, thanks to the control parameter $\mathbf{K}$, this error can be reduced, reducing as well the tracking position error of the agents, as shown in Figs. 3 and 4.

Remark 1 *In the definition of the control law* (8)*, it can be seen that the input variable u_i is multiplied by $1/\delta$, which corresponds to the inverse of the norm of the artificial error vector $\boldsymbol{\delta}$. Therefore, if one wishes to achieve almost perfect tracking, then the control input u_i may reach large values. Conversely, if one desires to limit the amplitude of the control input u_i, then one must accept larger tracking errors. There is therefore a tradeoff between reducing the position error and bounding the control input u_i.*

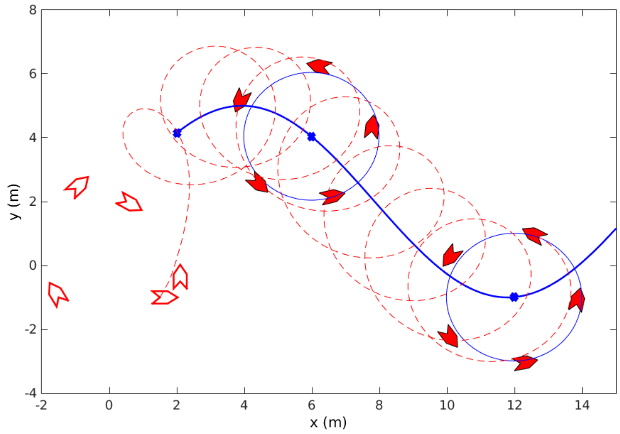

Fig. 2. *Target encircling with a group of 5 agents governed by control law* (8) *(target velocity is unknown). The figure shows three snapshots: the initial condition represented by the void red agents and two states at $t = 20s$ and $t = 50s$, respectively.*

The control law (8) presents several advantages with respect to the time-varying circular control law proposed in [20]. Firstly, the new control law allows for the direct computation of the inputs (v_i, u_i) for each vehicle i instead of requiring the design of a dynamic control law as in [20] where, in order to deal with time-varying references for the center $\mathbf{c}(t)$, $\dot{v}_i$ is viewed as a control input. Furthermore, the singularities of the control law presented in [20] are avoided with the new approach. Secondly, implementation of the new control law in (8) only requires access to the relative position vector $\mathbf{r}_i - \mathbf{c}$ expressed in the body frame. This is in striking contrast with the requirements of the dynamic controller in [20], which depends on absolute measurements of the center position $\mathbf{c}$, velocity $\dot{\mathbf{c}}$, and acceleration $\ddot{\mathbf{c}}$, as well as the absolute position of each vehicle $\mathbf{r}_i$. The proposed control law relies only on local information and compared to the control strategy presented in [22], the target velocity $\dot{\mathbf{c}}(t)$ is considered unknown. Hence, despite the issue discussed in Remark 1, the control law (8) presents several advantages for tracking and encircling a time-varying target.

Remark 2 *The position error depends directly on the bound on the target's velocity as well as on the control parameter $\boldsymbol{K}$, as stated in Theorem 1. There is a trade-off between reducing the position error and bounding the control inputs. As shown in Eq.* (9)*, for large values of the minimum eigenvalue of $\boldsymbol{K}$, the radius of the ε-stability is reduced and then the agents converge to a circular motion whose center is really close to the target position. However, for large values of $\boldsymbol{K}$ both control inputs (v_i, u_i) may reach large values. In conclusion, both the control*

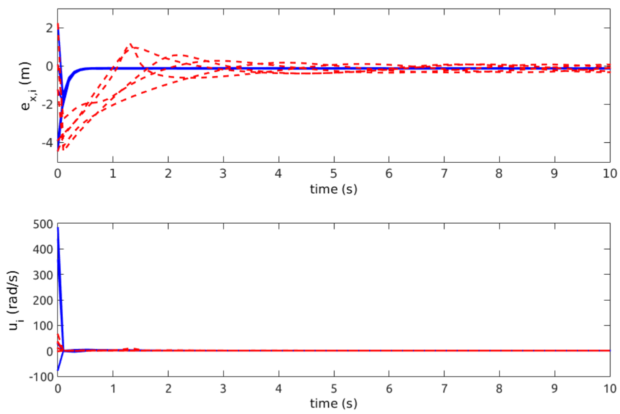

Fig. 3. *Evolution of the x-component of the position error $\boldsymbol{e}_{x,i}$ (top) and the control input u_i (bottom) for a simulation of 5 agents encircling a moving target with unknown velocity. Two simulations with different values of parameter $\boldsymbol{K}$ are displayed: $\boldsymbol{K} = \boldsymbol{I}_2$ (red dashed lines) and $\boldsymbol{K} = 10\boldsymbol{I}_2$ (blue lines).*

parameters $\boldsymbol{K}$ and δ must be chosen with a view to strike a compromise between a small enough tracking error and a suitable bound on the control inputs.

3.2 Simulation example

To illustrate the performance of the proposed strategy, a numerical simulation is presented in which a circular formation of vehicles is tracking a moving target. In order to easily compare the different results proposed in this paper, in all simulations the trajectory of the time-varying target expressed in meters is defined by $\mathbf{c}(t) = [2 + 0.2t,\ 2 + 3\sin(0.08t + 2)]^T$ and the parameters of the exosystem control law (4) are chosen as $\omega_0 = 1rad/s$, $R_0 = 2m$, $\kappa = 1$, and $\kappa_u = 0.1$.

We present the case in which the velocity of the target $\dot{\mathbf{c}}$ is unknown and thus, the only information available for each agent i is its relative position with respect to the target, $\mathbf{r}_i - \mathbf{c}$. In this situation the convergence of the error position depends strongly on the control parameter $\mathbf{K}$. Note that the unknown target velocity is bounded by $\|\dot{\mathbf{c}}(t)\| \leq \gamma_{\mathbf{c}} = 0.3124m/s$, $\forall t$.

Fig. 2 shows a simulation with 5 agents modeled with unicycle dynamics (1) driven by (8) with the control parameters $\mathbf{K} = \mathbf{I}_2$, $\delta = 0.1m$ and following a time-varying target trajectory whose velocity is unknown. The influence of the control parameter $\mathbf{K}$ is analyzed in Fig. 3. The evolution of the position errors $\mathbf{e}_{x,i}$ (top) and the evolution of the control inputs u_i (bottom) are shown for a simulation of 5 agents governed by (8) with $\delta = 0.1m$ and two different values of $\mathbf{K}$. The red dashed lines display the case in which $\mathbf{K} = \mathbf{I}_2$ and the blue solid ones the case $\mathbf{K} = 10\mathbf{I}_2$. From Theorem 1 we know that at steady-state $\|\mathbf{e}_i - \boldsymbol{\delta}\| \leq \epsilon_{\mathbf{c}}$ for all i, from which it follows

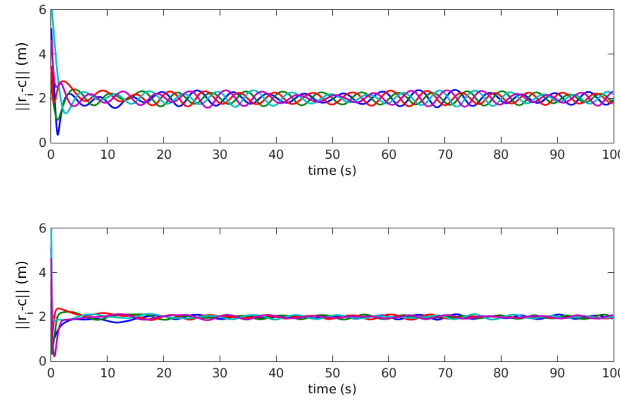

Fig. 4. *Evolution of the relative distance of each agent with respect to the target $\|\boldsymbol{r}_i(t) - \mathbf{c}(t)\|$ for a simulation of 5 agents following and encircling the time-varying target $\mathbf{c}(t)$ when its velocity is unknown, for two values of parameter $\boldsymbol{K}$: $\boldsymbol{K} = \boldsymbol{I}_2$ (top) and $\boldsymbol{K} = 10\boldsymbol{I}_2$ (bottom).*

that

$$-\delta - \epsilon_{\mathbf{c}} \leq \mathbf{e}_{x,i}(\infty) \leq -\delta + \epsilon_{\mathbf{c}},\ \forall i.$$

Numerically, for all i we get $-0.4124m \leq \mathbf{e}_{x,i}(\infty) \leq 0.2124m$ for the first case $\mathbf{K} = \mathbf{I}_2$ and $-0.1312m \leq \mathbf{e}_{x,i}(\infty) \leq -0.0688m$ when $\mathbf{K} = 10\mathbf{I}_2$. The bound on the control input depends strongly on the control parameter $\mathbf{K}$, as shown in Table 1. This figure illustrates that despite the unknown target velocity, the position error can be reduced by increasing the value of $\mathbf{K}$. However, this value plays an important role in the bound of the control input, as stated in Remark 2. To analyze the influence of the control parameter $\mathbf{K}$ in the speed of convergence we define the convergence time t_s such that $\forall t \geq t_s$, $|\mathbf{e}_{x,i}(t) + \delta| \leq \epsilon_{\mathbf{c}}$ for all the agents i. As expected, larger values of the control parameter $\mathbf{K}$ imply faster convergence. Table 1 presents the convergence time in seconds for the simulations shown in Fig. 3.

$\mathbf{K}$	$\varepsilon_{\mathbf{c}}$	t_s	R	$\|u\|_\infty$
$\mathbf{I}_2$	0.31240	3.5	[1.7876, 2.4124]	63.35
$10\mathbf{I}_2$	0.03124	0.6	[2.0688, 2.1312]	482.03

Table 1: Influence of control parameter $\mathbf{K}$ on the ε-stability radius, the convergence time t_s, the final radius R of the circular formation, and the control input u_i bound for the target tracking control law of Theorem 1 when the target velocity is unknown.

Fig. 4 displays the evolution of the relative distances between each agent and the target, $\|\mathbf{r}_i(t) - \mathbf{c}(t)\|$, for a 5 agents simulation governed by (8) with $\delta = 0.1m$ and two values of $\mathbf{K}$. The first simulation (top) displays the case in which $\mathbf{K} = \mathbf{I}_2$ and the second one (bottom) the case $\mathbf{K} = 10\mathbf{I}_2$. These figures show that the vehicles converge to a circular motion centered at the target position and whose final radius converges to the set $\mathcal{R}_{\mathbf{c}} = [R_0 - \delta - \varepsilon_{\mathbf{c}}, R_0 + \delta + \varepsilon_{\mathbf{c}}]$ as presented in Theorem 1. The numerical values of this interval are given in Table 1.

4 Robustness with respect to external disturbances

In this section, we analyze the robustness of the previous target tracking control law with respect to external disturbances.

4.1 Stability result

Consider a group of N agents modeled with unicycle kinematics subject to a simple non-holonomic constraint as stated previously in (1). Consider that the velocity vector of the agents is perturbed such that the model of the agents is now defined by:

$$\begin{aligned}\dot{\mathbf{r}}_i =& \mathbf{R}(\theta_i)\,[v_i,\,0]^T + \mathbf{w}_i \\ \dot{\theta}_i =& u_i\end{aligned} \qquad i = 1, \ldots, N \tag{12}$$

where $\mathbf{w}_i \in \mathbb{R}^2$ is a bounded perturbation vector for which there exists a positive scalar $\gamma_{\mathbf{w}}$ such that

$$\|\mathbf{w}_i\| \leq \gamma_{\mathbf{w}}, \quad \forall i. \tag{13}$$

This perturbation vector can describe several physical and realistic situations, for instance, the friction effect on the wheels of the vehicles, or the flow field in which the agents are moving (air and ocean currents for aerial and underwater scenarios, respectively).

Following the same ideas presented in the previous case in which the target velocity is unknown, the robustness of the proposed control law when external disturbances occur is studied in the sequel. In this situation, the error dynamics become

$$\dot{\mathbf{e}}_i = [v_i,\,0]^T - \mathbf{R}(\theta_i)^T(\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}} - \mathbf{w}_i) - u_i \mathbf{R}_{\frac{\pi}{2}} \mathbf{e}_i.$$

We introduce again the vector $\boldsymbol{\delta}$ and compute

$$\dot{\mathbf{e}}_i = \Delta[v_i,\,u_i]^T - \mathbf{R}(\theta_i)^T(\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}} - \mathbf{w}_i) - u_i \mathbf{R}_{\frac{\pi}{2}}(\mathbf{e}_i - \boldsymbol{\delta}).$$

Following the control design and analysis from Section 3 we can design a controller of the form

$$[v_i,\,u_i]^T = \Delta^{-1}\left(\mathbf{R}(\theta_i)^T(\dot{\hat{\mathbf{r}}}_i + \dot{\mathbf{c}} - \mathbf{w}_i) - \mathbf{K}(\mathbf{e}_i - \boldsymbol{\delta})\right).$$

It is clear that if both the target velocity $\dot{\mathbf{c}}$ and the perturbation vector $\mathbf{w}_i$ are known, the previous control law makes the position error $\mathbf{e}_i - \boldsymbol{\delta}$ converge to zero. However, it is more realistic to consider that only the relative position is measured, as in the previous section, and that the external disturbance is unknown because it cannot be measured or modeled. We aim then to study the robustness of our control strategy, particularly of the proposed control law (8), with respect to disturbances described by $\mathbf{w}_i$. Due to this perturbation term, the error dynamics cannot be driven to zero. We use again the notion of ε-stability to prove the robustness of the proposed target tracking control with respect to external disturbances.

Theorem 2 *Consider a differentiable function $\boldsymbol{c}(t) : \mathbb{R} \to \mathbb{R}^2$ with bounded derivative satisfying (7). Let $R_0 > 0$ be the radius of the desired circular motion, $\omega_0 \neq 0$, $\kappa > 0$ and $\kappa_u > 0$ be three control parameters and $\boldsymbol{K} \in \mathbb{R}^{2\times 2}$ be a symmetric positive definite matrix. Then, the control law (8) where the exosystem $\hat{\boldsymbol{r}}_i$ is defined by (3) and the control law $\hat{u}_i$ defined by (4) makes all the agents defined by (12) converge to a circular motion of radius R, and whose center converges to a ball centered at the time-varying target position $\boldsymbol{c}(t)$ and with radius*

$$\varepsilon_{\boldsymbol{w}} = (\gamma_{\boldsymbol{c}} + \gamma_{\boldsymbol{w}})/\lambda_{min}(\boldsymbol{K}), \tag{14}$$

where $\lambda_{min}(\boldsymbol{K})$ denotes the minimum eigenvalue of matrix $\boldsymbol{K}$. The direction of rotation is determined by the sign of ω_0. Moreover, the relative distance between each agent i and the moving target, i.e., $\|\boldsymbol{r}_i(t) - \boldsymbol{c}(t)\|$, converges to the set $\mathcal{R}_{\boldsymbol{w}} = [R_0 - \delta - \varepsilon_{\boldsymbol{w}}, R_0 + \delta + \varepsilon_{\boldsymbol{w}}]$.

Proof 2 *Following the reasoning used in Theorem 1, we know that the autonomous exosystem is stabilized to a fixed circular motion and thanks to the circular control law (4), when $t \to \infty$, the trajectories of the exosystem satisfy $\dot{\hat{\boldsymbol{r}}}_i(t) = \omega_0 \boldsymbol{R}_{\frac{\pi}{2}} \hat{\boldsymbol{r}}_i(t)$, $\forall i$. We propose the same Lyapunov function defined in (11) with a view to analyze the stability of the error system. Now, the agents' dynamics are described by (12) and therefore, the error dynamics are given by*

$$\begin{aligned}\dot{\boldsymbol{e}}_i =& \boldsymbol{R}(\theta_i)^T(\dot{\boldsymbol{r}}_i - (\dot{\hat{\boldsymbol{r}}}_i + \dot{\boldsymbol{c}})) - u_i \boldsymbol{R}_{\frac{\pi}{2}} \boldsymbol{e}_i \\ =& [v_i,\,0]^T - \boldsymbol{R}(\theta_i)^T(\dot{\hat{\boldsymbol{r}}}_i + \dot{\boldsymbol{c}} - \boldsymbol{w}_i) - u_i \boldsymbol{R}_{\frac{\pi}{2}} \boldsymbol{e}_i.\end{aligned}$$

Following the same control strategy we add the vector $\boldsymbol{\delta}$, yielding the following equation for the error dynamics:

$$\dot{\boldsymbol{e}}_i = \Delta[v_i,\,u_i]^T - \boldsymbol{R}(\theta_i)^T(\dot{\hat{\boldsymbol{r}}}_i + \dot{\boldsymbol{c}} - \boldsymbol{w}_i) - u_i \boldsymbol{R}_{\frac{\pi}{2}}(\boldsymbol{e}_i - \boldsymbol{\delta}).$$

According to the proposed control law (8) the error dynamics become

$$\dot{\boldsymbol{e}}_i = -u_i \boldsymbol{R}_{\frac{\pi}{2}}(\boldsymbol{e}_i - \boldsymbol{\delta}) - \boldsymbol{K}(\boldsymbol{e}_i - \boldsymbol{\delta}) - \boldsymbol{R}(\theta_i)^T(\dot{\boldsymbol{c}} - \boldsymbol{w}_i). \tag{15}$$

Differentiating the Lyapunov function $V(\boldsymbol{e}_i)$ defined by (11) along the solutions of the closed loop (15) yields

$$\begin{aligned}\dot{V}(\boldsymbol{e}_i) =& -(\boldsymbol{e}_i - \boldsymbol{\delta})^T \boldsymbol{K}(\boldsymbol{e}_i - \boldsymbol{\delta}) \\ & -(\boldsymbol{e}_i - \boldsymbol{\delta})^T \boldsymbol{R}(\theta_i)^T(\dot{\boldsymbol{c}} - \boldsymbol{w}_i).\end{aligned}$$

We thus have

$$\begin{aligned}\dot{V}(\boldsymbol{e}_i) \leq& -\lambda_{min}(\boldsymbol{K})\|\boldsymbol{e}_i - \boldsymbol{\delta}\|^2 + \|\boldsymbol{e}_i - \boldsymbol{\delta}\|(\|\dot{\boldsymbol{c}}\| + \|\boldsymbol{w}_i\|) \\ \leq& -\|\boldsymbol{e}_i - \boldsymbol{\delta}\|\left(\lambda_{min}(\boldsymbol{K})\|\boldsymbol{e}_i - \boldsymbol{\delta}\| - \gamma_{\boldsymbol{c}} - \gamma_{\boldsymbol{w}}\right).\end{aligned}$$

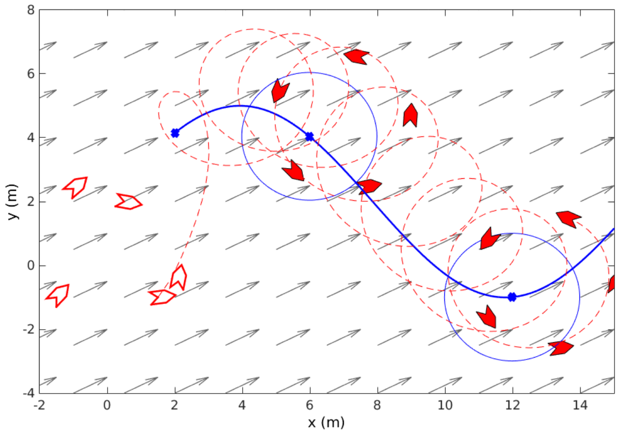

Fig. 5. *Target encircling with a group of 5 agents governed by control law (8) with $\delta = 0.1m$ and $\boldsymbol{K} = \boldsymbol{I}_2$ in an unknown uniform flowfield, $\boldsymbol{f} = [1,\, 0.5]^T m/s$ (grey arrows).*

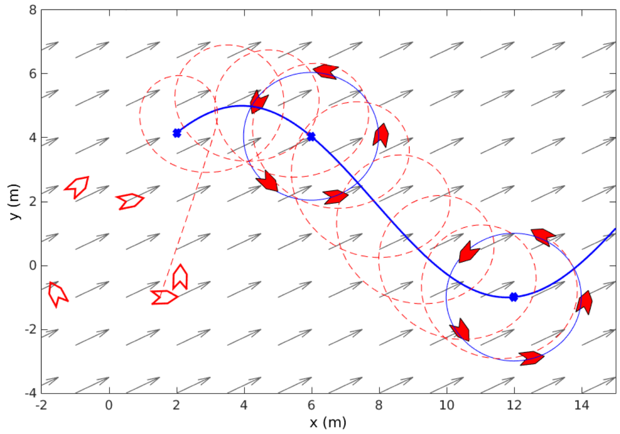

Fig. 6. *Target encircling with a group of 5 agents governed by control law (8) with $\delta = 0.1m$ and $\boldsymbol{K} = 10\boldsymbol{I}_2$ in an unknown uniform flowfield, $\boldsymbol{f} = [1,\, 0.5]^T m/s$ (grey arrows).*

To prove ε-stability of the error dynamics, consider a closed ball $B_{\boldsymbol{w}}$ centered at $\boldsymbol{e}_i = \boldsymbol{\delta}$ with radius $\varepsilon_{\boldsymbol{w}} = (\gamma_{\boldsymbol{c}} + \gamma_{\boldsymbol{w}})/\lambda_{min}(\boldsymbol{K})$. Let $\Omega_{\boldsymbol{w}} = \{\boldsymbol{e}_i - \boldsymbol{\delta} \,|\, V(\boldsymbol{e}_i) \leq \frac{1}{2}\varepsilon_{\boldsymbol{w}}^2\}$. Then $B_{\boldsymbol{w}}$ is contained in $\Omega_{\boldsymbol{w}}$ because

$$\|\boldsymbol{e}_i - \boldsymbol{\delta}\| \leq \varepsilon_{\boldsymbol{w}} \Rightarrow V(\boldsymbol{e}_i) = \frac{1}{2}\|\boldsymbol{e}_i - \boldsymbol{\delta}\|^2 \leq \frac{1}{2}\varepsilon_{\boldsymbol{w}}^2.$$

In conclusion, any solution starting in $\mathbb{R}^2 \setminus \Omega_{\boldsymbol{w}}$ satisfies $\dot{V}(\boldsymbol{e}_i) < 0$ (because $\|\boldsymbol{e}_i - \boldsymbol{\delta}\| \geq (\gamma_{\boldsymbol{c}} + \gamma_{\boldsymbol{w}})/\lambda_{min}(\boldsymbol{K})$) and thus enters $\Omega_{\boldsymbol{w}}$ in finite time and remains in there. Thus, the error dynamics converge asymptotically to a region centered at $\boldsymbol{e}_i = \boldsymbol{\delta}$ and radius $\varepsilon_{\boldsymbol{w}} = (\gamma_{\boldsymbol{c}} + \gamma_{\boldsymbol{w}})/\lambda_{min}(\boldsymbol{K})$. In other words, $\|\boldsymbol{e}_i - \boldsymbol{\delta}\| \leq \varepsilon_{\boldsymbol{w}}$. Following the same reasoning as in Theorem 1, the last inequality implies that the relative distance between each agent i and the target converges to the set $\mathcal{R}_{\boldsymbol{w}} = [R_0 - \delta - \gamma_{\boldsymbol{c}} - \gamma_{\boldsymbol{w}}, R_0 + \delta + \gamma_{\boldsymbol{c}} + \gamma_{\boldsymbol{w}}]$.

Remark 3 *Once again, there is a tradeoff between reducing the position error and bounding the control inputs. As analyzed in Theorem 2, the matrix $\boldsymbol{K}$ plays an important role in the final tracking error of the agents. As shown in Eq. (14), for large values of the minimum eigenvalue of $\boldsymbol{K}$ the radius of the ε-stability is reduced and the agents converge to a circular motion whose center is very close to the target position, despite external disturbances. However, in this situation, both control inputs $(v_i,\, u_i)$ may reach large values. The control parameter $\boldsymbol{K}$, as well as δ, must be chosen with a view to attain a compromise between a small enough tracking error and a suitable bound on the control inputs.*

4.2 Simulation example

In this example, a circular formation of vehicles tracks a moving target in the presence of a uniform unknown flowfield $\mathbf{f} = [1,\, 0.5]^T m/s$, which can be seen as a disturbance $\mathbf{w}_i = \mathbf{f}$, $\forall i$ in model (12). Note that the uniform flowfield is bounded, according to $\|\mathbf{f}\| \leq \gamma_{\mathbf{w}} = 1.118 m/s$.

Figures 5 and 6 show two simulations with 5 agents governed by (8) with $\delta = 0.1m$ and two different values of $\mathbf{K}$, in the presence of the uniform flowfield $\mathbf{f}$. These figures illustrate the influence of the control parameter $\mathbf{K}$ on the final radius of the convergence ball defined in (14) for the ε-stability proved in Theorem 2. The role of the control parameter $\mathbf{K}$ is highlighted in Table 2, where the numerical values of the ε-stability radius, the convergence time, the interval to which the final formation radius converges, and the control input bound are provided for both simulations with $\mathbf{K} = \mathbf{I}_2$ (Fig. 5) and with $\mathbf{K} = 10\mathbf{I}_2$ (Fig. 6).

$\mathbf{K}$	$\varepsilon_{\mathbf{w}}$	t_s	R	$\|u\|_\infty$
$\mathbf{I}_2$	1.1180	1.2	$[0.982, 3.218]$	65.02
$10\mathbf{I}_2$	0.1118	0.4	$[1.9882, 2.2118]$	483.71

Table 2: Influence of control parameter $\mathbf{K}$ on the ε-stability radius, the convergence time t_s, the final radius R of the circular formation, and the bound on the control input u_i for the target tracking control law (8) in the presence of an unknown flowfield.

Fig. 7 shows the evolution of the x-component of the position errors $\mathbf{e}_i$ on the top and of the evolution of the control inputs u_i at the bottom, for a simulation of 5 agents governed by (8) with $\delta = 0.1m$ and two different values of $\mathbf{K}$. The red dashed lines display the case in which $\mathbf{K} = \mathbf{I}_2$ and the blue solid ones the case $\mathbf{K} = 10\mathbf{I}_2$. As shown in the figure, the position error converges to a closed ball centered at $-\delta$, whose radius decreases for larger values of the control parameter $\mathbf{K}$. From Theorem 2 we know that at steady-state $\|\mathbf{e}_i - \delta\| \leq \epsilon_{\mathbf{w}}$ for all i, from which it can be deduced that

$$-\delta - \epsilon_{\mathbf{w}} \leq \mathbf{e}_{x,i}(\infty) \leq -\delta + \epsilon_{\mathbf{w}},\ \forall i.$$

Numerically, we obtain $-1.218m \leq \mathbf{e}_{x,i}(\infty) \leq 1.018m$ for the first case $\mathbf{K} = \mathbf{I}_2$ and $-0.2118m \leq \mathbf{e}_{x,i}(\infty) \leq 0.0118m$ when $\mathbf{K} = 10\mathbf{I}_2$, which is consistent with the

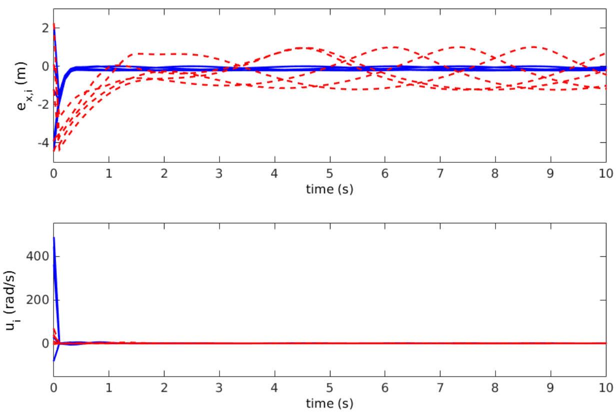

Fig. 7. *Evolution of the x-component of the position error $\mathbf{e}_{x,i}$ (top) and the control input u_i (bottom) for a simulation of 5 agents. Two simulations with different values of parameter $\boldsymbol{K}$ are displayed: $\boldsymbol{K} = \boldsymbol{I}_2$ (red dashed lines) and $\boldsymbol{K} = 10\boldsymbol{I}_2$ (blue lines).*

theoretical analysis. The bound on the control input depends strongly on the control parameter $\mathbf{K}$, as shown in Table 2. This figure illustrates that the larger the value of $\mathbf{K}$ the smaller the position error and the larger the control input are, as stated in Remark 3. As in previous section, we analyze the influence of the control parameter $\mathbf{K}$ on the speed of convergence. The convergence time t_s is now defined such that $\forall t \geq t_s$, $|\mathbf{e}_{x,i}(t_s) + \delta| \leq \epsilon_{\mathbf{w}}$ for all the agents i. As expected, larger values of the control parameter $\mathbf{K}$ imply faster convergence. Table 2 presents the convergence time in seconds for the simulations shown in Fig. 7.

Fig. 8 shows the evolution of the relative distances with respect to the target, $||\mathbf{r}_i(t) - \mathbf{c}(t)||$, for the previous two simulations with 5 agents, shown in Fig. 5 and Fig. 6. Two values of the control parameter $\mathbf{K}$ are considered. In both simulations, the vehicles converge to a circular motion centered at the target position with the final radius converging to the set $\mathcal{R}_{\mathbf{w}} = [R_0 - \delta - \gamma_{\mathbf{c}} - \gamma_{\mathbf{w}}, R_0 + \delta + \gamma_{\mathbf{c}} + \gamma_{\mathbf{w}}]$, as presented in Theorem 2.

5 Conclusions

In this paper, we presented a novel method to solve the target tracking problem with a fleet of unicycle vehicles in a circular configuration. The novelty of this method with respect to previous related work is the possibility to design a circular formation control law that relies only on measurements of the relative positions between the vehicles and the target expressed in the local frame of each vehicle. Currently available methods require at least access to the velocity of the target and in some cases also to the second derivative of the target trajectory. The only price to pay is the introduction of a small and controlled deviation from the desired circular formation. The proposed control strategy is robust with respect to external disturbances. The error between the

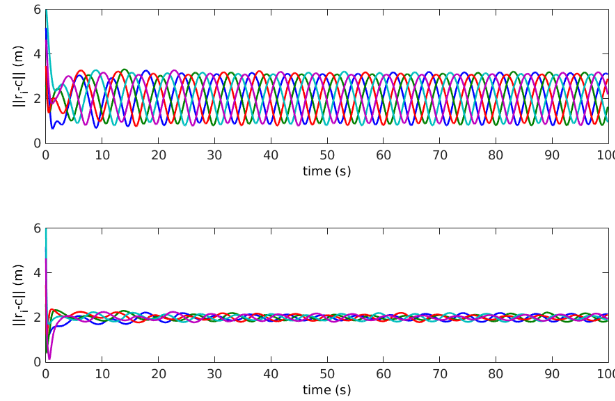

Fig. 8. *Evolution of the relative distance of each agent with respect to the target, $\|\boldsymbol{r}_i(t) - \mathbf{c}(t)\|$, for a simulation of 5 agents following and encircling the time-varying target $\mathbf{c}(t)$ in a flowfield for two values of parameter $\boldsymbol{K}$: $\boldsymbol{K} = \boldsymbol{I}_2$ (top) and $\boldsymbol{K} = 10\boldsymbol{I}_2$ (bottom).*

formation center and the target is bounded. In both situations, two control parameters can be tuned in order to strike a compromise between a small enough position error and a suitable bound on the control inputs.

Future research directions related to this work include cooperative control and the estimation of the target's position based on noisy measurements, as well as optimal controller parameter computation to obtain the minimum position error for a given admissible bound on the control input.

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