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Detection of time shift failures in $(\max, +)$ -linear systems with time intervals for assembly line

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Abstract—In this paper, we address the problem of failure detection in assembly lines modeled as Timed Event Graphs (TEG). The proposed method represents TEGs as $(\max, +)$ -linear systems with time intervals and aims at detecting time shift failures in the underlying assembly lines. To do so, we propose the definition of a set of indicators relying on the residuation theory on $(\max, +)$ linear systems that handle certain and uncertain observable outputs.

Index Terms— $(\max, +)$ -linear system, assembly line, fault detection, discrete event system, timed event graph

I. INTRODUCTION

In industrial systems such as assembly lines, fault diagnosis is usually automatized by using Discrete Event System (DES). Industry also requires that failures are rapidly identified, to avoid systems unavailability for too long. The various system failures can be loss of event information, loss of time information. Among those failures, timing issues can be a problem for instance, an assembly line that slows down will put out fewer pieces. STMicroelectronics is a company that develops, produces and commercializes microchips for electronic system. The semiconductor manufacturing process is extremely complex and constantly innovating. One of the challenges is the monitoring of time drifts for the supervision of an instrumented production chain. This means detecting as soon as possible the time differences between the production plan and the so-called Work In Progress (WIP for short) in order to be able to apply the corrections in WIP quickly without having too much delay in the delivery of the product.

This problem is a subclass of the problems called failure diagnosis in timed discrete event systems. One of the first diagnostic methods is the method extracted from [SSL⁺95] which is applied to timed automata [Tri02]. This diagnostic method refines decisions on the diagnosis by taking into account dated observations. In [GTY09] the diagnostic method uses time Petri net that models competition and/or system parallelism. Among the classes of Petri Nets, Timed Event Graph is a good candidate to represent assembly lines. TEGs are one of the subclasses of Petri nets where places are associated with a punctual duration. Recently, in the article [SLCP17] the method uses $(\max, +)$ algebra to model TEGs in a linear state representation and proposes a method to perform failure diagnosis. This method performs diagnosis on

a fixed time $(\max, +)$ -linear system. In this article, we extend the diagnostic method of the article [SLCP17] by dealing with time intervals in TEGs and by still using a representation with $(\max, +)$ algebra. We first propose a method for detecting time lags in system with certain outputs (i.e. we know exactly what are the output events of the system) that is secondly extended to deal with a method for detecting time lags in system with uncertain outputs (i.e. the occurrence date of the output events is within a given time interval).

The paper is organized as follows. Section II presents a motivation example inspired from the microchip industry. Section III summarizes the necessary mathematical background about $(\max, +)$ -linear systems. Section IV introduces the acceptable outputs in $(\max, +)$ -linear system with time intervals. Section V then defines detection in $(\max, +)$ -linear systems with time intervals with indicator for certain outputs and indicator for uncertain outputs.

II. MOTIVATION EXAMPLE

In the STMicroelectronics plant, several products are produced at the same time. For the production of products there is a several different production plans for the same product there are variations in production time. WIP may change during manufacture depending on equipment availability. These changes may cause delays depending on the delivery date of the product, the WIP have to be corrected. The purpose of the proposed method is to detect when the delay becomes significant and that it is absolutely necessary to make a correction of the WIP in order not to have a delay on the delivery.

Example 1. Figure 1 shows a part of such a plant. It is an assembly line composed of equipments. It is represented as a Time Event Graph where places hold time intervals. Equipment 1 and 2 do the same treatment but not will the same treatment time. The treatment on Equipment 3 need a sufficient number of wafers so you have a synchronization between equipments to do the treatment on Equipment 3. Input u_1 is a flow of timed events corresponding to the arrivals of the wafers on Equipment 1, for the place p_1 the time corresponds to the arrival time of the wafers in front of the equipment. If the equipment is ready, this corresponds to a token in the place

p_3 , the processing of the equipment is carried out on place p_4 and the process duration is between 2 and 5 hours. Input u_2 is a flow of timed events corresponding to the arrivals of the wafers on Equipment 2. For the place p_2 the time corresponds to the arrival time of the wafers in front of the equipment. If the equipment is ready, this corresponds to a token in the place p_6 , the processing of Equipment 2 is carried out on place p_5 and the process duration is between 1 and 3 hours. The processing of Equipment 3 corresponds to place p_{10} and the process lasts exactly 2 hours. This operation can only be processed if there is a sufficient number of wafers coming from Equipments 1 and 2. The time on places p_8 and p_7 corresponds to the arrival times where process lasts 3 hours for p_8 and 5 hours for p_7 . Now suppose that on the input u_1 we have a wafer at $t=0$ then a wafer at $t=1$ then a wafer at $t=2$ and finally a wafer at $t=3$. Suppose now that it is the same on input u_2 . Imagine now that final products are available on the output y respectively at time 12, 15, 18 and 23, is the schedule respected according the TEG of Figure 1 or is there a time drift within the assembly line.

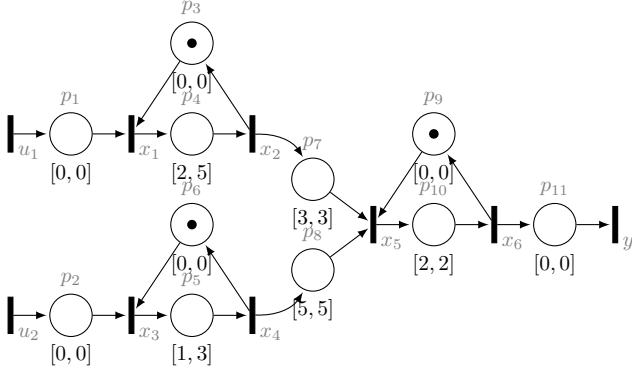


Fig. 1. Representation of assembly line as a Time Event Graph with time intervals

Time Event Graphs, as the one presented in Figure 1, can be formally defined as $(\max, +)$ linear systems that are introduced in the next section. This formalization will be used to the definition of the indicators of time shift failures as the one of Example 1.

III. SCIENTIFIC BACKGROUND

This section recalls the mathematical background that will be used in this paper for describing $(\max, +)$ linear systems with time intervals. For this section, interested reader is invited to peruse [BCOQ92], [Max91], [BHMR] and [LHCJ04].

A. Dioid theory

Definition 1. A dioid \mathcal{D} is a set composed of two internal operations \oplus and \otimes . The addition \oplus is associative, commutative, idempotent (i.e. $\forall a \in \mathcal{D}, a \oplus a = a$) and has a neutral element denoted ε . The multiplication \otimes is associative, distributive on the right and the left over the addition \oplus and has a neutral element denoted e . When there is no ambiguity, the symbol \otimes is omitted.

Definition 2. A dioid is complete if it is closed for infinite sums and if the multiplication \otimes is distributive over infinite sums.

Example 2. The dioid $\mathbb{R}_{\max} = (\mathbb{R} \cup -\infty)$ with the max operation for the law \oplus and the addition for the law \otimes and with $\varepsilon = -\infty$ and $e = 0$. \mathbb{R}_{\max} is not complete because $+\infty$ does not belong to the set \mathbb{R}_{\max} so the infinite sum is not set to $+\infty$. By adding $+\infty$ to the dioid \mathbb{R}_{\max} , we get the complete dioid $\overline{\mathbb{R}}_{\max}$.

Theorem 1. Let \mathcal{D} be a complete dioid, $x = a^*b$ is a solution of the equation $x = ax \oplus b$, where $x = a^*b$, where $a^* = \bigoplus_{i \geq 0} a^i$ the Kleene star operator with $a^0 = e$.

Definition 3. For a dioid \mathcal{D} , \preceq denotes the order relation such that $\forall a, b \in \mathcal{D}, a \preceq b \Leftrightarrow a \oplus b = b$.

Example 3. The complete dioid $\mathbb{B}[\gamma, \delta]$ is the set of formal series with two commutative variables γ and δ with boolean coefficients in $\{\varepsilon, e\}$ and exponents in \mathbb{Z} . A series $s \in \mathbb{B}[\gamma, \delta]$ is written $s = \bigoplus_{n, t \in \mathbb{Z}} s(n, t) \gamma^n \delta^t$ where $s(n, t) = e$ or ε . $\bigoplus_{n, t \in \mathbb{Z}} \gamma^n \delta^t$ and $e = \gamma^0 \delta^0$ are the neutral elements.

Graphically, a series of $\mathbb{B}[\gamma, \delta]$ is described by a collection of point of coordinates (n, t) in \mathbb{Z}^2 with γ as horizontal axis and δ as vertical axis. For instance, Figure 2 shows a couple of series $\underline{p} = \gamma^3 \delta^0 \oplus \gamma^4 \delta^1 \oplus \gamma^5 \delta^2$ and $\underline{p} = \gamma^0 \delta^1 \oplus \gamma^2 \delta^3 \oplus \gamma^4 \delta^5$.

Example 4. The complete dioid $\mathcal{M}_{in}^{ax}[\gamma, \delta]$ is the quotient dioid $\mathbb{B}[\gamma, \delta]$ by the modulo $\gamma^*(\delta^{-1})^*$. The complete dioid $\mathcal{M}_{in}^{ax}[\gamma, \delta]$ with $\forall a, b \in \mathcal{M}_{in}^{ax}[\gamma, \delta]: ab \Leftrightarrow a\gamma^*(\delta^{-1})^* = b\gamma^*(\delta^{-1})^*$. The internal operations are the same for $\mathbb{B}[\gamma, \delta]$ and neutral elements ε and e are identical to those of $\mathbb{B}[\gamma, \delta]$.

Definition 4. Let $s \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$ be a series, the date function of s is the non-decreasing function $D_s(n)$ from $\mathbb{Z} \mapsto \mathbb{Z}$ such that $s = \bigoplus_{n \in \mathbb{Z}} \gamma^n \delta^{D_s(n)}$.

Example 5. Considering the example 1, for the input u_1 must start operating on a wafer at time $t=0$ then on a second wafer at time $t=1$ then on a third at time $t=2$ and finally on a fourth wafer at $t=3$. The date function of this input is written $u_1 = \gamma^0 \delta^0 \oplus \gamma^1 \delta^1 \oplus \gamma^2 \delta^2 \oplus \gamma^3 \delta^3 \oplus \gamma^4 \delta^{+\infty}$ where $D_{u_1}(0) = 0$, $D_{u_1}(1) = 1$, $D_{u_1}(2) = 2$ and $D_{u_1}(3) = 3$.

B. Residuation theory

Residuation is a general notion in lattice theory which allows for the definition of "pseudo-inverse" of some isotone maps. In particular, the residuation allows to find the biggest solution of inequalities such as $\pi(x) \preceq b$.

Definition 5. Let $\pi: \mathcal{D} \mapsto \mathcal{C}$ be an isotone mapping, where \mathcal{D} and \mathcal{C} are complete dioid. The largest solution from $\pi(x) = b$ and noted π^\sharp is called the residual of π . Where π is residuated, π^\sharp is the unique isotone mapping such that $\pi \circ \pi^\sharp \preceq I_{\mathcal{C}}$ and $\pi^\sharp \circ \pi \succeq I_{\mathcal{D}}$ where $I_{\mathcal{C}}$ and $I_{\mathcal{D}}$ are respectively the identity mapping on \mathcal{C} and \mathcal{D} .

Example 6. The mapping $L_a \mapsto a \otimes x$ and $R_a \mapsto x \otimes a$ defined over a complete dioid \mathcal{D} are both residuated. Their residuals are denoted, respectively, by $L_a^\#(x) = a \backslash x = \frac{x}{a}$, $R_a^\#(x) = x \not\backslash a = \frac{x}{a}$.

Thanks to the residuals defined above we will be able to define time comparison between series.

Definition 6. Let $a, b \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$ and their respective dater function \mathcal{D}_a and \mathcal{D}_b , the time shift function representing the time shift between a and b for each $n \in \mathbb{Z}$ is defined by $\mathcal{T}_{a,b}(n) = \mathcal{D}_a - \mathcal{D}_b$.

Theorem 2. Let $a, b \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$, the time shift function can be bounded by:

$$\forall n \in \mathbb{Z}, \quad \mathcal{D}_{b \not\backslash a}(0) \leq \mathcal{T}_{a,b}(n) \leq -\mathcal{D}_{a \not\backslash b}(0).$$

This theorem defines some minimum and maximum bounds for the time shift function. The extraction of the bound is done on the series $a \not\backslash b$ and $b \not\backslash a$. The $\mathcal{D}_{b \not\backslash a}(0)$ bound is obtained from the $\gamma^0 \delta^{\mathcal{D}_{b \not\backslash a}(0)} \in b \not\backslash a$. The $\mathcal{D}_{a \not\backslash b}(0)$ bound is obtained from the $\gamma^0 \delta^{\mathcal{D}_{a \not\backslash b}(0)} \in a \not\backslash b$.

Definition 7. Let $a, b \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$, the time shift between series a and b is

$$\Delta(a, b) = [\mathcal{D}_{b \not\backslash a}(0); -\mathcal{D}_{a \not\backslash b}(0)], \quad (1)$$

where $\gamma^0 \delta^{\mathcal{D}_{b \not\backslash a}(0)} \in b \not\backslash a$ and $\gamma^0 \delta^{\mathcal{D}_{a \not\backslash b}(0)} \in a \not\backslash b$. In this interval, the reference series, that is the series from which the time offset is measured is the series a .

Example 7. Let us consider two different observable outputs y from the system modeled by Figure 1: $y_1 = \gamma^0 \delta^{12} \oplus \gamma^1 \delta^{15} \oplus \gamma^2 \delta^{18} \oplus \gamma^3 \delta^{21} \oplus \gamma^4 \delta^{+ \infty}$ (i.e delivery of final products at time 12, 15, 18, 23), and $y_2 = \gamma^0 \delta^{12} \oplus \gamma^1 \delta^{15} \oplus \gamma^2 \delta^{19} \oplus \gamma^3 \delta^{23} \oplus \gamma^4 \delta^{+ \infty}$ (i.e delivery of final products at time 12, 15, 19, 23). The minimal time shift between y_1 and y_2 is $\mathcal{D}_{y_2 \not\backslash y_1}(0) = 0$ and is found in $y_2 \not\backslash y_1 = \gamma^0 \delta^0 \oplus \gamma^1 \delta^3 \oplus \gamma^2 \delta^7 \oplus \gamma^3 \delta^{11} \oplus \gamma^4 \delta^{+ \infty}$. The maximal time shift is $-\mathcal{D}_{y_1 \not\backslash y_2}(0) = 2$ and is found in the monomial where the degree of γ is 0 of $y_1 \not\backslash y_2 = \gamma^0 \delta^{-2} \oplus \gamma^1 \delta^2 \oplus \gamma^2 \delta^6 \oplus \gamma^3 \delta^9 \oplus \gamma^4 \delta^{+ \infty}$.

After having introduced the residuation and comparison between series them we interval dioid theory that will allow to introduce (max,+)-linear systems with time intervals.

C. Interval dioid theory

Definition 8. A closed interval in dioid \mathcal{D} is a set of the form $\mathbf{x} = \{t \in \mathcal{D} | \underline{x} \preceq t \preceq \bar{x}\}$ denoted by $\mathbf{x} = [\underline{x}, \bar{x}]$.

Definition 9. The set of intervals over \mathcal{D} denoted by $I(\mathcal{D})$ endowed with the following element-wise algebraic operations $\mathbf{x} \oplus \mathbf{y} = [\underline{x} \oplus \underline{y}, \bar{x} \oplus \bar{y}]$ and $\mathbf{x} \otimes \mathbf{y} = [\underline{x} \otimes \underline{y}, \bar{x} \otimes \bar{y}]$ with $\mathbf{x} = \{x \in \mathcal{D} | \underline{x} \preceq x \preceq \bar{x}\}$ and $\mathbf{y} = \{y \in \mathcal{D} | \underline{y} \preceq y \preceq \bar{y}\}$ where the interval $\varepsilon = [\varepsilon, \varepsilon]$ is neutral element of \oplus and the interval $e = [e, e]$ is neutral element of \otimes .

The order relation \preceq in $I(\mathcal{D})$ induced by the additive law \oplus is such that

$$\mathbf{x} \oplus \mathbf{y} = \mathbf{y} \Leftrightarrow \mathbf{x} \preceq \mathbf{y} \Leftrightarrow \begin{cases} \underline{x} \preceq \underline{y} \text{ in } \mathcal{D} \\ \bar{x} \preceq \bar{y} \text{ in } \mathcal{D} \end{cases}$$

Let $I(\mathcal{M}_{in}^{ax}[\gamma, \delta])$ denote the set of intervals of the dioid $\mathcal{M}_{in}^{ax}[\gamma, \delta]$. An interval defined over $I(\mathcal{M}_{in}^{ax}[\gamma, \delta])$ corresponds to all the series between the series of the minimum bound and the series of the maximum bound. For this purpose, the representation of the series in $\mathcal{M}_{in}^{ax}[\gamma, \delta]$ is used for the plotting of the series of the lower and upper bounds.

Example 8. Back to Figure 2 the couple of polynomials represents the interval

$$\mathbf{p} = [\underline{p}, \bar{p}] = [\gamma^3 \delta^0 \oplus \gamma^4 \delta^1 \oplus \gamma^5 \delta^2, \gamma^0 \delta^1 \oplus \gamma^2 \delta^3 \oplus \gamma^4 \delta^5].$$

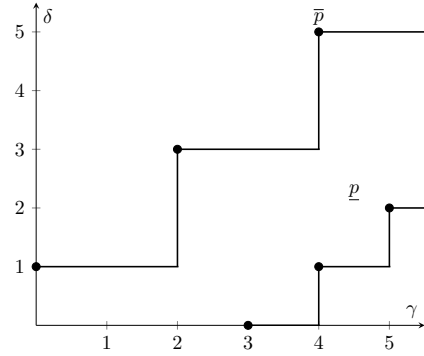


Fig. 2. Interval representation of $\mathbf{p} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])$

D. Models of (max,+)-linear systems with time intervals

The objective is to define a Time Event Graph with time intervals (see Figure 1) as a (max,+)-linear system with time intervals. The elements of the TEG will be represented by equations in $I(\mathcal{M}_{in}^{ax}[\gamma, \delta])$. The equations can be grouped into a set of matrices \mathbf{A} , \mathbf{B} and \mathbf{C} called the state representation of the system that defines the relations between any set of input event flows u and the state x and the relations between the state x and the output event flows y . Let $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])^{p \times 1}$ be the input vector of size p , $\mathbf{x} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])^{n \times 1}$ be the state vector of size n and $\mathbf{y} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])^{q \times 1}$ be the output vector of size q . The state representation is:

$$\begin{cases} \mathbf{x} = \mathbf{Ax} \oplus \mathbf{Bu}, \\ \mathbf{y} = \mathbf{Cx}. \end{cases}$$

where $\mathbf{A} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])^{n \times n}$, $\mathbf{B} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])^{n \times p}$ and $\mathbf{C} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])^{q \times n}$.

$\mathbf{x} = \mathbf{Ax} \oplus \mathbf{Bu}$ can be transformed to $\mathbf{x} = \mathbf{A}^* \mathbf{Bu}$ thanks to Theorem 1 so we have

$$\mathbf{y} = \mathbf{CA}^* \mathbf{Bu}.$$

The matrix $\mathbf{H} = \mathbf{CA}^* \mathbf{B}$ represents the transfer function of the TEG.

Example 9. For the system of Figure 1 the matrices $\mathbf{A} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])^{6 \times 6}$, $\mathbf{B} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])^{6 \times 2}$ and $\mathbf{C} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])^{1 \times 6}$ of the state representation are:

$$\begin{aligned}
A(1;2) &= [\gamma^1\delta^0, \gamma^1\delta^0]; & A(2;1) &= [\gamma^0\delta^2, \gamma^0\delta^5]; \\
A(3;4) &= [\gamma^1\delta^0, \gamma^1\delta^0]; & A(4;3) &= [\gamma^0\delta^1, \gamma^0\delta^3]; \\
A(5;2) &= [\gamma^0\delta^3, \gamma^0\delta^3]; & A(5;4) &= [\gamma^0\delta^5, \gamma^0\delta^5]; \\
A(5;6) &= [\gamma^1\delta^0, \gamma^1\delta^0]; & A(6;5) &= [\gamma^0\delta^2, \gamma^0\delta^2].
\end{aligned}$$

$$B(1;1) = [\gamma^0\delta^0, \gamma^0\delta^0]; \quad B(3;2) = [\gamma^0\delta^0, \gamma^0\delta^0];$$

$$C(1;6) = [\gamma^0\delta^0, \gamma^0\delta^0];$$

Any other elements in the matrices correspond to ε .

In any element of the matrices, the exponent n of γ represents the backward event shift between transitions (the $n + 1^{\text{th}}$ firing of x_1 depend on the n^{th} firing of x_2) and the exponent of δ represents the backward time shift between transition (the firing date of x_2 depend on the firing date of x_1 and time between 5 and 7).

IV. ACCEPTABLE OUTPUT IN (MAX,+)-LINEAR SYSTEMS WITH TIME INTERVALS

This section present the tool using to define acceptable output of this system. This section introduces the acceptable outputs of a (max,+)-linear system with time intervals. Acceptable outputs correspond to the observable outputs that are included in the interval of the predicted output. The hypothesis for the different definitions that we will define inputs are observable and outputs are observable.

A. Time comparison between output in (max,+)-linear systems with time intervals

For a (max,+)-linear system with time interval with $\mathbf{H} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{q \times p})$ as a transfer function and $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{p \times 1})$ as an observed input one can obtain the predicted outputs of the system $\mathbf{y}_p = \mathbf{H}\mathbf{u} = [\underline{y}_p, \bar{y}_p]$. The predicted outputs are the observable trajectories obtained with the knowledge of the system. With the time shift bounds given in Definition 9 we define the time shifts of the (max,+)-linear systems as time interval: that is the difference between \bar{y}_p and \underline{y}_p .

Definition 10. Let $\mathbf{y}_p = \mathbf{H}\mathbf{u} = [\underline{y}_p, \bar{y}_p]$, with $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{p \times 1})$, $\mathbf{H} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{q \times p})$, with \mathbf{u} a degenerated input and \mathbf{H} a transfer function. The system gap with \bar{y}_p in reference is:

$$\Delta_{\bar{y}_p} = [\mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0); -\mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0)] = [\underline{\Delta}_{\bar{y}_p}, \bar{\Delta}_{\bar{y}_p}].$$

The system gap with system gap with \underline{y}_p in reference is:

$$\Delta_{\underline{y}_p} = [\mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0); -\mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0)] = [\underline{\Delta}_{\underline{y}_p}, \bar{\Delta}_{\underline{y}_p}].$$

The system gap with \bar{y}_p in reference corresponds to the gap form \underline{y}_p to \bar{y}_p . Since \mathbf{y}_p is an interval $\bar{y}_p \succeq \underline{y}_p$ so $\mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0) \leq 0$ and $-\mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0) \leq 0$.

The system gap system gap with \underline{y}_p in reference corresponds to the gap of \bar{y}_p from \underline{y}_p . Since \mathbf{y}_p is an interval we have $\bar{y}_p \succeq \underline{y}_p$ so $\mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0) \geq 0$ and $-\mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0) \geq 0$.

Example 10. Given the TEG of Figure 1, suppose that Equipement 1 (resp. 2), this results in the following observable inputs $\mathbf{u}_1 = [\underline{u}_1, \bar{u}_1] = [\gamma^0\delta^0 \oplus \gamma^1\delta^1 \oplus \gamma^2\delta^2 \oplus \gamma^3\delta^3 \oplus \gamma^4\delta^{+\infty}, \gamma^0\delta^0 \oplus \gamma^1\delta^1 \oplus \gamma^2\delta^2 \oplus \gamma^3\delta^3 \oplus \gamma^4\delta^{+\infty}]$ when $\underline{u}_1 = \bar{u}_1$ and $\mathbf{u}_2 = [\underline{u}_2, \bar{u}_2] = [\gamma^0\delta^0 \oplus \gamma^1\delta^1 \oplus \gamma^2\delta^2 \oplus \gamma^3\delta^3 \oplus \gamma^4\delta^{+\infty}, \gamma^0\delta^0 \oplus \gamma^1\delta^1 \oplus \gamma^2\delta^2 \oplus \gamma^3\delta^3 \oplus \gamma^4\delta^{+\infty}]$ when $\underline{u}_2 = \bar{u}_2$. With this observable inputs we get the predictive output $\mathbf{y}_p = [\underline{y}_p, \bar{y}_p] = [\gamma^0\delta^9 \oplus \gamma^1\delta^{11} \oplus \gamma^2\delta^{13} \oplus \gamma^3\delta^{15} \oplus \gamma^4\delta^{+\infty}, \gamma^0\delta^{11} \oplus \gamma^1\delta^{16} \oplus \gamma^2\delta^{21} \oplus \gamma^3\delta^{26} \oplus \gamma^4\delta^{+\infty}]$. The system gap with \bar{y}_p in reference is $\Delta_{\bar{y}_p} = [\mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0); -\mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0)] = [\underline{\Delta}_{\bar{y}_p}, \bar{\Delta}_{\bar{y}_p}] = [-11, -2]$ where $\mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0)$ found in $\underline{y}_p \not\prec \bar{y}_p = \gamma^0\delta^{-11} \oplus \gamma^1\delta^{-6} \oplus \gamma^2\delta^{-1} \oplus \gamma^3\delta^4 \oplus \gamma^4\delta^{+\infty}$ and $\mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0)$ found in $\bar{y}_p \not\prec \underline{y}_p = \gamma^0\delta^2 \oplus \gamma^1\delta^7 \oplus \gamma^2\delta^{12} \oplus \gamma^3\delta^{17} \oplus \gamma^4\delta^{+\infty}$. The system gap with \underline{y}_p in reference is $\Delta_{\underline{y}_p} = [\mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0); -\mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0)] = [\underline{\Delta}_{\underline{y}_p}, \bar{\Delta}_{\underline{y}_p}] = [2, 11]$ where $\mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0)$ found in $\bar{y}_p \not\prec \underline{y}_p = \gamma^0\delta^5 \oplus \gamma^1\delta^7 \oplus \gamma^2\delta^{12} \oplus \gamma^3\delta^{17} \oplus \gamma^4\delta^{+\infty}$ and $-\mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0)$ found in $\underline{y}_p \not\prec \bar{y}_p = \gamma^0\delta^{-11} \oplus \gamma^1\delta^{-6} \oplus \gamma^2\delta^{-1} \oplus \gamma^3\delta^4 \oplus \gamma^4\delta^{+\infty}$.

B. Acceptable outputs

Definition 11. Let $\mathbf{H} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{q \times p})$ transfer function, $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{p \times 1})$ a observable interval input, $\mathbf{H}\mathbf{u} = \mathbf{y}_p = [\underline{y}_p, \bar{y}_p]$ the observable predicted output, a observable output y_{acc} is a output acceptable if y_{acc} must respect the following condition $\underline{y}_p \preceq y_{acc} \preceq \bar{y}_p$. The set of acceptable outputs denote $\mathcal{Y}_{acc} \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$.

Proposition 1. $y_{acc} \in \mathcal{Y}_{acc}$ is acceptable output if and only if the following 4 conditions hold:

$$\mathcal{D}_{y_{acc} \not\prec \bar{y}_p}(0) \in [\underline{\Delta}_{\bar{y}_p}, 0], \quad (2)$$

$$-\mathcal{D}_{\bar{y}_p \not\prec y_{acc}}(0) \in [\bar{\Delta}_{\bar{y}_p}, 0], \quad (3)$$

$$\mathcal{D}_{y_{acc} \not\prec \underline{y}_p}(0) \in [0, \underline{\Delta}_{\underline{y}_p}], \quad (4)$$

$$-\mathcal{D}_{\underline{y}_p \not\prec y_{acc}}(0) \in [0, \bar{\Delta}_{\underline{y}_p}]. \quad (5)$$

Proof. For the superior bound case $y_{acc} = \bar{y}_p$, the time shift between $\mathbf{y}_p = [\underline{y}_p, \bar{y}_p]$ and the observable output y_{acc} is $\mathcal{D}_{y_{acc} \not\prec \bar{y}_p}(0) = 0$ and $\mathcal{D}_{\bar{y}_p \not\prec y_{acc}}(0) = 0$ because $y_{acc} = \bar{y}_p$. It follows that $\mathcal{D}_{y_{acc} \not\prec \underline{y}_p}(0) = \mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0)$ and $\mathcal{D}_{\underline{y}_p \not\prec y_{acc}}(0) = \mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0)$.

For the lower bound case $y_{acc} = \underline{y}_p$, the time shift is $\mathcal{D}_{y_{acc} \not\prec \underline{y}_p}(0) = 0$ and $\mathcal{D}_{\underline{y}_p \not\prec y_{acc}}(0) = 0$ because $y_{acc} = \underline{y}_p$. Thus, $\mathcal{D}_{y_{acc} \not\prec \bar{y}_p}(0) = \mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0)$ and $\mathcal{D}_{\bar{y}_p \not\prec y_{acc}}(0) = \mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0)$.

As $\underline{y}_p \preceq y_{acc} \preceq \bar{y}_p$, $\mathcal{D}_{y_{acc} \not\prec \bar{y}_p}(0) \in [\underline{\Delta}_{\bar{y}_p}, 0]$ because y_{acc} evolves between \underline{y}_p and \bar{y}_p so $\mathcal{D}_{y_{acc} \not\prec \bar{y}_p}(0)$ can only evolves between limit cases $\mathcal{D}_{y_{acc} \not\prec \bar{y}_p}(0) = \mathcal{D}_{\underline{y}_p \not\prec \bar{y}_p}(0)$ and $\mathcal{D}_{y_{acc} \not\prec \bar{y}_p}(0) = 0$. We get as $-\mathcal{D}_{\bar{y}_p \not\prec y_{acc}}(0) \in [\bar{\Delta}_{\bar{y}_p}, 0]$ y_{acc} evolves between \underline{y}_p and \bar{y}_p so $-\mathcal{D}_{\bar{y}_p \not\prec y_{acc}}(0)$ can only evolves between limit cases $\mathcal{D}_{\bar{y}_p \not\prec y_{acc}}(0) = \mathcal{D}_{\bar{y}_p \not\prec \underline{y}_p}(0)$ and $\mathcal{D}_{\bar{y}_p \not\prec y_{acc}}(0) = 0$. Following the same reasoning, $\mathcal{D}_{y_{acc} \not\prec \underline{y}_p}(0)$

and $-\mathcal{D}_{\underline{y}_p \neq y_{acc}}(0)$. So if $y_{acc} \in \mathcal{Y}_{acc}$ then conditions 2, 3, 4 and 5 hold.

If $\mathcal{D}_{y_{acc} \neq \bar{y}_p}(0) \in [\underline{\Delta}_{\bar{y}_p}, 0]$, it involve the worst case $\mathcal{D}_{y_{acc} \neq \bar{y}_p}(0) = \underline{\Delta}_{\bar{y}_p}$ that gives us $y_{acc} = \underline{y}_p$ or $\mathcal{D}_{y_{acc} \neq \bar{y}_p}(0) = 0$ that gives us $y_{acc} = \bar{y}_p$ so $\underline{y}_p \preceq y_{acc} \preceq \bar{y}_p$. If $-\mathcal{D}_{\bar{y}_p \neq y_{acc}}(0) \in [\underline{\Delta}_{\bar{y}_p}, 0]$, it involve the worst case $-\mathcal{D}_{\bar{y}_p \neq y_{acc}}(0) = \underline{\Delta}_{\bar{y}_p}$ that gives use $y_{acc} = \underline{y}_p$ where $-\mathcal{D}_{\bar{y}_p \neq y_{acc}}(0) = 0$ that gives us $y_{acc} = \bar{y}_p$ so $\underline{y}_p \preceq y_{acc} \preceq \bar{y}_p$. If $\mathcal{D}_{y_{acc} \neq \underline{y}_p}(0) \in [0, \underline{\Delta}_{\underline{y}_p}]$ it involve the worst case $\mathcal{D}_{y_{acc} \neq \underline{y}_p}(0) = 0$ that gives us $y_{acc} = \underline{y}_p$ where $\mathcal{D}_{y_{acc} \neq \underline{y}_p}(0) = \underline{\Delta}_{\underline{y}_p}$ that gives us $y_{acc} = \bar{y}_p$ so $\underline{y}_p \preceq y_{acc} \preceq \bar{y}_p$. If $-\mathcal{D}_{\underline{y}_p \neq y_{acc}}(0) \in [0, \underline{\Delta}_{\underline{y}_p}]$ it involve the worst case $-\mathcal{D}_{\underline{y}_p \neq y_{acc}}(0) = 0$ that gives us $y_{acc} = \underline{y}_p$ where $-\mathcal{D}_{\underline{y}_p \neq y_{acc}}(0) = \underline{\Delta}_{\underline{y}_p}$ that gives us $y_{acc} = \bar{y}_p$. So if the 4 conditions 2, 3, 4 and 5 are true then $y_{acc} \in \mathcal{Y}_{acc}$. \square

Example 11. $\mathbf{y}_p = [\underline{y}_p, \bar{y}_p] = [\gamma^0\delta^7 \oplus \gamma^1\delta^9 \oplus \gamma^2\delta^{11} \oplus \gamma^3\delta^{13} \oplus \gamma^4\delta^{+}, \gamma^0\delta^{10} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{20} \oplus \gamma^3\delta^{25} \oplus \gamma^4\delta^{+}]$ is represented by the series in plain line in figure 3 with the system gap $\underline{\Delta}_{\bar{y}_p} = [-12, -3]$ and $\underline{\Delta}_{\underline{y}_p} = [3, 12]$.

The observable output $y_1 = \gamma^0\delta^8 \oplus \gamma^1\delta^{11} \oplus \gamma^2\delta^{14} \oplus \gamma^3\delta^{17} \oplus \gamma^4\delta^{+}$ is represented by the series in dashed line and the observable output $y_2 = \gamma^0\delta^7 \oplus \gamma^2\delta^9 \oplus \gamma^3\delta^{13} \oplus \gamma^4\delta^{+}$ is represented by the series in dotted line.

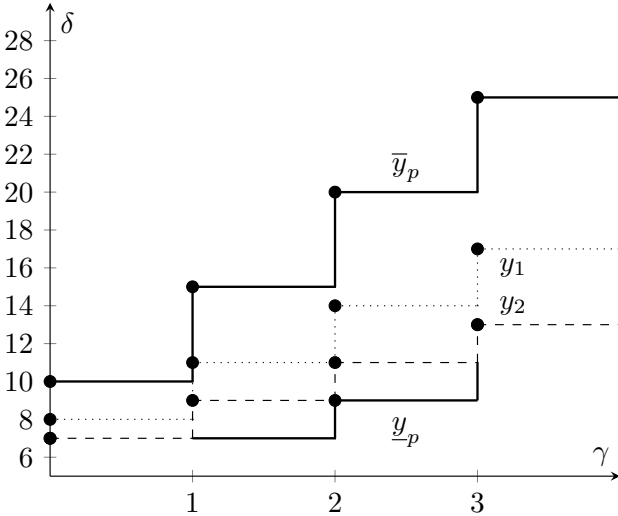


Fig. 3. Graphical representation of the acceptable output y_1 and the non-acceptable output y_2 with respect to interval $\mathbf{y}_p = [\underline{y}_p, \bar{y}_p]$

The output y_1 belongs to the set of acceptable outputs, it indicates $-8 \in [-12, 0]$ and $-2 \in [-3, 0]$ and $1 \in [0, 3]$ and $4 \in [0, 12]$. The output y_2 does not belongs to the set of acceptable outputs, it indicates $-12 \in [-12, 0]$ and $-3 \in [-3, 0]$ and $-3 \notin [0, 3]$ and $0 \in [0, 12]$.

V. FAULT DETECTION IN (MAX,+)-LINEAR SYSTEMS WITH TIME INTERVAL

In this section we will present indicators to detect faults in (max,+)-linear systems with time interval. Thanks to what we

learned in the section IV we can tell if any observable output belongs to the set of acceptable outputs or not so that we can make an indicator to tell us if a detectable failure is detected.

A. Indicator for certain output

For the certain output indicator, the outputs and inputs are observable. To be able to make the diagnosis it is necessary to detect the failures for that it is necessary to define the detectable failures.

Definition 12. A system failure is detectable if the observations of the system it generates do not correspond to the expected observations.

Definition 13. Let $(\max, +)$ -linear system with $H \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{q \times p})$ the transfer function, $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{p \times 1})$ the observed inputs, $y_o \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{q \times 1}$ the observed output. The indicator $I_I(\mathbf{u}, y_{obs})$ of this system is a function

$$I_I(\mathbf{u}, y_o) = \begin{cases} \text{false} & \text{if for } \mathbf{y}_p = H\mathbf{u}, -\mathcal{D}_{\bar{y}_p \neq y_o}(0) \in [\underline{\Delta}_{\bar{y}_p}, 0] \\ & \text{and } \mathcal{D}_{y_o \neq \underline{y}_p}(0) \in [0, \underline{\Delta}_{\underline{y}_p}] \\ \text{true} & \text{otherwise.} \end{cases} \quad (6)$$

Proposition 2. The indicator returns true if and only if the system receive a detectable failure.

Proof. Let a system sudden a detectable failure, this implies that the observed output is not in the set of acceptable outputs of the system and is therefore one of the conditions 2, 3, 4 and 5 is true. Yet if 2 is true with 4 is true and if 5 is true with 3 is true. Suppose the indicator returns false then 4 and 3 are false: the indicator cannot therefore be false at the same time as the system is with detectable failure.

Let be a system to which the indicator refers true. The indicator that returns true implies that 4 and 3 are false. Suppose that the system not sudden a detectable failure then the output is in the set of acceptable outputs of the system so the conditions 2, 3, 4 and 5 are true. So a system without detectable failure this indicator don't cannot be true. \square

Example 12. Back to Figure 1, consider the scenario that is defined by the inputs of Example 10 and suppose that in reality there was an incident on Equipement 1 the operation lasts longer will a processing time of 6 hours in place P1. The TEG has thus the following system gap: $\underline{\Delta}_{\bar{y}_p} = [-11, -2]$ and $\underline{\Delta}_{\underline{y}_p} = [2, 11]$. The incident on Equipement 1 gives us the following observable output $y_o = \gamma^0\delta^{12} \oplus \gamma^1\delta^{16} \oplus \gamma^2\delta^{21} \oplus \gamma^3\delta^{26} \oplus \gamma^4\delta^{+}$. Computations of $y_o \neq \underline{y}_p$ and $\bar{y}_p \neq y_o$ give for y_o : $y_o \neq \underline{y}_p = \gamma^0\delta^3 \oplus \gamma^1\delta^9 \oplus \gamma^2\delta^{15} \oplus \gamma^3\delta^{21} \oplus \gamma^4\delta^{+}$ and $\bar{y}_p \neq y_o = \gamma^0\delta^{-4} \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^8 \oplus \gamma^3\delta^{14}$. The residuals gives us the following results $\mathcal{D}_{y_o \neq \underline{y}_p}(0) = 3 \notin [0, \underline{\Delta}_{\underline{y}_p}] = [0, 2]$ and $-\mathcal{D}_{\bar{y}_p \neq y_o}(0) = -4 \in [\underline{\Delta}_{\bar{y}_p}, 0] = [-11, 0]$. This indicator $I_I(\mathbf{u}, y_o)$ indicates false because $\mathcal{D}_{y_o \neq \underline{y}_p}(0) = 3 \notin [0, \underline{\Delta}_{\underline{y}_p}] = [0, 2]$ is false.

B. Indicator for uncertain output

This section present indicator with uncertain output. For the uncertain output indicator, the outputs and inputs are observable. We will have to define the output sets acceptable intervals.

Definition 14. Let $(\max, +)$ -linear system with $H \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{q \times p})$ the transfer function, $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta]^{p \times 1})$ the observed inputs, $\mathbf{y}_o \in I(\mathcal{M}_{in}^{ax}[\gamma, \delta])$ the observed outputs. The indicator $I_I(\mathbf{u}, \mathbf{y}_{obs})$ of this system is a function:

$$I_I(\mathbf{u}, \mathbf{y}_o) = \begin{cases} \text{false if for } \mathbf{y}_p = \mathbf{H}\mathbf{u}, & -\mathcal{D}_{\bar{\mathbf{y}}_p \neq \bar{\mathbf{y}}_o}(0) \in [\bar{\Delta}_{\bar{\mathbf{y}}_p}, 0] \\ & \text{and } \mathcal{D}_{\mathbf{y}_o \neq \mathbf{y}_p}(0) \in [0, \underline{\Delta}_{\mathbf{y}_p}]. \\ \text{true otherwise.} \end{cases} \quad (7)$$

Proposition 3. The indicator returns true if and only if the system receive a detectable failure.

Proof. Let a system sudden a detectable failure, this implies that the observed output is not in the set of acceptable outputs of the system so either $\underline{\mathbf{y}}_o \prec \underline{\mathbf{y}}_p$, either $\bar{\mathbf{y}}_p \prec \bar{\mathbf{y}}_o$ or $\underline{\mathbf{y}}_o \prec \underline{\mathbf{y}}_p \preceq \bar{\mathbf{y}}_p \prec \bar{\mathbf{y}}_o$. When $\underline{\mathbf{y}}_o \prec \underline{\mathbf{y}}_p$ compare with the acceptable outputs of the proposition 1 it gives us as a result 4 false. When $\bar{\mathbf{y}}_p \prec \bar{\mathbf{y}}_o$ compare with the acceptable outputs of the proposition 1 it gives us as a result 3 false. Suppose the indicator returns false then $-\mathcal{D}_{\bar{\mathbf{y}}_p \neq \bar{\mathbf{y}}_o}(0) \in [\bar{\Delta}_{\bar{\mathbf{y}}_p}, 0]$ and $\mathcal{D}_{\mathbf{y}_o \neq \mathbf{y}_p}(0) \in [0, \underline{\Delta}_{\mathbf{y}_p}]$ is true: the indicator cannot therefore be false at the same time as the system is with detectable failure. Let be a system to which the indicator refers true. The indicator that returns true implies that $-\mathcal{D}_{\bar{\mathbf{y}}_p \neq \bar{\mathbf{y}}_o}(0) \in [\bar{\Delta}_{\bar{\mathbf{y}}_p}, 0]$ and $\mathcal{D}_{\mathbf{y}_o \neq \mathbf{y}_p}(0) \in [0, \underline{\Delta}_{\mathbf{y}_p}]$ are false. Suppose that the system not sudden a detectable failure then the output is in the set of acceptable outputs of the system so $\underline{\mathbf{y}}_p \preceq \underline{\mathbf{y}}_o \preceq \bar{\mathbf{y}}_o \preceq \bar{\mathbf{y}}_p$ which gives us $-\mathcal{D}_{\bar{\mathbf{y}}_p \neq \bar{\mathbf{y}}_o}(0) \in [\bar{\Delta}_{\bar{\mathbf{y}}_p}, 0]$ and $\mathcal{D}_{\mathbf{y}_o \neq \mathbf{y}_p}(0) \in [0, \underline{\Delta}_{\mathbf{y}_p}]$ true. So a system without detectable failure this indicator don't cannot be true. \square

Example 13. Let a TEG in example 1 with the system gap following $\bar{\Delta}_{\bar{\mathbf{y}}_p} = [-11, -2]$ and $\underline{\Delta}_{\mathbf{y}_p} = [2, 11]$ for the same inputs of example 10. For our TEG we will observe outputs \mathbf{y}_o (i.e delivery of final products at time between 9 and 12, 11 and 16, 13 and 21, 15 and 21). $\mathbf{y}_o = [\gamma^0\delta^9 \oplus \gamma^1\delta^{11} \oplus \gamma^2\delta^{13} \oplus \gamma^3\delta^{15} \oplus \gamma^4\delta^{+\infty}; \gamma^0\delta^{12} \oplus \gamma^1\delta^{16} \oplus \gamma^2\delta^{21} \oplus \gamma^3\delta^{26} \oplus \gamma^4\delta^{+\infty}]$. Computation of $\underline{\mathbf{y}}_o \neq \underline{\mathbf{y}}_p$ and $\bar{\mathbf{y}}_p \neq \bar{\mathbf{y}}_o$ give for \mathbf{y}_o : $\underline{\mathbf{y}}_o \neq \underline{\mathbf{y}}_p = \gamma^0\delta^3 \oplus \gamma^1\delta^9 \oplus \gamma^2\delta^{15} \oplus \gamma^3\delta^{21} \oplus \gamma^4\delta^{+\infty}$ and $\bar{\mathbf{y}}_p \neq \bar{\mathbf{y}}_o = \gamma^0\delta^2 \oplus \gamma^1\delta^7 \oplus \gamma^2\delta^{12} \oplus \gamma^3\delta^{17} \oplus \gamma^4\delta^{+\infty}$. The residuals gives us the following results $\mathcal{D}_{\underline{\mathbf{y}}_o \neq \underline{\mathbf{y}}_p}(0) = 3 \notin [0, \underline{\Delta}_{\mathbf{y}_p}] = [0, 2]$ and $-\mathcal{D}_{\bar{\mathbf{y}}_p \neq \bar{\mathbf{y}}_o}(0) = -2 \in [\bar{\Delta}_{\bar{\mathbf{y}}_p}, 0] = [-2, 0]$. This indicator $I_I(\mathbf{u}, \mathbf{y}_o)$ indicate false because $\mathcal{D}_{\underline{\mathbf{y}}_o \neq \underline{\mathbf{y}}_p}(0) = 3 \notin [0, \underline{\Delta}_{\mathbf{y}_p}] = [0, 2]$ is false.

VI. CONCLUSION

In this article, we have extended the problem of time failure diagnosis in TEG by using time intervals. Using $(\max, +)$ algebraic techniques we have proposed indicators that detect time shift with certain or uncertain outputs. The indicators are

based on the dioid $I(\mathcal{M}_{in}^{ax}[\gamma, \delta])$ to define our system but the indicator calculations are finally reduced to comparing the max and min bounds of the outputs. This study is motivated by the development of algorithms for time shift failure detection in assembly lines.

We have several perspectives for the use of indicators on a real production line such as that of STMicroelectronics. We will set up the C++ coding of this indicator in the *MaxPlus-Diag* library of the article [LCSP18]. We plan to extend this method by adding failure localisation and isolation modules to refine the diagnosis. The distances measured between the expected and observed output can give us useful information for locating a fault in a system. Another perspective is also to deal with event uncertainty (interval of tokens in a Place of the TEG).

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