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Multi-emission discrimination using IIR Volterra Filtering

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Abstract:

Keywords: .

1 Introduction.

The aim of this paper is to build a test algorithm devoted to detect when several independent emissions are using the same carrier and, in such a situation, to determine the number of them.

Let us consider the case of phasis or frequency modulation. If there is only one modulation in the observed signal, its module remains constant with respect to time and the fluctuations of the module of the observed signal is only due to additive white noise. If now, one consider the sum of two modulations, as the phasis of each modulations are assumed to be independents, the module of the observed signal will be fluctuating. As a consequence of this remark, the algorithm which is proposed is based on the prediction of the module of the observed signal conditionally to the number of modulation supposed. The innovations involved are then used to construct a sufficient statistic for the detection in the sense that its expectation maximize some contrast criterium.

2 Statistics of a multi-emission.

In our problem, the hypothesis to be tested is the number n of emissions in the observed complex signal y_t obtain after two channels demodulation. It can be written:

$$y_t = s_t^n + v_t \quad (2.1)$$

where:

$$s_t^n = \sum_{i=1}^n x_t^i \quad (2.2)$$

is the sum of n independent modulations and $v_t = v_t^I + jv_t^2$ is a complex white noise with real and imaginary assumed to be independent and of variance R . The moments of the white noise v_t are all zero except the following:

$$E[|v_t|^{2k}] = R^k \sum_{p=0}^k C_k^p \alpha_p \alpha_{k-p} \quad (2.3)$$

with:

$$C_k^p = \frac{k!}{p!(k-p)!} \text{ and } \alpha_p = 1 \times 3 \times \dots \times (2p-1) \quad (2.4)$$

Let us examine the statistics of multi-emission in the particular case of a digital phasis modulation. Each signal can be represented, after demodulation, as the complex signal:

$$x_t = x_0 \exp(j\varphi_t) \quad (2.5)$$

with $x_0 = A_0 \exp(j\varphi_0)$ where A_0 is the (constant) amplitude of the signal and φ_0 the initial phasis. φ_t is the current phasis carrying the information to be send. Let Δt be the sample period. Between to samples, the evolution of the signal is defined as:

$$x_{t+\Delta t} = x_t \exp(j\Delta\varphi_t) \quad (2.6)$$

where $\Delta\varphi_t$ designates the phasis increment. As the transmitted information is unknown, $\Delta\varphi_t$ may be viewed as a white noise with characteristics depending of the kind of modulation used. Let $w_t = \exp(j\Delta\varphi_t)$.

For an analogical modulation, the density of probability of $\Delta\varphi_t$ may be assumed gaussian. In such a situation, if the variance of $\Delta\varphi_t$ is Q , the moments of w_t are defined by:

$$\begin{aligned} \forall k \neq l, E[w_t^k (w_t^*)^l] &= \exp\left(-\frac{1}{2}(k-l)^2 Q\right) \\ \forall k, E[w_t^k (w_t^*)^k] &= 1 \end{aligned} \quad (2.7)$$

where w^* is complex conjugate of w .

For a digital modulation, the phasis jumps time of occurrence may be modelised with a Poisson process of frequency λ and the amplitude uniformly distributed on m points on the unit circle. In that case, the density of probability of $\Delta\varphi_t$ is:

$$p(\Delta\varphi_t) = (1 - \lambda\Delta t)\delta_0(\Delta\varphi_t) + \frac{\lambda\Delta t}{m-1} \sum_{p=1}^{m-1} \delta_{\frac{2\pi p}{m}}(\Delta\varphi_t) \quad (2.8)$$

where $\delta_a(\Delta\varphi)$ is the Dirac distribution centered on a . The resulting moments of w_t are then defined by:

$$\begin{aligned} \forall k \neq l, E[w_t^k (w_t^*)^l] &= 1 - \frac{m}{m-1} \lambda\Delta t \\ \forall k, E[w_t^k (w_t^*)^k] &= 1 \end{aligned} \quad (2.9)$$

One may of course consider other kind of modulations but it is important to note that, for all types of modulations, one has always the same property:

$$\begin{aligned} \forall k \neq l, E[w_t^k (w_t^*)^l] &= a \\ \forall k, E[w_t^k (w_t^*)^k] &= 1 \end{aligned} \quad (2.10)$$

with $a < 1$.

Let us now analyze the evolution of moments of one modulation. According to (2.10), one has:

$$\begin{aligned} \forall k \neq l, E[x_{t+\Delta t}^k (x_{t+\Delta t}^*)^l] &= a E[x_t^k (x_t^*)^l] \\ \forall k, E[x_{t+\Delta t}^k (x_{t+\Delta t}^*)^k] &= E[x_t^k (x_t^*)^k] \end{aligned} \quad (2.11)$$

As a consequence, because $a < 1$, it is clear that all moments of x_t are asymptotically zero except the set $\{E[x_t^k (x_t^*)^k], k \geq 1\}$ which are constant.

The first moment of a multi-emission can now be computed using (2.11). One may derive that all moments of degree ≤ 4 are asymptotically zero except the following:

$$E[s_t^n s_t^{n*}] = \sum_{k=1}^n E[x_t^k x_t^{k*}] \quad (2.12)$$

$$E[(s_t^n s_t^{n*})^2] = \sum_{k=1}^n E[(x_t^k x_t^{k*})^2] + 2 \sum_{\substack{k \neq l \\ k, l = 1}}^n E[x_t^k x_t^{k*}] E[x_t^l x_t^{l*}] \quad (2.13)$$

Let us compare these moments when the global power P is the same but when the number of modulation is different. On has:

$$\forall k, E[x_t^k x_t^{k*}] = P/n \quad (2.14)$$

and the fourth order moment is:

$$E[(s_t^n s_t^{n*})^2] = \frac{1 + 2(n-1)}{n} (E[s_t^n s_t^{n*}])^2 \quad (2.15)$$

This relation means that the number of signals is detectable as the ratio within the fourth order moment and the square of the second order moment depends on n . This property will be exploited in the sequel to construct the detection test based on predictors of the signal square module.

3 The IIR Volterra quadratic predictor.

Let us applied the quadratic Volterra predictor as defined in [2] to our problem. When one deals with real numbers, the purely quadratic IIR predictor of any function ϕ_t of the state has the following recursive structure:

$$\begin{aligned} p_{t+\Delta t} &= A_t^1 p_t + A_t^0 y_t \\ z_{t+\Delta t} &= A_t^2 z_t + p_{t+\Delta t} y_t \end{aligned} \quad (3.1)$$

where z_t is the output and $\{A_t^i\}$ are time variant parameters computed to minimize the mean square prediction error, that is:

$$J_t = \frac{1}{2} E[\|\phi_t - z_t\|^2] \quad (3.2)$$

The optimal parameters are computed using the following orthogonalities:

$$E \left[(\phi_{t+\Delta t} - z_{t+\Delta t}) \begin{bmatrix} y_t^2 \\ p_t^T y_t \\ z_t^T \end{bmatrix} \right] = 0 \quad (3.3)$$

which leads to the following linear system:

$$\begin{aligned} A_t^2 E[z_t(y_t)^2] + A_t^1 E[p_t(y_t)^3] + A_t^0 E[(y_t)^4] &= E[\phi_{t+\Delta t}(y_t)^2] \\ A_t^2 E[z_t p_t^T y_t] + A_t^1 E[z_t^T p_t^T (y_t)^2] + A_t^0 E[p_t^T (y_t)^3] &= E[\phi_{t+\Delta t} p_t^T y_t] \\ A_t^2 E[z_t z_t^T] + A_t^1 E[p_t z_t^T (y_t)^2] + A_t^0 E[z_t^T (y_t)^2] &= E[\phi_{t+\Delta t} z_t^T] \end{aligned} \quad (3.4)$$

The optimal parameters are then computable if all the expectations in (3.4) are computable. Note that this condition is achieved when the prediction system of ϕ_t is bilinear.

Let us generalize this estimator to the complex number case. Let $\phi_t = |s_t^n|^2$ to be estimated. It is easy to verify that, according to (2.13), the only non-zero quadratic projection of ϕ_t is the projection over the term $y_t y_t^*$. As a consequence, the structure of the predictor must be as following:

$$\begin{aligned} p_{t+\Delta t} &= A_t^1 p_t + A_t^0 y_t \\ z_{t+\Delta t} &= A_t^2 z_t + p_{t+\Delta t} y_t^* + p_{t+\Delta t}^* y_t + B_t \end{aligned} \quad (3.5)$$

with $z_t \in R$. The term B_t makes sure that the predictor is not biased.

The stochastic observability concept introduced in [2] is used here to determine the dimension of the predictor, that is the dimension of z_t . In the general case, the variable $\Pi_{t+\Delta t}$ is stochastically observable if the following relation:

$$\Pi_{t+\Delta t} \rightarrow \left\{ E([y_{\tau_1} y_{\tau_2}^* | \Pi_{t+\Delta t}], \forall \tau_1, \tau_2 \leq t) \right\} \quad (3.6)$$

is one to one. To verify this property, let us compute first the conditional expectation:

$$\begin{aligned} E[y_t y_t^* | \Pi_{t+\Delta t}] &= \sum_{k, l=1}^n E[x_t^k x_t^{l*} | \Pi_{t+\Delta t}] + 2R \\ &= \sum_{k=1}^n E[x_t^k x_t^{k*} | \Pi_{t+\Delta t}] + \sum_{k \neq l}^n E[x_t^k x_t^{l*} | \Pi_{t+\Delta t}] + 2R \end{aligned} \quad (3.7)$$

According to (2.11), one may write:

$$E[y_t y_t^* | \Pi_{t+\Delta t}] = \sum_{k=1}^n E[x_{t+\Delta t}^k x_{t+\Delta t}^{k*} | \Pi_{t+\Delta t}] + \frac{1}{a} \sum_{k \neq l}^n E[x_{t+\Delta t}^k x_{t+\Delta t}^{l*} | \Pi_{t+\Delta t}] + 2R \quad (3.8)$$

In the same way, one has:

$$E[y_{t-\Delta t} y_{t-\Delta t}^* | \Pi_{t+\Delta t}] = \sum_{k=1}^n E[x_{t+\Delta t}^k x_{t+\Delta t}^{k*} | \Pi_{t+\Delta t}] + \frac{1}{a} \sum_{k \neq l}^n E[x_{t+\Delta t}^k x_{t+\Delta t}^{l*} | \Pi_{t+\Delta t}] + 2R$$

Let:

$$\Pi_t = \left[\sum_{k=1}^n x_t^k x_t^{k*}, \sum_{k \neq l}^n x_t^k x_t^{l*} \right] \quad (3.9)$$

It is clear that the relation $\Pi_{t+\Delta t} \rightarrow \{E[y_t y_t^* | \Pi_{t+\Delta t}], E[y_{t-\Delta t} y_{t-\Delta t}^* | \Pi_{t+\Delta t}]\}$ is one to one. As a consequence, the dimension of z_t must be at least two. Note that it is equivalent to consider Π_t and the more natural following set:

$$\Psi_t = \left[\sum_{k=1}^n x_t^k x_t^{k*}, \left| \sum_{k=1}^n x_t^k \right|^2 \right] \quad (3.10)$$

Note moreover that if $n = 1$, the two components of Ψ_t are identical and the dimension of the quadratic predictor is reduced to 1.

4 Detection test.

The detection test used here takes one's inspiration from the linear gaussian case. Recall that in that classical case, one constructs the optimal predictor $\hat{x}^n_{(k+1)\Delta t|k\Delta t}$ of the signal under each hypothesis H_n . One then computes the likelihood of each hypothesis H_n that has the following expression:

$$L_t^n = \sum_{k=0}^{t/\Delta t} \frac{(y_{(k+1)\Delta t} - \hat{x}^n_{(k+1)\Delta t|k\Delta t})^2}{E[(y_{(k+1)\Delta t} - \hat{x}^n_{(k+1)\Delta t|k\Delta t})^2]} + \log(E[(y_{(k+1)\Delta t} - \hat{x}^n_{(k+1)\Delta t|k\Delta t})^2]) \quad (4.1)$$

The decision process is then defined as: if $L_t^n < L_t^{m_2}$, the hypothesis n is chosen.

Our test is based on the innovation processes generated by the signal square module prediction. Let $\hat{\phi}^n_{t+\Delta t|t}$ be the one step predictor of ϕ_t under hypothesis "n modulations in the signal". The test variable S_t is imposed to have the following form:

$$S_t = \sum_{k=0}^{t/\Delta t} (|y_{(k+1)\Delta t}|^2 - \hat{\phi}^{n_1}_{(k+1)\Delta t|k\Delta t})^2 + \alpha_k (|y_{(k+1)\Delta t}|^2 - \hat{\phi}^{n_2}_{(k+1)\Delta t|k\Delta t})^2 + \beta_k \quad (4.2)$$

where α_k and β_k are parameters devoted to normalize the contrast within the two hypothesis. A natural way to satisfied this condition is to impose:

$$\begin{aligned} E[S_t | H_{n_1}] &= t \\ E[S_t | H_{n_2}] &= -t \end{aligned} \quad (4.3)$$

assuring that the expectation of the distance within the test variable under each hypothesis increases linearly with the number of data treated. The test is then reduced to:

$$S_t < 0 \rightarrow n_1 \text{ and } S_t > 0 \rightarrow n_2 \quad (4.4)$$

The constraints (4.3) leads to the following values of α_k, β_k :

$$\begin{aligned} \alpha_k &= \frac{2 - J_k^{23} + J_k^{22}}{J_k^{33} - J_k^{32}} \\ \beta_k &= 1 - J_k^{23} - \alpha_k J_k^{33} \end{aligned} \quad (4.5)$$

where:

$$J_k^{ij} = E[(|y_{(k+1)\Delta t}|^2 - \hat{\phi}^{n_i}_{(k+1)\Delta t|k\Delta t})^2 | H_{n_j}] \quad (4.6)$$

Note that all the variances J_k^{ij} may be computed a priori since the system is bilinear.

5 Simulation results.

In all the following simulations, the signal to noise ratio is fixed to $10dB$.

5.1 Detection within 1 and 2 modulations.

The first step of construction of the test is the computation of the predictor of the signal square module. One shows on figure 1 the estimation of the signal square module under each hypothesis when only one signal is present with power equal one. It appears clearly that the estimator which deals with H_1 , the good hypothesis, is much more accurate than the estimator which deals with H_2 . The situation is inverted when two signals are present as it appears on figure 2.

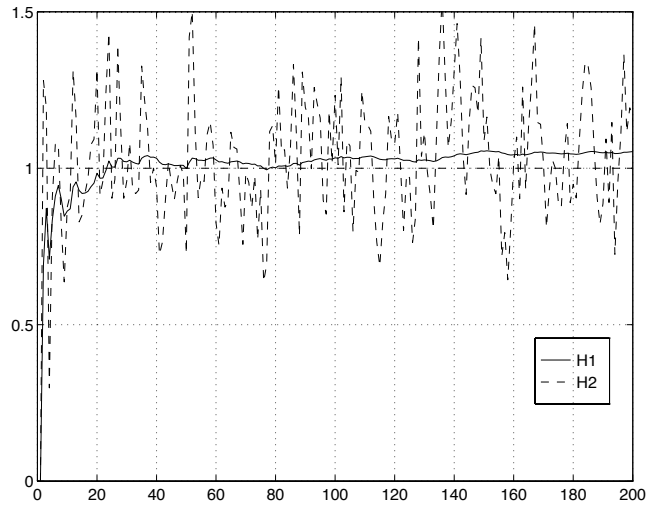


figure 1: Module estimation - 1 modulation.

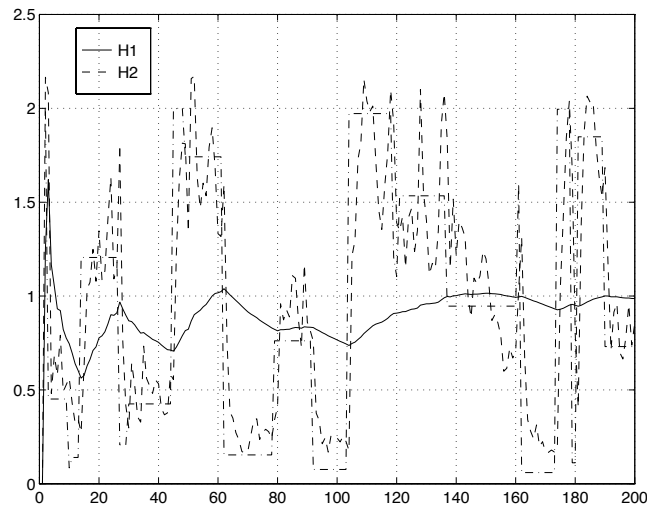


figure 2: Module estimation - 2 modulations.

The test variable S_t as defined by (4.2) is presented on figure 3. The dotted lines are the expectations of S_t under each hypothesis. One may observe that the detection is very fast.

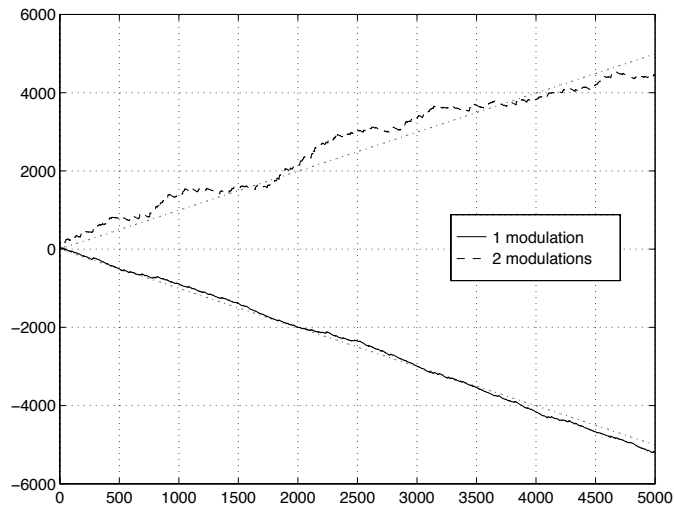


figure 3: Detection within 1 and 2 modulations.

5.2 Detection within 2 and 3 modulations.

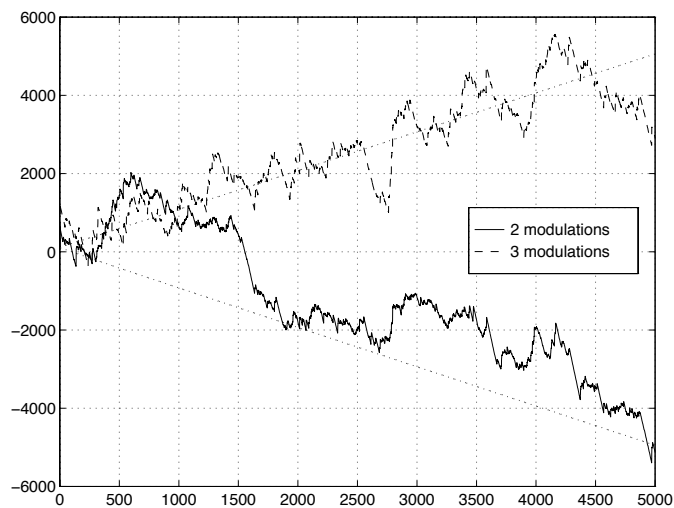


figure 4: Dtection within 2 and 3 modulations

6 References.

- [1] H. L. Van Trees
- [2] A. Monin, G. Salut, *IIR Volterra Filtering With Application to Bilinear Systems*. IEEE Trans. on Signal Processing, vol. 44, n°9, pp. 2209-2221, 1996.