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► **To cite this version:**

Claire Paya, Euriell Le Corrond, Yannick Pencolé, Philippe Vialletelle. Observer-based detection of time shift failures in (max,+)-linear systems. The 31st International Workshop on Principles of Diagnosis (DX-2020), Sep 2020, Nashville, United States. hal-03023250

**HAL Id: hal-03023250**

**<https://laas.hal.science/hal-03023250>**

Submitted on 25 Nov 2020

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# Observer-based detection of time shift failures in $(\max,+)$ -linear systems

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## Abstract

In this paper, we address the problem of failure detection in production lines modeled as Timed Event Graphs (TEG). The proposed method represents TEGs as  $(\max,+)$ -linear systems with disturbance and aims at detecting time shift failures in the underlying production lines. To do so, we will reconstruct the state of the observed system and define an indicator relying on the residuation theory on  $(\max,+)$ -linear systems.

## 1 Introduction

In industry, Discrete Event Systems (DES) can be used to model and solve fault diagnosis problems in automated production lines. The objective is to detect, localize and ultimately identify failures as soon as possible to avoid further equipment unavailability. In systems like production lines, failures can be not only caused by complete equipment breakdowns but also by the occurrence of time shifts so the production line can dramatically slow down and not be able to comply with the specified production objectives. This paper addresses the problem of detecting and localizing the source of such time shifts. One of the first methods used to diagnose failures in DES is proposed in [SSL<sup>+</sup>95] on untimed automata. Then, [Tri02] defines the diagnosis problem on timed automata so that diagnostic decisions are refined by using timed observations. In [GT09], the diagnosis is based on time Petri Nets that is well-suited to model competition and parallelism within the system. One subclass of time Petri Nets, called Timed Event Graph (TEG), can also be used to specifically represent systems like production lines. In TEGs, places are associated with a punctual duration and they can be modeled by  $(\max,+)$  algebra as introduced in [BCOQ92, Max91]. [KLBvdB18] presents the history of DES with the use of  $(\max,+)$  algebra. For example, [KL15] uses  $(\max,+)$  algebra to control wafer delays in cluster tools for semiconductor production. The problem of failure diagnosis by the use of  $(\max,+)$  algebra has been introduced in [SLCP17] where the proposed detection method relies on the residuation theory and compares observable outputs with expected ones to detect output time shifts. Failure localisation is then performed by an adhoc structural analysis of the underlying TEG that does not use  $(\max,+)$  algebra. The objective of this paper is to design a new set of time shift failure indicators that are not based on the observable outputs of the system only but on the estimation of the internal state of the system so that the fail-

ure localisation problem is also solved in an algebraic way. To do so, the proposed failure indicator will rely on an observer that is proposed in [HMCL10] and aim at rebuilding system's states based on the observations.

The paper is organized as follows. Section 2 presents a motivation example inspired from the semiconductor industry. Section 3 summarizes the necessary mathematical background about  $(\max,+)$ -linear systems. Section 4 describes how the time shift failure problem in system can be turned into the problem of state estimation in a system with input disturbance and gives the construction of an observer of such a system. Section 5 finally defines the proposed time shift failure indicators.

## 2 Motivation example

The problem that we address is motivated by a real production line that is at STMicroelectronics Crolles300 plant. STMicroelectronics is among the world's largest semiconductor companies, serving all electronics segments. Semiconductor manufacturing is complex and one of its most important challenges is to succeed in detecting production drifts before they have real impact on production plan. STMicroelectronics has complex production lines of wafer batches with many pieces of equipment running in parallel. One of the objectives is to detect as soon as possible that an equipment is late to ensure that products (wafer batches) are delivered on time or at least with minimal delays.

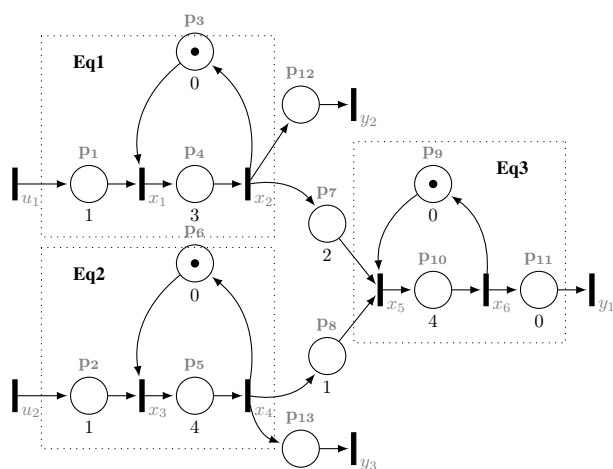


Figure 1: Fault free model

Figure 1 presents a fault-free behavioural model of such a

production line defined by a TEG. This production line corresponds to three pieces of equipment (namely Eq1, Eq2, Eq3). Eq1 is modeled with a couple of places  $p_3, p_4$ : it is available (i.e. no current processing) if a token is in place  $p_3$  while it is processing its input if a token is in place  $p_4$ . The process of Eq1 is carried out in 3 hours. Similarly Eq2 and Eq3 are respectively modeled by the couple of places  $p_6, p_5$  (processing time: 4 hours) and  $p_9, p_{10}$  (processing time 4 hours). Places  $p_7, p_8$  model wafer batch transportation between Eq1, Eq2 and Eq3. From Eq1 to Eq3 it takes 2 hours while from Eq2 to Eq3 it takes only 1 hour. Eq3 requires processed wafer batches from both Eq1 and Eq2 to operate. It is modeled by a transition  $x_5$  that can be fired only when Eq3 is available and wafer batches from Eq1 and Eq2 are finally in front of Eq3. For  $i = \{1, 2\}$ , a trigger of an input transition  $u_i$  represents the occurrence of an event from sensors on the production line that indicates the arrival of unprocessed wafer batches in front of Eq $i$ . The output to the production line is a stream of fully processed wafer batches modeled by firing transition  $y_1$ . Outputs  $y_2$  and  $y_3$  provide observable information about the end of the process of Eq1 and Eq2. Suppose now that the underlying production line is faulty, i.e. it does not comply with the behaviour defined by Figure 1. Consider that a stream of 7 wafer batches arrive at Eq1 (input  $u_1$ ) respectively at time  $t \in \{1, 2, 3, 4, 5, 6, 7\}$  and a similar stream with the same dates arrive at Eq2 (input  $u_2$ ). Then, suppose that processed wafer batches are successively available at time  $\{12, 17, 22, 27, 32, 37, 42\}$  (output  $y_1$ ), and process information is available at time  $\{5, 8, 11, 14, 17, 20, 23\}$  for output  $y_2$  and at time  $\{7, 12, 17, 22, 27, 32, 37\}$  for output  $y_3$ . Then, the question is: based on the fault-free model of Figure 1, can we detect and localize a time shift failure in the underlying production line?

If the line is not faulty, the wafer batch that arrives at  $t=1$  on Eq2 (resp. Eq1) is processed in 4 hours (resp. 3 hours) and then takes 1 hour (2 hours) to arrive in front of Eq3. Synchronization is done at  $t=7$  and the batches are processed by Eq3 in 4 hours. So, the fully processed batch should come out at  $t=11$ . However, the first real output comes out one hour later: a time drift has happened. Following the same idea, considering output  $y_3$ , the batch that arrives at  $t=1$  on Eq2, should be processed in 4 hours and process information should come at  $t=6$ , hence another time drift.

In production lines, such time drifts are considered as time shift failures. This paper aims at designing a  $(\max, +)$ -algebraic indicator that detects them based on a model as the one presented in Figure 1. TEGs can be formally defined as  $(\max, +)$ -linear systems that are introduced in the next section. It relies on the dioid  $\mathcal{M}_{in}^{ax}[\gamma, \delta]$  where the successive fires of a transition  $x$  of the TEG are characterized by a series representing the events  $\gamma$  and the fire timings  $\delta$ .

### 3 Mathematical background

This section recalls the mathematical background used in this paper for describing  $(\max, +)$ -linear systems [BCOQ92, Max91].

#### 3.1 Dioid theory

The dioid theory is used to describe the inputs and the behavior of the studied system. In particular, series of a specific dioid are defined to obtain the trajectories of inputs and states flows of timed events.

**Definition 1.** A dioid  $\mathcal{D}$  is a set composed of two internal operations  $\oplus$  and  $\otimes$ . The addition  $\oplus$  is associative, commutative, idempotent (i.e.  $\forall a \in \mathcal{D}, a \oplus a = a$ ) and has a neutral element  $\varepsilon$ . The multiplication  $\otimes$  is associative, distributive on the right and the left over the addition  $\oplus$  and has a neutral element  $e$ . Element  $\varepsilon$  is absorbing by  $\otimes$ . When there is no ambiguity, the symbol  $\otimes$  is omitted.

**Definition 2.** A dioid is complete if it is closed for infinite sums and if  $\otimes$  is distributive over infinite sums.

**Example 1.** The dioid  $\mathbb{Z}_{max} = (\mathbb{Z} \cup -\infty)$  endowed with the max operation as addition  $\oplus$  and the addition as multiplication  $\otimes$  with neutral element denoted  $\varepsilon = -\infty$  and  $e = 0$ . The dioid  $\mathbb{Z}_{max}$  is not complete because  $+\infty$  does not belong to the set  $\mathbb{Z}_{max}$  so the infinite sum is not set to  $+\infty$ . By adding  $+\infty$  to the dioid  $\mathbb{Z}_{max}$ , we get the complete dioid  $\overline{\mathbb{Z}}_{max}$  where  $(-\infty) + (+\infty) = (-\infty)$ .

**Theorem 1** ([BCOQ92]). Let  $\mathcal{D}$  be a complete dioid,  $x = a^*b$  is the solution of  $x = ax \oplus b$ , where  $x = a^*b$ , and  $a^* = \bigoplus_{i \geq 0} a^i$  is the Kleene star operator with  $a^0 = e$  and  $a^{i+1} = a \otimes a^i$ .

**Definition 3.** For a dioid  $\mathcal{D}$ ,  $\preceq$  denotes the order relation such that  $\forall a, b \in \mathcal{D}, a \preceq b \Leftrightarrow a \oplus b = b$ .

**Example 2.** The complete dioid  $\mathbb{B}[\gamma, \delta]$  is the set of formal series with two commutative variables  $\gamma$  and  $\delta$  with Boolean coefficients in  $\{\varepsilon, e\}$  and exponents in  $\mathbb{Z}$ . A series  $s \in \mathbb{B}[\gamma, \delta]$  is written  $s = \bigoplus_{n, t \in \mathbb{Z}} s(n, t) \gamma^n \delta^t$

$s(n, t) = e$  or  $\varepsilon$  (respectively representing the presence or the absence of the monomial). The neutral elements are  $\varepsilon = \bigoplus_{n, t \in \mathbb{Z}} \varepsilon \gamma^n \delta^t$  and  $e = \gamma^0 \delta^0$ .

Graphically, a series of  $\mathbb{B}[\gamma, \delta]$  is described by a collection of point of coordinates  $(n, t)$  in  $\mathbb{Z}^2$  with  $\gamma$  as horizontal axis and  $\delta$  as vertical axis. For instance, Figure 2 shows a series  $u_1 = u_2 = \gamma^0 \delta^1 \oplus \gamma^1 \delta^2 \oplus \gamma^2 \delta^3 \oplus \gamma^3 \delta^4 \oplus \gamma^4 \delta^5 \oplus \gamma^5 \delta^6 \oplus \gamma^6 \delta^7 \oplus \gamma^7 \delta^{+\infty}$  (monomials with  $e$  as Boolean coefficient).

In the following, we will consider the dioid  $\mathcal{M}_{in}^{ax}[\gamma, \delta]$ . It

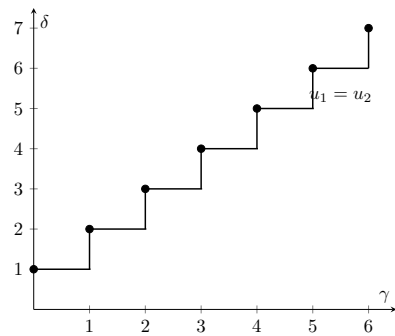


Figure 2: Representation of inputs  $u_1 = u_2$

is the quotient of the dioid  $\mathbb{B}[\gamma, \delta]$  by the modulo  $\gamma^*(\delta^{-1})^*$ . The dioid  $\mathcal{M}_{in}^{ax}[\gamma, \delta]$  is a complete dioid with  $\forall a, b \in \mathcal{M}_{in}^{ax}[\gamma, \delta]: a = b \Leftrightarrow a \gamma^*(\delta^{-1})^* = b \gamma^*(\delta^{-1})^*$ . The internal operations are the same as in  $\mathbb{B}[\gamma, \delta]$  and neutral elements  $\varepsilon$  and  $e$  are identical to those of  $\mathbb{B}[\gamma, \delta]$ .

**Definition 4.** Let  $s \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$  be a series, the dater function of  $s$  is the non-decreasing function  $D_s(n)$  from  $\mathbb{Z} \mapsto \overline{\mathbb{Z}}$  such that  $s = \bigoplus_{n \in \mathbb{Z}} \gamma^n \delta^{D_s(n)}$ .

Considering the TEG of Figure 1 and the associated scenario defined in Section 2, inputs  $u_1$  and  $u_2$  are represented by the series  $u_1 = u_2 = \gamma^0\delta^1 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^3 \oplus \gamma^3\delta^4 \oplus \gamma^4\delta^5 \oplus \gamma^5\delta^6 \oplus \gamma^6\delta^7 \oplus \gamma^7\delta^{+\infty}$  from the  $\mathcal{M}_{in}^{ax}[\gamma, \delta]$  dioid. The absence of an 8<sup>th</sup> wafer batch in the scenario is indicated by  $+\infty$  in monomial  $\gamma^7\delta^{+\infty}$ . Figure 2 is the graphical representation of  $u_1 = u_2$ . This series is composed of monomials  $\gamma\delta$ , where  $\gamma$  represents the events of the series, and  $\delta$  represents the dates of the series. Series  $u_1$  has for dater function  $D_{u_1}(0) = 1, D_{u_1}(1) = 2, D_{u_1}(2) = 3, D_{u_1}(3) = 4, D_{u_1}(4) = 5, D_{u_1}(5) = 6$  and  $D_{u_1}(6) = 7$ . This dater function lists all the dates of the event occurrences.  $u_2$  has obviously the same dater function (as  $u_1 = u_2$ ).

**Definition 5.** Let  $s \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$  be a series, its canonical form is

$$s = \bigoplus_{k=0}^K \gamma^{n_k} \delta^{t_k} \text{ with } K \in \mathbb{N} \cup \{+\infty\} \text{ and } \begin{cases} n_0 < n_1 < \dots \\ t_0 < t_1 < \dots \end{cases}$$

**Definition 6.** Let  $\Pi : \mathcal{D} \mapsto \mathcal{C}$  an application defined on ordered sets. We say:  $\Pi$  isotone  $\equiv \forall s, s' \in \mathcal{D} \ s \preceq s' \Rightarrow \Pi(s) \preceq \Pi(s')$ .

**Definition 7.** Let  $\Pi : \mathcal{D} \mapsto \mathcal{C}$  be an isotone mapping, where  $\mathcal{D}$  and  $\mathcal{C}$  are complete dioids. The largest solution of  $\Pi(x) = b$ , if it exists, is called the residual of  $\Pi$  and is denoted  $\Pi^\sharp$ . When  $\Pi$  is residuated,  $\Pi^\sharp$  is the unique isotone mapping such that  $\Pi \circ \Pi^\sharp \preceq I_{\mathcal{D}\mathcal{C}}$  and  $\Pi^\sharp \circ \Pi \succeq I_{\mathcal{D}}$  where  $I_{\mathcal{D}\mathcal{C}}$  and  $I_{\mathcal{D}}$  are respectively the identity mappings on  $\mathcal{C}$  and  $\mathcal{D}$ .

**Example 3.** The mappings  $L_a : x \mapsto a \otimes x$  and  $R_a : x \mapsto x \otimes a$  defined over a complete dioid  $\mathcal{D}$  are both residuated. Their residuals are denoted by  $L_a^\sharp(x) = a \backslash x$  and  $R_a^\sharp(x) = x \phi a$ .

**Theorem 2** ([Max91]). Let  $\mathcal{D}$  be a complete dioid and  $A \in \mathcal{D}^{n \times m}$  be a matrix. Then,

$$A \backslash A = (A \backslash A)^* \quad (1)$$

Time comparison between series can then be defined based on residuals.

**Definition 8.** Let  $a, b \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$  and their respective dater functions  $\mathcal{D}_a$  and  $\mathcal{D}_b$ . The time shift function representing the time shift between  $a$  and  $b$  for each  $n \in \mathbb{Z}$  is defined by  $\mathcal{T}_{a,b}(n) = \mathcal{D}_a - \mathcal{D}_b$ .

**Theorem 3** ([Max91]). Let  $a, b \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$ , the time shift function  $\mathcal{T}_{a,b}(n)$  can be bounded by:

$$\forall n \in \mathbb{Z}, \mathcal{D}_{b\phi a}(0) \leq \mathcal{T}_{a,b}(n) \leq -\mathcal{D}_{a\phi b}(0),$$

where  $\mathcal{D}_{b\phi a}(0)$  is obtained from monomial  $\gamma^0\delta^{\mathcal{D}_{b\phi a}(0)}$  of series  $b\phi a$  and  $\mathcal{D}_{a\phi b}(0)$  is obtained from  $\gamma^0\delta^{\mathcal{D}_{a\phi b}(0)}$  of series  $a\phi b$ .

**Definition 9.** Let  $a, b \in \mathcal{M}_{in}^{ax}[\gamma, \delta]$ , the time shift between series  $a$  and  $b$  is

$$\Delta(a, b) = [\mathcal{D}_{b\phi a}(0); -\mathcal{D}_{a\phi b}(0)], \quad (2)$$

where  $\gamma^0\delta^{\mathcal{D}_{b\phi a}(0)} \in b\phi a$  and  $\gamma^0\delta^{\mathcal{D}_{a\phi b}(0)} \in a\phi b$ . In this interval, the series from which the time offset is measured is the series  $a$ . It is called the reference series of the interval.

From this definition, if the time shift interval needs to be defined with series  $b$  as the reference series, the interval will be  $\Delta(b, a) = [\mathcal{D}_{a\phi b}(0); -\mathcal{D}_{b\phi a}(0)]$ .

**Example 4.** Generally speaking, let us consider two different series  $a = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{18} \oplus \gamma^3\delta^{21} \oplus \gamma^4\delta^{+\infty}$  and  $b = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{19} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{+\infty}$ . The minimal time shift between  $a$  and  $b$  is  $\mathcal{D}_{b\phi a}(0) = 0$  and is found in the monomial where the degree of  $\gamma$  is 0 in  $b\phi a = \gamma^0\delta^0 \oplus \gamma^1\delta^3 \oplus \gamma^2\delta^7 \oplus \gamma^3\delta^{11} \oplus \gamma^4\delta^{+\infty}$ . The maximal time shift is  $-\mathcal{D}_{a\phi b}(0) = 2$  and is found in  $\gamma^0\delta^{-2}$  from  $a\phi b = \gamma^0\delta^{-2} \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^9 \oplus \gamma^4\delta^{+\infty}$ . The time shift interval is  $\Delta(a, b) = [0; 2]$ . The distance between  $a$  and  $b$  is a minimum of 0 and a maximum of 2 hours meaning that series  $a$  is faster than  $b$ .

### 3.2 Models of (max,+)-linear systems

The elements of the TEG are represented by equations in  $\mathcal{M}_{in}^{ax}[\gamma, \delta]$ . The equations can be grouped into a set of matrices  $A, B$  and  $C$  that contain information about the structure of the TEG. The state representation defines relations between any set of input event flows  $u$  and the state  $x$ , and the relations between the state  $x$  and the output event flows  $y$ . Let  $u \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{p \times 1}$  be the input vector of size  $p$ ,  $x \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{n \times 1}$  be the state vector of size  $n$  and  $y \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{q \times 1}$  be the output vector of size  $q$ . The state representation is:

$$\begin{cases} x = Ax \oplus Bu, \\ y = Cx, \end{cases}$$

where  $A \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{n \times n}$ ,  $B \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{n \times p}$  and  $C \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{q \times n}$ . Equality  $x = Ax \oplus Bu$  can be transformed to  $x = A^*Bu$  thanks to Theorem 1 so we have

$$y = CA^*Bu.$$

Matrix  $H = CA^*B$  represents the transfer function of the TEG, that is the dynamic of the system between the inputs and the outputs.

For the system of Figure 1 the matrices  $A \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{6 \times 6}$ ,  $B \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{6 \times 2}$  and  $C \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{3 \times 6}$  of the state representation are:

$$A = \begin{pmatrix} \gamma^0\delta^3 & \gamma^1\delta^0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \gamma^1\delta^0 & \cdot & \cdot \\ \cdot & \cdot & \gamma^0\delta^4 & \cdot & \cdot & \cdot \\ \cdot & \gamma^0\delta^2 & \cdot & \gamma^0\delta^1 & \cdot & \gamma^1\delta^0 \\ \cdot & \cdot & \cdot & \cdot & \gamma^0\delta^4 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix},$$

$$B = \begin{pmatrix} \gamma^0\delta^1 & \cdot \\ \cdot & \gamma^0\delta^1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}, \quad C = \begin{pmatrix} \cdot & \gamma^0\delta^0 & \cdot & \cdot & \cdot & \gamma^0\delta^0 \\ \cdot & \cdot & \cdot & \gamma^0\delta^0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}.$$

The exponent  $n$  of  $\gamma$  represents the backward event shift between transitions (the  $n + 1$ <sup>th</sup> firing of  $x_1$  depends on the  $n$ <sup>th</sup> firing of  $x_2$ ) and the exponent of  $\delta$  represents the backward time shift between transition (the firing date of  $x_2$  depends on the firing date of  $x_1$  and time between 2 and 5).

## 4 How can a (max,+) observer be sensitive to time shift failures?

The objective of the paper is to propose a method that detects time shift failures as proposed in Section 2 and that uses an observer as introduced in [HMCL10] and [HMCSM10]. As later detailed in Section 4.2, this observer aims at computing a reconstructed state from the observation of the inputs and outputs of the system that is sensitive to a specific type of disturbance. These disturbances are characterized as new inputs  $w$  that slow down the system. Based on this characterisation, the system will then be assumed to behave with respect to the following state representation.

$$\begin{cases} x = Ax \oplus Bu \oplus Rw, \\ y = Cx. \end{cases} \quad (3)$$

Section 4.1 describes how time shift failures can be characterized as such disturbances. Section 4.2 then introduces the observer that will be used in the proposed detection method.

### 4.1 Time shift failures as input disturbances

Throughout this paper, we consider that time shift failures are permanent phenomena that can occur at any step of the production. Formally speaking, a time shift failure is characterized by an unexpected and unknown delay  $d > 0$  that is added to the normal duration time  $t$  of a place  $p$ .

As shown on Figure 3, this place is characterized by a transition upstream  $x_{i-1}$ , a duration  $t$ , a number of tokens  $o$  and a transition downstream  $x_i$ . Let  $x_{i-1} = \bigoplus_{n=0}^K \gamma^{s_n} \delta^{h_n}$  (see Definition 5), where  $s_n$  is the transition firing number,  $h_n$  is the firing date and  $K$  the number of firing events. The normal downstream transition is  $x_i = \bigoplus_{n=0}^K \gamma^{s_n+o} \delta^{h_n+t}$ .

When a time shift failure  $d > 0$  holds in a place, the downstream transition then becomes:  $x_i = \bigoplus_{n=0}^K \gamma^{s_n+o} \delta^{h_n+t+d}$ .

To characterize the same time shift failure over the place  $p$  by a disturbance, we will first modify the TEG. We add to the downstream transition  $x_i$  an input  $w_i$ , as shown in Figure 4, which slows down this transition. This new input  $w_i$  is not observed because it is related to a failure in an equipment. To get the same effect of an offset  $d > 0$  in the downstream transition, input  $w_i$  has to be defined as

$$w_i = \bigoplus_{n=0}^k \gamma^{s_n+o} \delta^{h_n+t+d}. \quad (4)$$

Back to Figure 1 where the place  $p_5$  has a duration of  $t = 4$ . To characterize an offset, meaning a time shift failure, of  $d = 1$ , we add a disturbance  $w_4$  to the transition  $x_4$  after the place  $p_5$  in the same configuration as Figure 4. Suppose that  $x_3 = \gamma^0 \delta^2 \oplus \gamma^1 \delta^6 \oplus \gamma^2 \delta^{10} \oplus \gamma^3 \delta^{14} \oplus \gamma^4 \delta^{18} \oplus \gamma^5 \delta^{22} \oplus \gamma^6 \delta^{26} \oplus \gamma^7 \delta^{+}$ . Since an offset of 1 time unit is present on  $p_5$ ,  $x_4 = \gamma^0 \delta^{2+4+1} \oplus \gamma^1 \delta^{6+4+1} \oplus \gamma^2 \delta^{10+4+1} \oplus \gamma^3 \delta^{14+4+1} \oplus \gamma^4 \delta^{18+4+1} \oplus \gamma^5 \delta^{22+4+1} \oplus \gamma^6 \delta^{26+4+1} \oplus \gamma^7 \delta^{+}$ . By setting the disturbance  $w_4 = x_4 = \gamma^0 \delta^7 \oplus \gamma^1 \delta^{12} \oplus \gamma^2 \delta^{17} \oplus \gamma^3 \delta^{22} \oplus \gamma^4 \delta^{27} \oplus \gamma^5 \delta^{32} \oplus \gamma^6 \delta^{37} \oplus \gamma^7 \delta^{+}$ , the firing of transition  $x_4$  is slowed down.

Based on this characterization, the faulty system that we consider will behave based on Equation (3) and input disturbances as defined by Equation (4). Let  $w \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{l \times 1}$



Figure 3: Representation of a place

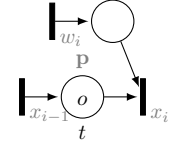


Figure 4: Representation of a place with disturbance

be the input vector of disturbances of size  $l$ . The input  $w$  corresponds to the transition that will be disturbed. Matrix  $R \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{n \times l}$  is filled with  $\gamma^0 \delta^0$  monomials that represent the connections between disturbances and internal disturbed transitions. All the other entries are set to  $\varepsilon$ . Equality  $x = Ax \oplus Bu \oplus Rw$  can be transformed to  $x = A^*Bu \oplus A^*Rw$  thanks to Theorem 1 so we have

$$y = CA^*Bu \oplus CA^*Rw.$$

In the example of Section 2, all the internal transitions in Figure 1 will be disturbed so  $R$  is the matrix  $R \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{6 \times 6}$ :

$$R = \begin{pmatrix} \gamma^0 \delta^0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \gamma^0 \delta^0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \gamma^0 \delta^0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \gamma^0 \delta^0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \gamma^0 \delta^0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \gamma^0 \delta^0 \end{pmatrix}$$

### 4.2 Observer synthesis

In this paper we use the definition of an observer from the articles [HMCSM10], [HMCL10]. Figure 5 shows the system with disturbances  $w$  and from which we can observe the outputs  $y_o$ . The observer is a new model obtained from the fault-free model and that will estimate the states of the system  $x_r$  in the presence of such disturbances.

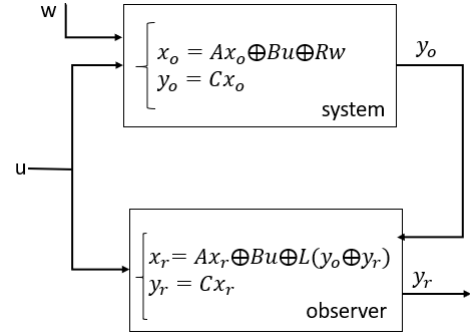


Figure 5: Observer structure with disturbance

From articles [HMCSM10], [HMCL10] we get the following observer's equations:

$$\begin{cases} x_r = Ax_r \oplus Bu \oplus L(y_r \oplus y_o) \\ = (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw, \\ y_r = Cx_r. \end{cases} \quad (5)$$

To obtain the estimated vector  $x_r$  as close as possible to real state  $x$ , the observer relies on the largest matrix  $L \in \mathcal{M}_{in}^{ax}[\gamma, \delta]^{n \times q}$  such that:

$$(A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw \stackrel{\preceq}{=} x_o \stackrel{\preceq}{=} A^*Bu \oplus A^*Rw$$

which is given by:

$$L = (A^* B \phi C A^* B) \wedge (A^* R \phi C A^* R).$$

The observer matrix  $L$  of the TEG of Figure 1 is

$$L = \begin{pmatrix} \cdot & \gamma^1 \delta^0 (\gamma^1 \delta^3)^* & \cdot \\ \cdot & \gamma^0 \delta^0 (\gamma^1 \delta^3)^* & \cdot \\ \cdot & \cdot & \gamma^1 \delta^0 (\gamma^1 \delta^4)^* \\ \gamma^1 \delta^0 (\gamma^1 \delta^4)^* & \gamma^0 \delta^2 (\gamma^1 \delta^4)^* & \gamma^0 \delta^0 (\gamma^1 \delta^4)^* \\ \gamma^0 \delta^0 (\gamma^1 \delta^4)^* & \gamma^0 \delta^6 (\gamma^1 \delta^4)^* & \gamma^0 \delta^5 (\gamma^1 \delta^4)^* \end{pmatrix}$$

Based on the previous observer, suppose that the system behaves with respect to the inputs  $u_1$  and  $u_2$  defined in Section 3 but transition  $x_4$  disturbed with  $w_4 = \gamma^0 \delta^7 \oplus \gamma^1 \delta^{12} \oplus \gamma^2 \delta^{17} \oplus \gamma^3 \delta^{22} \oplus \gamma^4 \delta^{27} \oplus \gamma^5 \delta^{32} \oplus \gamma^6 \delta^{37} \oplus \gamma^7 \delta^{+ \infty}$  then the reconstructed state is  $x_r = (A \oplus LC)^* B u \oplus (A \oplus LC)^* L C A^* R w$  which is the vector  $x_r = [x_{r1}, \dots, x_{r6}]^T =$

$$\begin{bmatrix} \gamma^0 \delta^2 \oplus \gamma^1 \delta^5 \oplus \gamma^2 \delta^8 \oplus \gamma^3 \delta^{11} \oplus \gamma^4 \delta^{14} \oplus \gamma^5 \delta^{17} \oplus \gamma^6 \delta^{20} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^5 \oplus \gamma^1 \delta^8 \oplus \gamma^2 \delta^{11} \oplus \gamma^3 \delta^{14} \oplus \gamma^4 \delta^{17} \oplus \gamma^5 \delta^{20} \oplus \gamma^6 \delta^{23} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^2 \oplus \gamma^1 \delta^7 \oplus \gamma^2 \delta^{12} \oplus \gamma^3 \delta^{17} \oplus \gamma^4 \delta^{22} \oplus \gamma^5 \delta^{27} \oplus \gamma^6 \delta^{32} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^7 \oplus \gamma^1 \delta^{12} \oplus \gamma^2 \delta^{17} \oplus \gamma^3 \delta^{22} \oplus \gamma^4 \delta^{27} \oplus \gamma^5 \delta^{32} \oplus \gamma^6 \delta^{37} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^8 \oplus \gamma^1 \delta^{13} \oplus \gamma^2 \delta^{18} \oplus \gamma^3 \delta^{23} \oplus \gamma^4 \delta^{28} \oplus \gamma^5 \delta^{33} \oplus \gamma^6 \delta^{38} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^{12} \oplus \gamma^1 \delta^{17} \oplus \gamma^2 \delta^{22} \oplus \gamma^3 \delta^{27} \oplus \gamma^4 \delta^{32} \oplus \gamma^5 \delta^{37} \oplus \gamma^6 \delta^{42} \oplus \gamma^7 \delta^{+ \infty} \end{bmatrix}$$

The state  $x_r$  estimated by the observer takes into account the disturbance  $w_4$ . If the disturbance  $w_4$  were not present, the estimated state would be  $x_{r4} = \gamma^0 \delta^6 \oplus \gamma^1 \delta^{10} \oplus \gamma^2 \delta^{14} \oplus \gamma^3 \delta^{18} \oplus \gamma^4 \delta^{22} \oplus \gamma^5 \delta^{26} \oplus \gamma^6 \delta^{30} \oplus \gamma^7 \delta^{+ \infty}$  (no time shift: monomial  $\gamma^0 \delta^6$  instead of  $\gamma^0 \delta^7$ ).

## 5 Time shift failure detection in (max,+)-linear systems with observer

Figure 6 shows how the proposed set of indicators is designed: the system is ruled by the observable inputs  $u$ , the unobservable disturbances  $w$  and produces the observable outputs  $y_o$ ; the observer estimates the states  $x_r$  based on the observation of  $u$  and  $y_o$ . States  $x_s$  result from the simulation of the fault-free model (as in Figure 1) based on  $u$ , the proposed indicator then relies on a series comparison denoted  $\Delta(x_{ri}, x_{si})$  (see Definition 9) for every transition  $x_i$ .

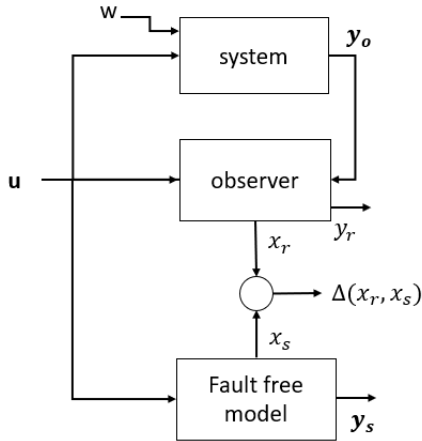


Figure 6: Detection method structure

**Definition 10.** The indicator for state  $x_i$  is  $I_{x_i}(u, y_o)$  defined as the Boolean function:

$$I_{x_i}(u, y_o) = \begin{cases} \text{false} & \text{if } \Delta(x_{ri}, x_{si}) = [0; 0], \\ \text{true} & \text{otherwise,} \end{cases}$$

with

$$x_s = [x_{s1} \dots x_{sn}]^T = A^* B u$$

$$x_r = [x_{r1} \dots x_{rn}]^T = A x_r \oplus B u \oplus L C x_r \oplus L y_o$$

$$\Delta(x_{ri}, x_{si}) = [\mathcal{D}_{x_{ri} \phi x_{si}}(0); -\mathcal{D}_{x_{si} \phi x_{ri}}(0)].$$

**Theorem 4.** The indicator  $I_{x_i}(u, y_o)$  returns true only if a time shift failure involving  $x_i$  with  $\Delta(x_{ri}, x_{si}) \neq [0, 0]$  has occurred in the system. A time shift failure involves a transition  $x_i$  if the time shift failure occurs in a place of the TEG that is in the upstream<sup>1</sup> of transition  $x_i$ .

To prove the result, we show that if the system has no failure in the places in the upstream of  $x_i$  then the indicator  $I_{x_i}(u, y_o)$  necessary returns false. Suppose the system does not have such a time shift failure, it means by definition of the observer that the estimated state  $x_{ri}$  is the same as the fault-free model state  $x_{si}$  as no place in the upstream of  $x_i$  is disturbed. If  $x_{si} = x_{ri}$ , then we have  $x_{si} \phi x_{ri} = x_{ri} \phi x_{si} = x_{ri} \phi x_{ri}$  but  $x_{ri} \phi x_{ri} = (x_{ri} \phi x_{ri})^*$  according to Theorem 2 and with Definition 1 of the Kleene star:  $(x_{ri} \phi x_{ri})^* = e \oplus \dots = \gamma^0 \delta^0 \oplus \dots$ . So if  $x_{ri} = x_{si}$ , one has  $\mathcal{D}_{x_{ri} \phi x_{si}}(0) = -\mathcal{D}_{x_{si} \phi x_{ri}}(0) = 0$ .

In the example of Section 2, based on the previous observer, suppose that the system behaves with respect to the inputs  $u_1$  and  $u_2$  defined in Section 3. Suppose that in reality there was an incident on Equipment 2: the operation lasts longer with a processing time of 5 hours in  $p_5$  instead of 4 hours (see Figure 1). The real system is then characterized by Equation (3) with the disturbance  $w_4$  that is defined in Section 4.1. The estimated state is the same as given at the end of Section 4.2. In particular,  $x_{r3}$  is represented with plain line in Figure 7. The expected state  $x_s$  is the vector  $[x_{s1}, \dots, x_{s6}]^T =$

$$\begin{bmatrix} \gamma^0 \delta^2 \oplus \gamma^1 \delta^5 \oplus \gamma^2 \delta^8 \oplus \gamma^3 \delta^{11} \oplus \gamma^4 \delta^{14} \oplus \gamma^5 \delta^{17} \oplus \gamma^6 \delta^{20} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^5 \oplus \gamma^1 \delta^8 \oplus \gamma^2 \delta^{11} \oplus \gamma^3 \delta^{14} \oplus \gamma^4 \delta^{17} \oplus \gamma^5 \delta^{20} \oplus \gamma^6 \delta^{23} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^2 \oplus \gamma^1 \delta^6 \oplus \gamma^2 \delta^{10} \oplus \gamma^3 \delta^{14} \oplus \gamma^4 \delta^{18} \oplus \gamma^5 \delta^{22} \oplus \gamma^6 \delta^{26} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^6 \oplus \gamma^1 \delta^{10} \oplus \gamma^2 \delta^{14} \oplus \gamma^3 \delta^{18} \oplus \gamma^4 \delta^{22} \oplus \gamma^5 \delta^{26} \oplus \gamma^6 \delta^{30} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^7 \oplus \gamma^1 \delta^{11} \oplus \gamma^2 \delta^{15} \oplus \gamma^3 \delta^{19} \oplus \gamma^4 \delta^{23} \oplus \gamma^5 \delta^{27} \oplus \gamma^6 \delta^{31} \oplus \gamma^7 \delta^{+ \infty} \\ \gamma^0 \delta^{11} \oplus \gamma^1 \delta^{15} \oplus \gamma^2 \delta^{19} \oplus \gamma^3 \delta^{23} \oplus \gamma^4 \delta^{27} \oplus \gamma^5 \delta^{31} \oplus \gamma^6 \delta^{35} \oplus \gamma^7 \delta^{+ \infty} \end{bmatrix}$$

Series  $x_{s3}$  is represented with dotted line in Figure 7. The

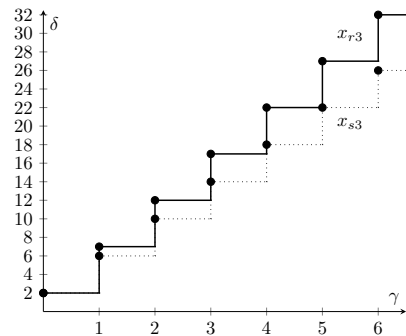


Figure 7: Graphical representation of  $x_{r3}$  and  $x_{s3}$

intervals computed by the set of indicators are:

$$\Delta(x_{r1}, x_{s1}) = [\mathcal{D}_{x_{r1} \phi x_{s1}}(0), -\mathcal{D}_{x_{s1} \phi x_{r1}}(0)] = [0, 0],$$

$$\Delta(x_{r2}, x_{s2}) = [\mathcal{D}_{x_{r2} \phi x_{s2}}(0), -\mathcal{D}_{x_{s2} \phi x_{r2}}(0)] = [0, 0],$$

<sup>1</sup>A place  $p$  is in the upstream of a transition  $x$  in a TEG if there is a path of arcs from  $p$  to  $x$ .

$$\begin{aligned}\Delta(x_{r3}, x_{s3}) &= [\mathcal{D}_{x_{r3} \neq x_{s3}}(0), -\mathcal{D}_{x_{s3} \neq x_{r3}}(0)] = [0, 6], \\ \Delta(x_{r4}, x_{s4}) &= [\mathcal{D}_{x_{r4} \neq x_{s4}}(0), -\mathcal{D}_{x_{s4} \neq x_{r4}}(0)] = [1, 7], \\ \Delta(x_{r5}, x_{s5}) &= [\mathcal{D}_{x_{r5} \neq x_{s5}}(0), -\mathcal{D}_{x_{s5} \neq x_{r5}}(0)] = [1, 7], \\ \Delta(x_{r6}, x_{s6}) &= [\mathcal{D}_{x_{r6} \neq x_{s6}}(0), -\mathcal{D}_{x_{s6} \neq x_{r6}}(0)] = [1, 7].\end{aligned}$$

The set of indicators that return true are associated with transitions  $x_3, x_4, x_5, x_6$ . Indicators for transitions  $x_1, x_2$  return false. Now, if we assume that there is only one type of time shift failure in the system (as it is the case in the proposed scenario where there is only one incident on Equipement 2), Proposition 4 ensures that the time shift failure occurs in a place that is in the upstream of every transition  $x_3, x_4, x_5, x_6$ . It follows that the time shift failure occurs in Eq2, either in place  $p_2$  (transportation delay before the arrival in front of Eq2), or in place  $p_6$  (processing start of Eq2 is delayed), or in place  $p_5$  (process of Eq2 longer than expected, which is the real case).

## 6 Conclusion

In this paper, we define a method for detecting time shift failures in systems modeled as Timed-Event Graphs using an observer that estimates the real states of the system based on the observations. Our work is motivated by the monitoring and the detection of time shift in production lines like in semiconductor manufacturing industry. The method defines a formal (max,+) algebraic indicator on the residuation theory. The proposed indicator is able to detect the presence of time shift failures as soon as it returns true and provides first localisation results. As a perspective, we aim at improving the accuracy of this indicator to better exploit the quantitative information contained in the interval  $\Delta(x_{ri}, x_{si})$ . We expect that a further analysis about the bounds of the intervals may actually provide more information about failure localization and identification. Indeed, observer matrix  $L$  actually represents the connections between the observed output  $y_o$  and the internal transitions  $x_r$  of the observer. In the matrix  $L$  of Section 4.2, we can notice that all the rows are filled which means that  $L$  is able to provide an estimate of any of the states  $x_i$ 's (one row per  $x_i$ ). Looking at the first monomial  $\gamma^j \delta^l$  of a series  $\gamma^j \delta^l (\dots)^*$  it should be possible to know when a state  $x_i$  starts to be effectively reconstructed by  $L$ . A monomial like  $\gamma^j \delta^l$  asserts that the estimation of the corresponding state only starts after the  $j^{th} + 1$  event. By using this property of  $L$ , our perspective would be to make the indicators more accurate and take benefit of the quantitative information from the computed intervals to estimate the delay of the time failures.

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