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# Event-triggered Dynamic Output Feedback Controller for Discrete-time LPV Systems with Constraints<sup>\*</sup>

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**Abstract:** This paper examines an event-triggered control design approach for discrete-time linear parameter-varying (LPV) systems under control constraints. A parameter-dependent dynamic output-feedback controller with an event-triggering condition is designed to ensure the regional asymptotic stability of the closed-loop system while minimizing a quadratic cost function. Sufficient conditions are derived in terms of linear matrix inequalities (LMIs) thanks to the use of a parameter-dependent quadratic Lyapunov function. Also, convex optimization schemes are proposed using these conditions to minimize an upper bound of a given cost function or to maximize the size of the region of closed-loop stability. Examples illustrate the proposal.

*Keywords:* Linear parameter-varying systems. Event-triggered control. Saturation. Dynamic controller. Discrete-time systems.

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## 1. INTRODUCTION

Event-triggered control (ETC) has attracted a lot of attention in recent years due to its significant benefits in terms of reduced usage of the communication and computational resources compared to traditional control systems (Sandee et al., 2005; Aström, 2008; Heemels et al., 2012; Postoyan et al., 2015). The idea behind the event-triggered control consists of performing control tasks after the occurrence of an event, generated by some well-designed event-triggering mechanism, rather than the elapse of a certain fixed period of time, as in conventional periodic sampled-data control. ETC is then capable of reducing effectively the control task executions, while ensuring a satisfactory closed-loop performance. Various ETC strategies can be found in the literature: see, for example, (Wu et al., 2014a; Zhang and Han, 2014; Khashooei et al., 2017) for continuous-time framework and (Eqtami et al., 2010; Tallapragada and Chopra, 2012; Wu et al., 2016) for discrete-time counterpart. However, most of the prior results have been obtained for linear and nonlinear systems, without taking into account the presence of measurable parameters or control constraints.

Besides, Linear Parameter-Varying (LPV) systems represent a very important class of dynamical systems whose

dynamics depends on a priori unknown, but on-line measurable time-varying parameters. Due to their effectiveness in modeling and control of nonlinear systems, LPV systems have been extensively studied in the literature (Mohammadpour and Scherer, 2012). However, a few studies have been done on event-triggered control of LPV systems, in particular in the discrete-time context. In (Hu et al., 2012) an event-triggered guaranteed cost control is proposed for systems with norm-bounded uncertainties and time-varying transmission delays. Li et al. (2015) propose an event-triggered  $\mathcal{H}_\infty$  control by jointly designing an event-triggering mechanism with two given threshold (proportional, additional). In (Li and Xu, 2013; Golabi et al., 2016) the problem of the co-design of an event-triggering condition and a state-feedback controller is addressed.

Additionally, saturation is a common phenomenon in practical control systems. However, its presence in a control loop can cause performance degradation or even instability. Recently, the ETC problem under actuator saturation was addressed, for instance, by Kiener et al. (2014); Seuret et al. (2016); Liu and Yang (2017); Li et al. (2018) for continuous-time framework and by Wu et al. (2014b); Zuo et al. (2016); Groff et al. (2016); Ma et al. (2019) for discrete-time counterpart. More precisely, a procedure to design a state-feedback controller under a given event-triggering condition that maximizes the region of attraction of linear systems is proposed in (Wu et al., 2014b) and of linear piece-wise affine systems in (Ma et al., 2019). A cone complementary linearization algorithm is used in

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(Zuo et al., 2016) for solving the non-convex optimization procedure to treat the co-design of an event-triggering strategy and a controller under saturation. In (Groff et al., 2016), the co-design of a static state-feedback gain and an event-triggering function to stabilize discrete-time LTI systems subject to actuator saturation is proposed aiming at reducing the sampling activity.

Based on the above discussion, it is clear that the event-triggered control design remains an open issue for discrete-time LPV systems under saturating actuators. Thus, the main contributions of this paper aims at filling such gap providing the following features: i) *A convex procedure to design a parameter-dependent dynamic output-feedback controller with anti-windup action subject to a given event-triggering condition*; ii) *The proposed methodology includes the optimization of the control performance, while transmission reductions can be configured by adapting a given parameter in the event-generator*. The convex conditions are formulated in terms of LMIs applying the Lyapunov theory together with the S-procedure and the generalized sector condition, and guarantee the regional asymptotic stability of the closed-loop system. The usefulness of our design approach is illustrated by a numerical example and comparison with similar approach in the literature.

**Notation:** The set of real numbers is denoted by  $\mathbb{R}$ . The set of integer numbers belonging to the interval from  $a \in \mathbb{Z}$  up to  $b \in \mathbb{Z}$ ,  $b \geq a$ , is denoted by  $\mathcal{I}[a, b]$ . The set of matrices with real entries and dimensions  $m \times n$  is noted by  $\mathbb{R}^{m \times n}$ . A block-diagonal matrix  $A$  with blocks  $A_1$  and  $A_2$  is denoted as  $A = \text{diag}\{A_1, A_2\}$ . The  $\ell^{\text{th}}$  line of a vector or matrix  $A$  is indicated by  $A_{(\ell)}$ . The matrix  $\mathbf{0}$  stands for the null matrix of appropriate dimensions and  $\mathbf{I}_n$  corresponds to the identity matrix with dimensions  $n \times n$ . The symbol  $\star$  stands for symmetric blocks in the matrices.  $\bullet$  represents an element that has no influence on development.

## 2. PROBLEM FORMULATION

Consider the discrete-time system represented by:

$$\begin{aligned} x(k+1) &= A(\theta_k)x(k) + B(\theta_k)\text{sat}(u(k)), \\ y(k) &= Cx(k), \end{aligned} \quad (1)$$

where  $x(k) \in \mathbb{R}^n$  is the state vector,  $y(k) \in \mathbb{R}^p$  is the measurable output,  $u(k) \in \mathbb{R}^m$  is the control signal and  $\text{sat}(u(k))$  is a symmetric decentralized saturation function given by

$$\text{sat}(u_{(\ell)}(k)) = \text{sign}(u_{(\ell)}(k)) \min(|u_{(\ell)}(k)|, \bar{u}_{(\ell)}) \quad (2)$$

with  $\bar{u}_{(\ell)} > 0$ ,  $\ell \in \mathcal{I}[1, m]$ , being the  $\ell^{\text{th}}$  component of the symmetric saturation level  $\bar{u}$ .

The time-varying matrices  $A(\theta_k) \in \mathbb{R}^{n \times n}$  and  $B(\theta_k) \in \mathbb{R}^{n \times m}$  belong to a polytopic set given by the convex combination of known  $N$  vertices, i.e.

$$[A(\theta_k) \ B(\theta_k)] = \sum_{i=1}^N \theta_{k(i)} [A_i \ B_i], \quad (3)$$

where  $\theta_k \in \mathbb{R}^n$  is a vector of time-varying parameters that belongs to a unitary simplex

$$\Theta \triangleq \left\{ \sum_{i=1}^N \theta_{k(i)} = 1, \theta_{k(i)} \geq 0, i \in \mathcal{I}[1, N] \right\}. \quad (4)$$

To stabilize system (1), we adopt the following parameter-dependent dynamic output-feedback controller:

$$\begin{aligned} x_c(k+1) &= A_c(\theta_k)x_c(k) + B_c(\theta_k)\hat{y}(k) - E_c(\theta_k)\Psi(u(k)), \\ u(k) &= C_c(\theta_k)x_c(k) + D_c(\theta_k)\hat{y}(k), \end{aligned} \quad (5)$$

where  $x_c(k) \in \mathbb{R}^n$  is the state of the controller,  $\Psi(u(k)) : \mathbb{R}^m \rightarrow \mathbb{R}^m$  is the dead-zone non-linearity defined by  $\Psi(u(k)) = u(k) - \text{sat}(u(k))$ , and  $\hat{y}(k)$  is a signal defined as

$$\hat{y}(k) = \begin{cases} y(k), & \text{if } y(k) \text{ is updated,} \\ \hat{y}(k-1), & \text{if } y(k) \text{ is not updated.} \end{cases} \quad (6)$$

Note that we are employing an event-triggering mechanism to determine whether or not the current measurement  $y(k)$  should be transmitted to the controller. The idea is to reduce the number of packets exchanged between these nodes, so that we can save energy and communication bandwidth while ensuring the stability of the closed-loop system with a certain performance index. The updating of the output signal is performed ever that the following event-triggering condition is verified

$$\|\hat{y}(k-1) - \hat{y}(k)\|^2 > \sigma^2(\mu\|y(k)\|^2 + (1-\mu)\|u(k)\|^2) \quad (7)$$

for a given  $\sigma > 0$  and  $\mu \in \mathcal{I}[0, 1]$ .

Additionally, we assume the following linear structures for the matrices of the controller (5):

$$\begin{aligned} [A_c(\theta_k) \ B_c(\theta_k)] &= \sum_{i=1}^N \sum_{j=1}^N (1 + \varsigma_{ij}) \theta_{k(i)} \theta_{k(j)} \begin{bmatrix} \frac{A_{cij}}{2} & \frac{B_{cij}}{2} \end{bmatrix}, \\ [C_c(\theta_k) \ D_c(\theta_k)] &= \sum_{i=1}^N \theta_{k(i)} [C_{ci} \ D_{ci}], \ E_c(\theta_k) = \sum_{i=1}^N \theta_{k(i)} E_{ci}, \end{aligned}$$

with  $\theta_k \in \Theta$  and  $\varsigma_{ij} = 1$  if  $i \neq j$  and  $\varsigma_{ij} = 0$  otherwise.

In this technical note, we consider not only the stability of the discrete-time saturated LPV system by using a dynamical event-triggered control, but also the satisfaction of a certain level of performance measured by a quadratic cost function. The considered cost function is defined as follows

$$J_\infty = \sum_{k=0}^{\infty} J(k) = \sum_{k=0}^{\infty} x(k)' Q x(k) + u(k)' R u(k), \quad (8)$$

where  $Q \in \mathbb{R}^{n \times n}$  and  $R \in \mathbb{R}^{m \times m}$  are symmetric and positive definite matrices.

Since there is a non-linearity in the loop, the global stability of the origin may not be guaranteed. In this case, the region of attraction  $\mathcal{R}_A$ , defined in terms of the augmented state vector  $\xi(k) = [x(k)' \ x_c(k)']' \in \mathbb{R}^{2n}$ , must be considered. As the exact characterization of the  $\mathcal{R}_A$  is, generally, a hard task, it is important to determine subsets with a well-fitted analytical representation (see, for example, in (Tarbouriech et al., 2011)). By denoting  $\mathcal{R}_E$  the estimated attraction region, then we are interested in computing  $\mathcal{R}_E \subseteq \mathcal{R}_A$  and in optimizing its size with respect to some criteria (Tarbouriech et al., 2011).

Hence, the problem we intend to solve can be summarized as follows.

*Problem 1.* Given  $\sigma$  and  $\mu$ , design a parameter-dependent dynamic output-feedback controller (5) under the event-triggering condition (6)-(7), that ensures the regional

asymptotic stability of the closed-loop system, while minimizing the quadratic cost function (8).

### 3. MAIN RESULTS

#### 3.1 Preliminary Results

Define the error variable as

$$e(k) = \hat{y}(k) - y(k). \quad (9)$$

Based on the definition (7), the following inequality

$$\|e(k)\|^2 \leq \sigma^2(\mu\|y(k)\|^2 + (1-\mu)\|u(k)\|^2) \quad (10)$$

is always satisfied. Thus, with the control law (5) and  $\hat{y}(k) = e(k) + y(k)$ , the closed-loop system takes the form

$$\begin{aligned} \xi(k+1) &= \mathbb{A}(\theta_k)\xi(k) - \mathbb{B}(\theta_k)\Psi(u(k)) + \mathbb{E}(\theta_k)e(k) \\ u(k) &= \mathbb{K}(\theta_k)\xi(k) + D_c(\theta_k)e(k) \\ y(k) &= \mathbb{C}\xi(k) \end{aligned} \quad (11)$$

where  $\xi(k) = [x(k)' \ x_c(k)']' \in \mathbb{R}^{2n}$  and

$$\begin{aligned} \mathbb{A}(\theta_k) &= \begin{bmatrix} A(\theta_k) + B(\theta_k)D_c(\theta_k)C & B(\theta_k)C_c(\theta_k) \\ B_c(\theta_k)C & A_c(\theta_k) \end{bmatrix}, \\ \mathbb{B}(\theta_k) &= \begin{bmatrix} B(\theta_k) \\ E_c(\theta_k) \end{bmatrix}, \quad \mathbb{E}(\theta_k) = \begin{bmatrix} B(\theta_k)D_c(\theta_k) \\ B_c(\theta_k) \end{bmatrix}, \\ \mathbb{K}(\theta_k) &= [D_c(\theta_k)C \ C_c(\theta_k)] \quad \text{and} \quad \mathbb{C} = [C \ \mathbf{0}]. \end{aligned}$$

The time-varying matrices of the closed-loop system (11) also verify

$$\begin{aligned} [\mathbb{A}(\theta_k) \ \mathbb{E}(\theta_k)] &= \sum_{i=1}^N \sum_{j=i}^N (1 + \varsigma_{ij})\theta_{k(i)}\theta_{k(j)} \begin{bmatrix} \hat{\mathbb{A}}_{ij} & \hat{\mathbb{E}}_{ij} \\ 2 & 2 \end{bmatrix}, \\ [\mathbb{B}(\theta_k) \ \mathbb{K}(\theta_k)'] &= \sum_{i=1}^N \theta_{k(i)} [\mathbb{B}_i \ \mathbb{K}'_i], \end{aligned}$$

with  $\theta_k \in \Theta$  and  $\varsigma_{ij} = 1$  if  $i \neq j$  and  $\varsigma_{ij} = 0$  otherwise.

To investigate the regional asymptotic stability of the closed-loop system (11), we propose the following candidate Lyapunov function

$$V(k) = \xi(k)' P^{-1}(\theta_k)\xi(k), \quad (12)$$

where  $P(\theta_k) = \sum_{i=1}^N \theta_{k(i)} P_i$ , with  $\mathbf{0} < P_i = P'_i \in \mathbb{R}^{2n \times 2n}$  and  $\theta_k \in \Theta$ . Thus, we are searching for a dynamic controller such that the difference  $\Delta V(k) = V(k+1) - V(k)$  along the trajectories of the closed-loop system satisfies

$$\begin{aligned} \Delta V(k) &= \xi(k+1)' P^{-1}(\theta_{k+1})\xi(k+1) \\ &\quad - \xi(k)' P^{-1}(\theta_k)\xi(k) < -J(k). \end{aligned} \quad (13)$$

By summing (13) up from  $k=0$  and  $k=\infty$ , it yields to

$$J_\infty < \xi(0)' P^{-1}(\theta_0)\xi(0) < \text{tr}(P^{-1}(\theta_0))\|\xi(0)\|^2, \quad (14)$$

with  $J_\infty$  defined in (8). Thus, we can conclude that the upper bounded of the cost function  $J_\infty$  is related to the Lyapunov matrix  $P^{-1}(\theta_0)$  and the initial state  $\xi(0)$ . The estimation of the region of attraction of the origin for the closed-loop system is computed, in this case, by

$$\mathcal{R}_\mathcal{E} = \bigcap_{\theta_k \in \Theta} \mathcal{E}(P(\theta_k)^{-1}, 1) = \bigcap_{i \in \mathcal{I}[1, N]} \mathcal{E}(P_i^{-1}, 1) \quad (15)$$

with  $\mathcal{E}(P_i^{-1}, 1) = \{\xi(k) \in \mathbb{R}^{2n} : \xi(k)^\top P_i^{-1} \xi(k) \leq 1\}$  for all  $i \in \mathcal{I}[1, N]$ .

To deal with the saturating actuator, we use the following property directly derived from Tarbouriech et al. (2011).

*Lemma 1.* Consider a matrix  $G(\theta_k) = \sum_{i=1}^N \theta_{k(i)} G_i$  with  $G_i \in \mathbb{R}^{m \times 2n}$ ,  $i \in \mathcal{I}[1, N]$  and  $\theta_k \in \Theta$ . If  $\xi(k)$  belongs to  $\mathcal{S}(\bar{u})$  given by

$$\mathcal{S}(\bar{u}) \triangleq \{\xi(k) \in \mathbb{R}^{2n} : |G(\theta_k)\xi(k)| \leq \bar{u}\}, \quad (16)$$

then, for any diagonal positive definite matrix  $T \in \mathbb{R}^{m \times m}$ , the non-linearity  $\Psi(u(k))$  satisfies the following inequality.  $\Psi(u(k))' T (\Psi(u(k)) - (\mathbb{K}(\theta_k) - G(\theta_k))\xi(k) - D_c(\theta_k)e(k)) \leq 0$ . (17)

#### 3.2 Theoretical conditions

Inspired by Scherer et al. (1997), we define the real matrices  $X, Y, W$ , and  $Z \in \mathbb{R}^{n \times n}$  such that

$$U = \begin{bmatrix} X & \bullet \\ Z & \bullet \end{bmatrix}, \quad U^{-1} = \begin{bmatrix} Y & \bullet \\ W & \bullet \end{bmatrix}, \quad \text{and} \quad \Omega = \begin{bmatrix} Y & \mathbf{I}_n \\ W & \mathbf{0} \end{bmatrix}.$$

Therefore, we have

$$U\Omega = \begin{bmatrix} \mathbf{I}_n & X \\ \mathbf{0} & Z \end{bmatrix} \quad \text{and} \quad \hat{U} = \Omega' U \Omega = \begin{bmatrix} Y' & F' \\ \mathbf{I}_n & X \end{bmatrix}, \quad (18)$$

where, by construction

$$F' = Y'X + W'Z. \quad (19)$$

Furthermore, by using the partitioning  $P = \begin{bmatrix} P_{11} & P_{12} \\ \star & P_{22} \end{bmatrix}$ , we obtain

$$\hat{P} = \Omega' P \Omega = \begin{bmatrix} \hat{P}_{11} & \hat{P}_{12} \\ \star & \hat{P}_{22} \end{bmatrix} \quad (20)$$

with  $\hat{P}_{11} = Y'P_{11}Y + W'P'_{12}Y + Y'P_{12}W + W'P_{22}W$ ,  $\hat{P}_{12} = Y'P_{11} + W'P'_{12}$  and  $\hat{P}_{22} = P_{11}$ .

From that, the following result can be stated to design the dynamic controller (5) when the parameters of the event-generator (7)  $\mu$  and  $\sigma$  are assumed to be given.

*Theorem 1.* (Control Synthesis). Given  $\sigma > 0$ ,  $\mu > 0$  and  $\kappa > 0$ , consider that there exist symmetric positive definite matrices  $\hat{P}_i$ , positive definite diagonal matrix  $S$  and matrices  $X, Y, T, \hat{A}_{cij}, \hat{B}_{cij}, \hat{C}_{ci}, \hat{D}_{ci}$  and  $\hat{E}_{ci}$  of appropriate dimensions such that (21) (provided at the top of the next page) and the following LMI are feasible.

$$\begin{bmatrix} \hat{U} + \hat{U}' - \hat{P}_i & \star \\ H_{i(\ell)} & \bar{u}_{(\ell)}^2 \end{bmatrix} > \mathbf{0}, \quad \left\{ \begin{array}{l} i \in \mathcal{I}[1, N] \\ \ell \in \mathcal{I}[1, m]; \end{array} \right. \quad (22)$$

with

$$\begin{aligned} \Pi_{1ij} &= [(\hat{D}_{ci} + \hat{D}_{cj})C \ \hat{C}_{ci} + \hat{C}_{cj}], \\ \Pi_{2ij} &= \begin{bmatrix} Y'(A_i + A_j) + \hat{B}_{cij}C \\ A_i + A_j + (B_i \hat{D}_{cj} + B_j \hat{D}_{ci})C \end{bmatrix}, \\ \Pi_{3ij} &= \begin{bmatrix} \hat{A}_{cij} \\ B_i \hat{D}_{cj} + B_j \hat{D}_{ci} \end{bmatrix}, \quad \Pi_{4ij} = \begin{bmatrix} \hat{E}_{ci} + \hat{E}_{cj} \\ (B_j + B_i)S \end{bmatrix}, \\ \Pi_{5ij} &= \hat{Q}^{1/2} \begin{bmatrix} 2I_n & 2X \\ (\hat{D}_{ci} + \hat{D}_{cj})C \ \hat{C}_{ci} + \hat{C}_{cj} \end{bmatrix}, \quad \hat{Q} = \begin{bmatrix} Q & \mathbf{0} \\ \mathbf{0} & R + \beta_2 \mathbf{I}_m \end{bmatrix}, \\ \Pi_{6ij} &= \hat{Q}^{1/2} \begin{bmatrix} \mathbf{0} \\ \hat{D}_{ci} + \hat{D}_{cj} \end{bmatrix}, \quad \beta_1 = \kappa\sigma^2\mu, \quad \text{and} \quad \beta_2 = \kappa\sigma^2\bar{\mu}. \end{aligned}$$

Therefore, by choosing full rank matrices  $W$  and  $Z$  such that (19) holds, we have that the LPV system (1) under the dynamic output-feedback controller (5) with matrices

$$\begin{array}{c}
\left[ \begin{array}{c|c|c|c|c|c}
\hat{U} + \hat{U}' - \frac{1}{2}(\hat{P}_i + \hat{P}_j) & \star & \star & \star & \star & \star \\
\mathbf{0} & \kappa \mathbf{I}_p & \star & \star & \star & \star \\
\frac{1}{2}(H_i + H_j - \Pi_{1ij}) & -\frac{1}{2}(\hat{D}_{ci} + \hat{D}_{cj}) & 2S & \star & \star & \star \\
\frac{1}{2}\Pi_{2ij} & \frac{1}{2}\Pi_{3ij} & -\frac{1}{2}\Pi_{4ij} & \hat{P}_r & \star & \star \\
C \quad CX & \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{1}{\beta_1} \mathbf{I}_p & \star \\
\frac{1}{2}\Pi_{5ij} & \frac{1}{2}\Pi_{6ij} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{n+m}
\end{array} \right] > \mathbf{0}, \begin{cases} r, i \in \mathcal{I}[1, N] \\ j \in \mathcal{I}[i, N]; \end{cases} \quad (21)
\end{array}$$

$$\begin{aligned}
D_{ci} &= \hat{D}_{ci}, \\
C_{ci} &= (\hat{C}_{ci} - D_{ci}CX)Z^{-1}, \\
B_{cij} &= (W^{-1})'(\hat{B}_{cij} - Y'(B_iD_{cj} + B_jD_{ci})), \\
A_{cij} &= (W^{-1})'(\hat{A}_{cij} - Y'(A_i + A_j + (B_iD_{cj} + B_jD_{ci})C)X \\
&\quad - W'B_{cij}CX - Y'(B_iC_{cj} + B_jC_{ci})Z)Z^{-1}, \\
E_{ci} &= (W^{-1})'(\hat{E}_{ci}S^{-1} - Y'B_i), \quad (23)
\end{aligned}$$

is regionally asymptotically stable, and the region  $\mathcal{R}_{\mathcal{E}}$ , defined in (15), is an estimation of the region of attraction of the origin for the closed-loop system.

The proof of Theorem 1 is presented in the Appendix A.

### 3.3 Optimization procedures

We are more particularly interested in the minimization of the upper bound of the cost function  $J_{\infty}$ . In this case, the objective is to design the dynamic controller (5) that minimizes the cost function  $J_{\infty}$ . According (14),  $J_{\infty}$  is upper-bounded by  $\xi(0)'P^{-1}(\theta_0)\xi(0)$ . Then, considering (20), we have

$$J_{\infty} < \xi(0)'\Omega\hat{P}^{-1}(\theta_0)\Omega'\xi(0) \quad (24)$$

i.e.

$$J_{\infty} < \begin{bmatrix} x(0) \\ x_c(0) \end{bmatrix}' \begin{bmatrix} Y & \mathbf{I} \\ W & \mathbf{0} \end{bmatrix} \hat{P}^{-1}(\theta_0) \begin{bmatrix} Y' & W' \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x(0) \\ x_c(0) \end{bmatrix} \quad (25)$$

By setting  $x_c(0)$  equal to zero, the above inequality results in  $J_{\infty} < x(0)'[Y \ \mathbf{I}]\hat{P}^{-1}(\theta_0)[Y \ \mathbf{I}]'x(0)$ . So, we are interested in minimizing  $\text{tr}([Y \ \mathbf{I}]\hat{P}^{-1}(\theta_0)[Y \ \mathbf{I}]')$ , or, similarly, in minimizing the scalar  $\eta$  such that

$$\begin{bmatrix} \eta \mathbf{I}_n & Y \ \mathbf{I}_n \\ \star & \hat{P}(\theta_0) \end{bmatrix} \geq \mathbf{0} \quad (26)$$

Thus, the optimization procedure can be summarized as

$$\begin{aligned} &\min \quad \eta \\ &\text{subject to LMIs (21), (22), and (26).} \end{aligned} \quad (27)$$

Note that if  $\theta_0$  is not known *a priori*, we have to check the condition (26) for all  $\hat{P}_i$  with  $i \in \mathcal{I}[1, N]$ . In this case, the same kind of optimization procedure can be used to maximize the region of stability  $\mathcal{R}_{\mathcal{E}}$  when no cost function optimization is taken into account.

## 4. SIMULATION RESULTS

Consider the saturated LPV system (1) with the following data:

$$\begin{aligned}
A(\rho) &= (1 + \rho) \begin{bmatrix} 1.0018 & 0.01 \\ 0.36 & 1.0018 \end{bmatrix}, \quad B(\rho) = (1 + \rho) \begin{bmatrix} -0.001 \\ -0.184 \end{bmatrix}, \\
C &= [0.5 \ 1],
\end{aligned}$$

where the measured parameter is  $\|\rho\| \leq 0.05$  and symmetric saturation limit is  $\bar{u} = 1$ . This system can be cast in the polytopic form (3)-(4) with  $A_1 = A(-0.05)$ ,  $B_1 = B(-0.05)$ ,  $A_2 = A(0.05)$  and  $B_2 = B(0.05)$ .

We have run the optimization procedure (27) with  $\kappa = 15$ ,  $Q = 0.2\mathbf{I}$ ,  $R = 1$ , and different values of  $\sigma$  and  $\mu$  such that  $0 < \sigma < 0.2$  and  $0 < \mu < 1$ . Then, for each pair  $(\sigma, \mu)$ , we have simulated the respective closed-loop system with the initial condition  $\xi(0) = [0.02085 \ 0.1203 \ 0 \ 0]'$ , and an  $\theta_k$  sequence that leads the open-loop system to have unstable modes; thus requiring a greater control effort. In each simulation we computed the cost function  $J_{\infty}$  and percentage of the update rate. The  $J_{\infty}$  varies between 0.657 and 0.723 with no evident pattern. On the other hand, the percentage of update rate, as shown in the left-hand side of Figure 1, decreases with bigger values of  $\sigma$  and  $\mu$ . In this case, we can conclude that as  $\mu$  goes to one, it relieves the weight of the control signal, reducing the number of updates. For instance, for  $\sigma = 0.18$ , there was a decrease of 74.51% in the update rate when we increased  $\mu$  from 0.1 to 0.9.

Another similar set of tests was performed with  $\mu = 0.7$ ,  $12.5 \leq \kappa \leq 27.5$ , and  $0 < \sigma < 0.15$ . For each achieved controller design and the same previous initial condition, we have simulated the closed-loop and computed both the cost  $J_{\infty}$  and the percentage of rate update. The  $J_{\infty}$  is shown in the right hand-side of Figure 1, where we note the cost increasing with  $\kappa$ . We have observed no pattern in rate update in this case.

We illustrate the closed-loop behavior under the event-triggering condition with a design dismissing the optimal control law, i.e., with  $Q = 0$ ,  $R = 0$ . We chose  $\mu = \sigma = 0.1$ ,  $\kappa = 1$  and simulated the closed-loop response with  $\xi(0) = [0.1508 \ 0.5741 \ 0 \ 0]'$ . The states, the control signal, the events and the used parameter-varying  $\theta_k$  are showed in Figure 2, where we can see the closed-loop asymptotic stability despite the saturation of the control signal for  $0 \leq k \leq 13$ . This test leads to an mean update rate of 65.55%.

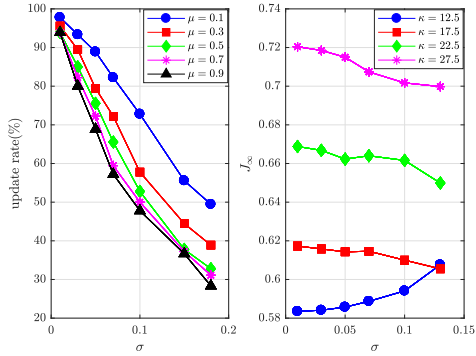


Fig. 1. Update rate (%) with  $\kappa = 15$  (left-side) and cost with  $\mu = 0.7$  (right).

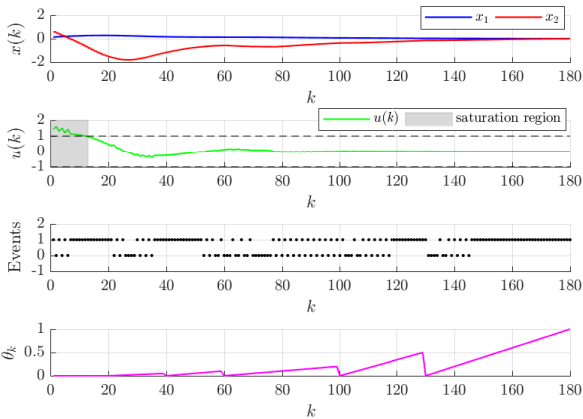


Fig. 2. Time evolution of event-triggered control system.

With a constant  $\rho = 0$ , replacing the output matrix  $C$  by the identity,  $Q = 0$ ,  $R = 0$ , and  $\mu = 1$  we compare our approach with the one in (Wu et al., 2014b). In this case, we set  $\kappa$  with the same value found in (Wu et al., 2014b). So, we show in Figure 3 two projections of  $\mathcal{R}_E$  on the  $x_k$  plan, one for  $\sigma = 0.1$  (line) and the other for  $\sigma = 0.02$  (dashed-line), illustrating how such a region can be sensitive to  $\sigma$  value. However, our approach (blue), with a dynamic output feedback controller, yields to almost the same  $\mathcal{R}_E$  as with the method in (Wu et al., 2014b) (green) which is based on state-feedback. Moreover, if we consider  $\mu \neq 1$ , which is not possible in (Wu et al., 2014b), our projections become even larger (magenta).

## 5. CONCLUSION

In this paper, we investigated an event-triggered control design approach. Our proposal allows synthesizing dynamic output-feedback controllers under event-triggering conditions for discrete-time LPV systems subject to actuator saturation. The convex conditions, in the form of LMIs, ensure the regional asymptotic stability of the closed-loop system while minimizing the upper bound of a quadratic cost function. The usefulness of the proposed methods was showed through numerical tests.

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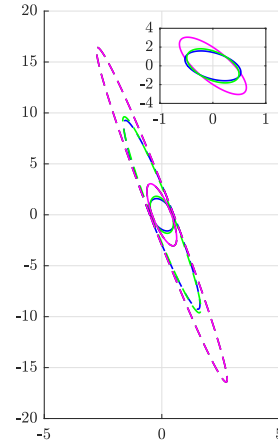


Fig. 3. Comparison of the estimated attractions regions obtained by Theorem 1 and the proposal in (Wu et al., 2014b) for  $\sigma = 0.1$  (line) and  $\sigma = 0.02$  (dashed line).

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$$\begin{aligned}
& \left( \begin{bmatrix} P^{-1}(\theta_k) & \star & \star \\ \mathbf{0} & \mathbf{0} & \star \\ -T(G(\theta_k) - \mathbb{K}(\theta_k)) & -TD_c(\theta_k) & 2T \end{bmatrix} - \begin{bmatrix} \mathbb{A}(\theta_k) \\ \mathbb{E}(\theta_k) \\ -\mathbb{B}(\theta_k) \end{bmatrix}' P^{-1}(\theta_{k+1}) \begin{bmatrix} \mathbb{A}(\theta_k) \\ \mathbb{E}(\theta_k) \\ -\mathbb{B}(\theta_k) \end{bmatrix} \right) \\
& - \left( \begin{bmatrix} \mathbb{I}' & \mathbb{K}(\theta_k)' \\ \mathbf{0} & D_c(\theta_k)' \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} Q & \mathbf{0} \\ \mathbf{0} & R \end{bmatrix} \begin{bmatrix} \mathbb{I} & \mathbf{0} & \mathbf{0} \\ \mathbb{K}(\theta_k) & D_c(\theta_k) & \mathbf{0} \end{bmatrix} \right) > \kappa \left( \begin{bmatrix} \sigma^2(\mu\mathbb{C}'\mathbb{C} + \bar{\mu}\mathbb{K}(\theta_k)'\mathbb{K}(\theta_k)) & \star & \star \\ \sigma^2\bar{\mu}D_c(\theta_k)'\mathbb{K} & \sigma^2\bar{\mu}D_c(\theta_k)'D_c(\theta_k) - \mathbf{I}_p & \star \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \right) \quad (28)
\end{aligned}$$

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## Appendix A. PROOF OF THEOREM 1

By supposing the feasibility of (21), it follows from block (1,1) that  $\hat{U} + \hat{U}' > \mathbf{0}$ , and consequently,  $\hat{U}$  has full rank. Thus, in view of (19), the matrix  $(T' - Y'X)$  has full rank. Then, it is always possible to choose full rank matrices  $W$  and  $Z$  such that (19) is verified. This shows that the gains (23) are well-defined.

Moreover, by considering (18), (20), and the change of variables  $\hat{A}_c, \hat{B}_c, \hat{C}_c, \hat{D}_c, \hat{E}_c, \hat{Q}, \beta_1$  and  $\beta_2$  in (23), pre- and post-multiply (21) by  $\text{diag}\{(\Omega')^{-1}, \mathbf{I}_p, (\Omega')^{-1}, \mathbf{I}_p, \mathbf{I}_{n+m}\}$  and its transpose, respectively. Next, multiply the left-hand side of the resulting inequality by  $\theta_{k(r+1)}, \theta_{k(i)}$  and  $\theta_{k(j)}$ , sum it up for  $r, i \in \mathcal{I}[1, N]$  and  $j \in \mathcal{I}[i, N]$ , and replace  $H(\theta_k)$  by  $G(\theta_k)U$ . Then, use the fact that  $[P(\theta_k) - U']P^{-1}(\theta_k)[P(\theta_k) - U] \geq \mathbf{0}$  to upper-bound  $U' + U - P(\theta_k)$  by  $U'P(\theta_k)^{-1}U$ , pre- and post-multiply the resulting inequality by  $\text{diag}\{(U')^{-1}, \mathbf{I}_p, (S')^{-1}, \mathbf{I}_{2n}, \mathbf{I}_p, \mathbf{I}_{n+m}\}$  and its transpose, respectively, and replace  $S^{-1} = T$ . After, apply Schur complement and rearrange the terms to obtain (28) (reported at the top of the this page).

Finally, pre- and post-multiply both sides of (28) by the augmented vector  $X(k) = [\xi(k)' e(k)' \Psi(u(k))']'$  and  $X(k)'$ , respectively, and replace  $\mathbb{A}(\theta_k)\xi(k) - \mathbb{B}(\theta_k)\Psi(u(k)) + \mathbb{E}(\theta_k)e(k)$  by  $\xi(k+1)$ , see (11), and  $\xi(k+1)'P^{-1}(\theta_{k+1})\xi(k+1) - \xi(k)'P^{-1}(\theta_k)\xi(k)$  by  $\Delta V(k)$  to get

$$\begin{aligned}
& \Delta V(k) - 2\Psi(u(k))'T(\Psi(u(k)) - (\mathbb{K}(\theta_k) - G(\theta_k))\xi(k) \\
& - D_c(\theta_k)e(k)) + J(k) < \kappa(\sigma^2(\mu y(k)'y(k) + (1-\mu)u(k)'u(k)) \\
& - e(k)'e(k)) \leq 0 \quad (\text{A.1})
\end{aligned}$$

Therefore, we conclude that the feasibility of (21) ensures the positivity of (12) and the negativity of  $\Delta V(k)$ . Also, by Lemma 1, we have that  $\xi(k) \in \mathcal{S}(\bar{u})$ , and by inequality (10), we have that the event-triggering condition (7) is always satisfied.

Now, by supposing the feasibility (22), pre- and post multiply it by  $\text{diag}\{(\Omega')^{-1}, \mathbf{I}\}$  and its transpose, respectively. Next, multiply the left-hand side of the resulting inequality by  $\theta_{k(i)}$ , sum it up for  $i \in \mathcal{I}[1, N]$  and replace  $H(\theta_k)_{(\ell)}$  by  $G(\theta_k)_{(\ell)}U$ . Then, use again the fact  $P(\theta_k) - U - U' \geq U'P^{-1}(\theta_k)U$ , and pre- and post-multiply the resulting inequality by  $\text{diag}\{(U')^{-1}, \mathbf{I}\}$  and its transpose, respectively. Finally, apply Schur complement and pre- and post multiply the resulting inequality by  $\xi(k)'$  and  $\xi(k)$ , to get

$$\xi(k)'(-P^{-1}(\theta_k) + G(\theta_k)_{(\ell)}'\bar{u}_{(\ell)}^{-2}G(\theta_k)_{(\ell)})\xi(k) \leq 0, \quad (\text{A.2})$$

which ensures that  $|G(\theta_k)\xi(k)| \leq \bar{u}^2$  and  $\xi(k)'P^{-1}(\theta_k)\xi(k) \leq 1$ , and consequently,  $\mathcal{R}_{\mathcal{E}} \subseteq \mathcal{S}(\bar{u})$ .