Non-Uniform Sampling Theory applied to Optical Feedback Interferometry for Displacement Sensors

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***Abstract*—In this paper, a method based on the non-uniform sampling theory is proposed to recover remote target's displacement from laser optical feedback interferometry (OFI) signals. The laser diode is operated in moderate optical feedback regime and is modulated with a sinusoidal dithering signal via laser diode current drive modulation. This dithering signal allows the recovery of sub-$\lambda/2$ displacements. The proposed method relies only on OFI's fringe detection to recover the target's displacement. The results are compared with mechanical dithering of the target. Using a laser diode emitting at 1550 nm, the measured white noise power spectral density is approximately 3.4 nm/$\sqrt{\text{Hz}}$.**

***Index Terms*—Optical feedback interferometry, self-mixing interferometry,non-uniform sampling, dithering, laser diode current modulation, displacement sensor**

I. Introduction

Optical feedback interferometry (OFI), also referred as self-mixing (SM), effect in laser diodes (LD) [1], [2] has been widely investigated for the last decades as it results in a self-aligned and cost effective sensing system. The resolution of a stationary OFI based displacement sensor depends on the employed signal processing techniques.

Displacement measurement with a basic resolution of half-wavelength (λ_0/2) can be easily achieved with an OFI sensor under moderate optical feedback regime by fringe counting [1]. The basic resolution can be improved by locking the laser phase to half-wavelength [3] or by fringe duplication [4], [5] or by utilizing phase unwrapping techniques. Different phase unwrapping techniques (based on time-domain OFI signal processing) have been proposed in literature [6]–[12] providing accuracy from λ_0/8 to λ_0/60. For accuracy exceeding λ_0/40, these methods [7], [8] require elaborate time-domain SM signal segmentations as well as estimations of key OFI parameters, such as optical feedback coupling parameter C. Except for the fringe-locking method [3], [13], [14], to the best of our knowledge, none of the previously mentioned methods exploiting the modulation of optical output power (OOP) by OFI have achieved precision down to the nanometer yet.

Using the OOP SM signal obtained with a LD operating in the moderate feedback regime ($1 < C < 4.6$), we do propose here a new open-loop approach that allows to recover sub-λ_0/2 displacement with nanometric precision for an LD of wavelength λ_0. This approach retains the inherent simplicity of OFI as the required hardware consists only in amplifying and acquiring the SM signal contrary to [15].

SM interferometer is here perceived as an inherent non-uniform sampling system with its own embedded phase level-crossing detector. Based on the non-uniform sampling (NUS) theory, we show that it is possible to reconstruct the target displacement based on fringe detection only, thereby without requiring phase unwrapping techniques. In addition, a phase dither Φ_d obtained by modulating the LD driving current, is added. This allows to recover the displacements without estimating C as well as sub-$\lambda/2$ displacements. Note that in [16], [17], the dithering technique was also employed to retrieve the displacement. However, the LDs were operated in very weak feedback regime and dithering was employed so that lock-in techniques can be used.

In the following section II, we present the non-uniform sampling theory applied on SM signals as well as the purpose of the dithering signal. Finally, in section III, different experimental test benches are described and results are analyzed to assess the system performances.

II. Proposed Method

A. OFI overview

In OFI, a portion of the laser beam can be back-scattered from a target placed at a distance D_0 from the laser (moving with displacement $D(t)$) and can thus re-enter the active laser cavity (Fig. 1). This causes a mixing of generated and phase-

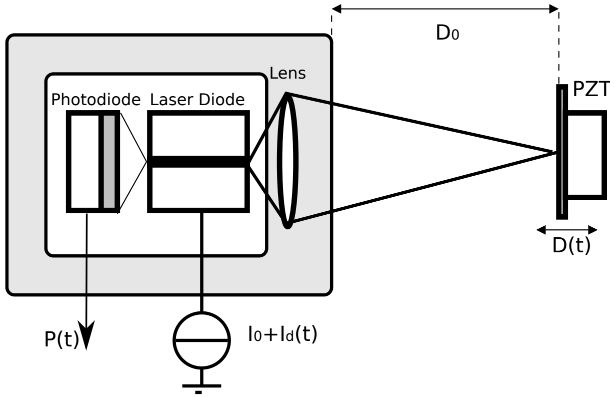

Fig. 1. Self-Mixing displacement sensor set-up with a piezoelectric transducer (PZT) used as a target. A dithering signal can be added either via the laser drive current $I_0 + I_d(t)$.

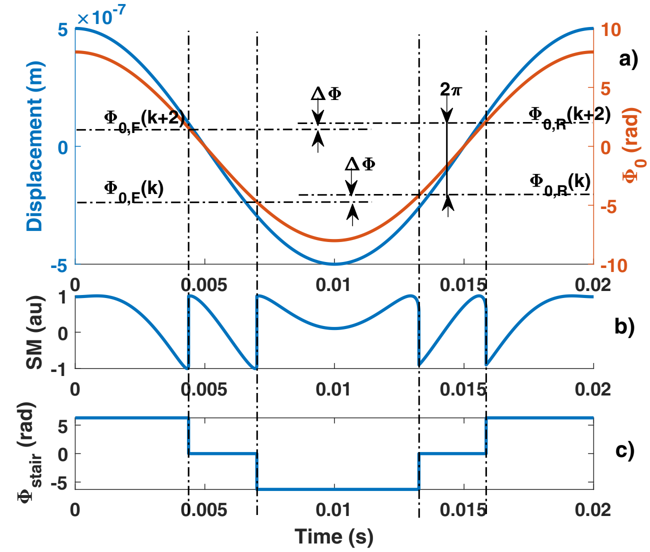

Fig. 2. Simulated typical Self-Mixing signal (b) obtained for (a) a 0.5μm sinusoidal displacement, a laser wavelength λ_0=785 nm and C=1.5 (in blue line) with its corresponding phase Φ_0 (in red) and (c) staircase phase Φ_{stair}.

shifted back-scattered beams. This "self-mixing" (SM) causes fluctuation in the laser OOP, denoted as $P(t)$, given by [1]:

$$P(t) = P_0 \left(1 + m \cos\left(\Phi_F(t)\right)\right) \tag{1}$$

where P_0 is the emitted optical power under free-running conditions, m is the modulation index and $\Phi_F(t)$ is the laser output phase in the presence of feedback. $\Phi_F(t)$ is related to the laser output phase without feedback $\Phi_0(t) = 4\pi D(t)/\lambda_0$ by:

$$\Phi_0(t) = \Phi_F(t) + C \sin\left(\Phi_F(t) + \arctan\alpha\right) \tag{2}$$

where α is referred to as the linewidth enhancement factor [1], [2]. Depending on C , the laser can operate into different regimes. SM sensing is generally performed under weak feedback regime ($C < 1$), moderate feedback regime ($1 < C < 4.6$), or strong feeback regime ($C > 4.6$). However, moderate feedback regime ($1 < C < 4.6$) is usually preferred as the apparently simple saw-tooth shaped SM fringes belonging to such a regime [18] intrinsically provide motion direction indication and require simplified SM fringe detection processing [19].

B. OFI as a non-uniform sampling system

Here, in this moderate feedback regime, based on (1) and (2), we propose to perceive SM interferometers as an inherent non-uniform sampling system with its own embedded phase level-crossing detector. By monitoring the OOP discontinuities, a phase domain level crossing every 2π can thus be obtained (Fig.2). However, as shown in Fig.2, these phase quantization levels (PQL) $\Phi_0(k)$ are slightly different (by an amount denoted $\Delta\Phi$) for an increasing and decreasing Φ_0 phase. These PQLs can thus be referred to as Φ_{0_R} and Φ_{0_F} when Φ_0 is increasing or decreasing respectively. They are completely defined by (2) with Φ_F as given in [20] whenever Φ_F has infinite slopes. It will be later shown that this $\Delta\Phi$ does not cause any issue for the proposed approach.

Similarly to level-crossing analog-to-digital converters [21], [22], the SM phase level-crossing detector outputs time-phase pairs $[t_n, \Phi_n]$. For each pair, t_n corresponds to the time instant when $\Phi_0(t)$ crosses one PQL which can be either $\Phi_{0_R}(k)$ or $\Phi_{0_F}(k)$. Further, due to the data acquisition system used, these t_n are quantized $Q(t_n)$ with a time resolution of $1/f_s$ (where f_s is the sampling frequency) to finally generate non-uniform samples (NUS) $[Q(t_n), \Phi(t_n)]$. As shown in [23], $D(t)$ can be reconstructed from these samples if the quantization sampling rate of $D(t)$ exceeds twice $D(t)$ bandwidth (Nyquist criterion). In addition, to be further processed, these $[Q(t_n), \Phi(t_n)]$ sets are usually fed to an interpolator to generate a uniformly sampled rate output signal. However, for sub-$\lambda_0/2$ displacements, none or only one level crossing (depending on the initial phase value of Φ_F) can be detected thereby leading to a poor displacement reconstruction.

C. Advantages of Dithering

Here, phase dither Φ_d (resulting in an equivalent Displacement dithering $D_d(t)$) is used for two main reasons:

- retrieve sub-λ/2 displacements.
- remove the $\Delta\Phi$ effect on the reconstructed displacement.

In a manner similar to approaches used in NUS ADCs, a phase dither Φ_d can be added to the phase Φ_0 so that both the number of crossed levels as well as the rate of level crossings can be increased. The resulting equivalent displacement $D_\Sigma(t)$ is obtained by summing the target displacement $D(t)$ to $D_d(t)$. Φ_d can be implemented either by directly vibrating the laser itself with $D_d(t)$ or indirectly by modulating the laser driving current, thereby modulating λ_0. Here, a sinusoidal equivalent vibration dithering obtained via current modulation has been chosen with an amplitude A_d and a frequency f_d. To achieve an accurate displacement reconstruction and fulfill the Nyquist criterion, f_d and A_d are chosen to be respectively greater than

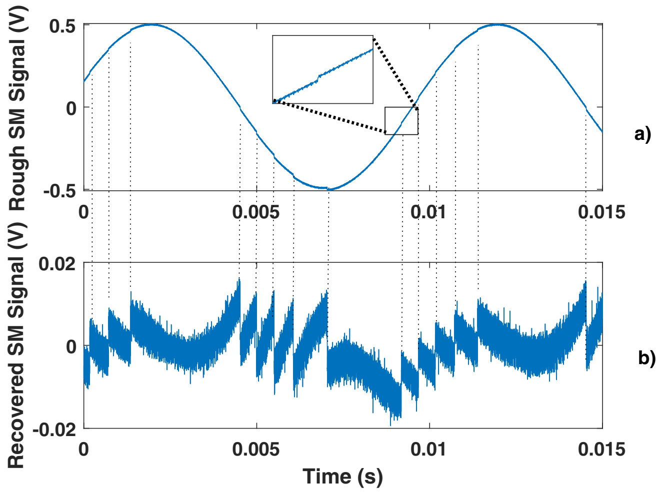

Fig. 3. Measured rough SM signal (a) obtained for a target vibrating at f_v= 10 Hz with a 150 nm amplitude. A dithering current signal modulates the LD drive current at 100 Hz. Note that the optical ouput power is directly modulated by the LD current dithering signal. In (b), recovered SM signal from a) after filtering out the dithering signal.

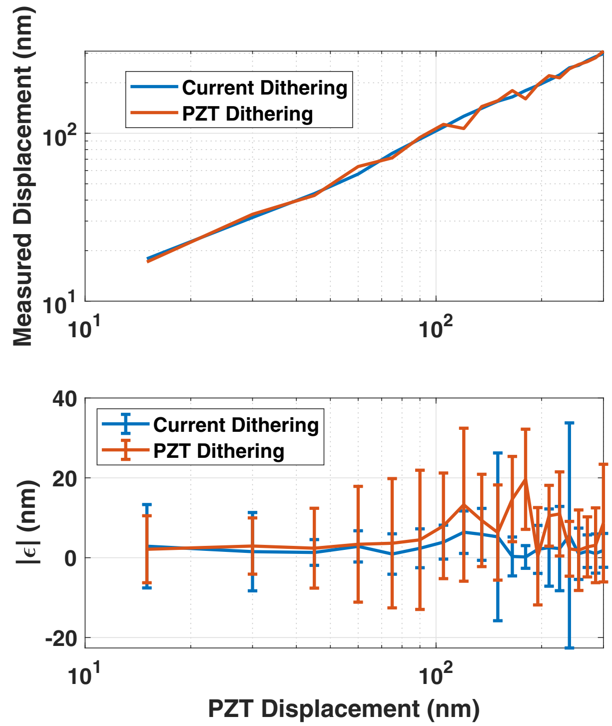

Fig. 4. Measured sub-$\lambda/2$ displacement amplitude at the target vibrating frequency with LD at 50 cm from the PZT using the Spline interpolator via FFT analysis : (1) in red line, PZT is used both as the target vibrating at 10 Hz and as the dithering reference vibration at 100 Hz and (2) in blue line, the LD driving current is modulated to generate the dithering vibration at 100 Hz. The lower plot presents the absolute error results between the reconstructed displacement amplitude and the reference motion provided by PZT's integrated capacitive sensor. The vertical error bars represent the standard deviation obtained for 20 measurements

the bandwidth of interest and than $\lambda_0/2$ (to induce at least two level crossings).

From the NUS samples, as previously mentioned, to achieve the reconstruction of $D\left(t\right)$, interpolators are required to convert these NUSs into uniform samples. Different interpolators can be employed such as: splines, polynomials [23], sinc-based polynomials [24]... Here, both the Consecutive Samples based Unwrapping (CSU) approach [11] that consists in adding the normalized SM signal to the staircase approximation Φ_{stair} of Φ_0 based on $\left[Q\left(t_n\right), \Phi\left(t_n\right)\right]$, and the Spline method are used as interpolators and their achieved performances will be compared in the Results section.

Finally, regarding the impact of $\Delta\Phi$ on the reconstructed displacement, it is interesting to look at this issue from a different perspective. Instead of considering the PQLs Φ_{0_R} and Φ_{0_F} to be different, they can be supposed to be equal if a virtual square displacement of amplitude $\lambda_0\Delta\Phi/8\pi$ is added on top of $D(t)$. In presence of a dithering signal, the change of direction of this virtual square will occur at f_d, which is out of the signal bandwidth of interest. Consequently, $\Delta\Phi$ has a direct impact on the retrieved amplitude of the dithering signal D_d but none on the desired displacement $D\left(t\right)$.

III. Results

A SM test bench was developed to assess the performances of the proposed approach through two main test procedures. The aim of the first one is to verify the principle of dithering. In this case, the target generates both the dithering displacement at 100 Hz and the sub-$\lambda_0/2$ displacement while no current modulation ($I_{LD} = I_0$) is performed. In the second case, as a proof of concept, the target only generates the sub-$\lambda_0/2$ displacement to be recovered while the LD driving current is modulated at 100 Hz ($I_{LD}(t) = I_0 + I_d(t)$) to generate the dithering signal. The LD, driven by I_0=20 mA, is emitting at λ_0=1550 nm. A piezoelectric transducer (PZT) from Physik Instrumente (P753.1CD) is used as a target positioned at 50 cm from the LD. It is equipped with an internal capacitive feedback position sensor with a 0.1 nm resolution and $\pm$1 nm repeatability. The laser beam is focused on the PZT via a lens. The data is acquired by a NI USB 6251 data acquisition system operating at 10^6sample/s with a 16 bit resolution.

Here, the data is firstly processed to remove the LD power modulation induced by $I_d\left(t\right)$ [3] (Fig. 3 a)). This is simply obtained by filtering out this unwanted signal at the frequency f_d from the SM signal FFT spectrum (Fig. 3 b)). Note also that the applied dithering signal induces 5 NEQLs (Fig. 3 b)). Then, a simple fringe detection algorithm (using the derivative of SM signal [6]) is applied on the recovered SM signal to generate the non-uniform sample set $\left[Q\left(t_n\right), \Phi\left(t_n\right)\right]$. From this set, using the NUS method (and using either the spline or the CSU as interpolator), a first estimate of D_Σ is achieved. Then, an FFT analysis is performed to filter out from this reconstructed displacement D_Σ all the signals out of the bandwidth of interest ($f > f_d$). Fig. 4 shows the measured amplitude of the displacement extracted from the SM signal acquired during 1 s in order to retrieve the sub-

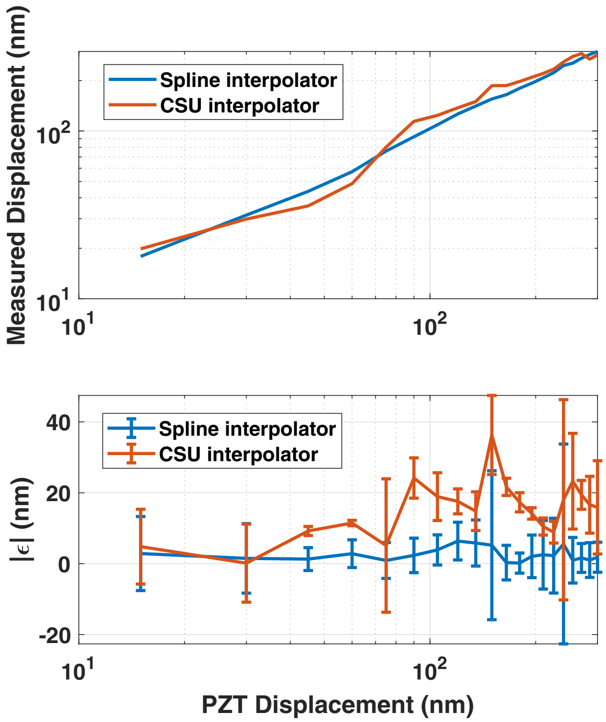

Fig. 5. Impact of type of interpolation: measured sub-$\lambda/2$ displacement amplitude at the target vibrating frequency with LD at 50 cm from the PZT using the Spline (blue line) and CSU (red line) interpolator via FFT analysis. PZT is used as the target vibrating at 10 Hz and the LD driving current is modulated to generate the dithering vibration at 100 Hz. The lower plot presents the absolute error results between the reconstructed displacement amplitude and the reference motion provided by PZT's integrated capacitive sensor. The vertical error bars represent the standard deviation obtained for 20 measurements

$\lambda_0/2$ displacement of a target vibrating at f_v=10 Hz while the dithering vibration is set at f_d=100 Hz. For each displacement amplitude, 20 sets of measurement are performed.

Fig. 4 shows that the proposed method can correctly recover sub-$\lambda_0/2$ displacement while the LD operates in the moderate feedback regime. Further, the results obtained with the current and mechanical dithering are similar. The average RMS error is approximately 8 nm and 11.5 nm for the current- and mechanical-dithering respectively.

Fig. 5 compares the performances obtained between the spline and CSU interpolators. The performances obtained with spline appear to be better with an absolute maximum error of 7 nm compared to 36 nm with the CSU interpolator while both achieved a similar average rms error of 8 nm. This can be explained by the fact that the measured SM signal is quite noisy as shown in Fig. 3 b). As opposed to the case of spline interpolator in which only the detected SM signal discontinuities are used, for the CSU interpolator, scaled SM signal is directly added to reconstructed staircase phase. Thus, CSU based interpolation performance is affected by this added noise.

Fig. 6 shows a typical spectrum of the reconstructed displacement signal together with the dithering signal when using

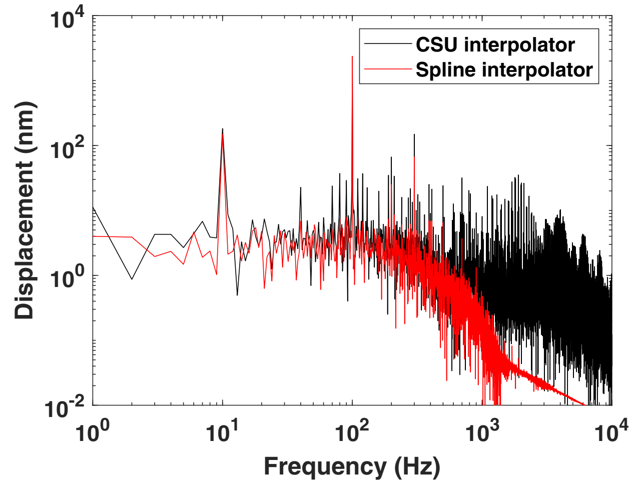

Fig. 6. FFT of the reconstructed displacement obtained with the spline (red line) and CSU (black) interpolation for a 10 Hz target vibration with a 150 nm amplitude and a 100 Hz dithering signal obtained by current modulation.

the CSU and spline interpolation. With the present set-up, the measured white noise power spectral density is approximately 3.4 nm/$\sqrt{\text{Hz}}$.

IV. Conclusion

We propose to interpret OFI as a non-uniform event based sampling system where the quantization levels are clearly defined by the signal discontinuities in the moderate optical feedback regime. A major result obtained by applying this NUS theory on SM signals consists in demonstrating that displacements can be successfully reconstructed from SM fringe detection only. In addition, based on the NUS theory, it was shown that by adding a laser diode current modulation based dithering signal, it was possible to successfully recover sub-$\lambda_0/2$ displacements without knowing C. It was also demonstrated that both Spline and CSU can be used as efficient interpolators.

As a result, the proposed sensor can measure sub-$\lambda_0/2$ displacement and achieve a high precision with a noise power spectral density $\approx$ 3.4 $nm/\sqrt{\text{Hz}}$ while based on both a relatively simple set-up and processing method compared to [15].

By directly removing most of the optical power modulation induced by the current modulation of the LD via hardware circuits, we expect to further improve the noise performances that can be achieved by the system. In addition, based on the NUS theory, the instants when quantization levels are crossed contain all the information. It is thus possible to envision the development of a system based only on fringe detection using an analog front-end similar to [25] and then process the acquired data. As a result, the amount of acquired data to be processed can be much less than that of a classical approach based on an ADC front-end, resulting in simpler and cheaper embedded monitoring systems.

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