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# Calculation of aggregation formulas for GSN argument types using belief functions

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## Abstract

This document is a technical report providing calculation details of aggregation rules for safety case confidence propagation. Belief theory is used to express and propagate this confidence, and safety cases are modeled with GSN (Goal Structuring Notation).

## 1 Introduction

Following the study published in [1], we propose in this technical report an update of the formulae for confidence propagation. In the same direction as [1], we use the same argument types ( $[\bigwedge_{i=1}^n p_i]$  infer  $C$ ) and  $\bigwedge_{i=1}^n [p_i]$  infer  $C$ , which express respectively the relation of conjunction and disjunction between premises. The inference between the premises and the conclusion can be expressed either by an equivalence operator [3, 2, 4, 5] or an implication operator. In this report, we focus on arguments that only use an implication operator. Therefore, we define three argument types for basic patterns that can be found in an argument modelled with GSN (Goal structuring Notation). Then, we calculate the formulae to compute belief and disbelief of the top conclusion of these arguments, through a combination rule aggregating pieces of information about premises and rules.

## 2 Argument types aggregation rules

In [1], we saw that the use of implication is more appropriate than equivalence. However, using only implication from premises to conclusion sets the degree of disbelief to zero, even if the degree of disbelief in premises is not null. Indeed modus ponens can only infer  $C$  from  $(p, p \Rightarrow C)$  and can never infer  $\neg C$ . To address this issue, we use two types of rules. The first type can only infer the acceptance of the conclusion, and the second can only infer its rejection (when the premises are false).

### 2.1 Direct and inverse rules

As we presented above, we introduce two types of rules for each argument type. The first will be named “direct rule” which only states that when the premise is true, the the conclusion is also true ( $r_{dir} = p_i \Rightarrow C$ ). The second will be named “inverse rule”. In the same way, it can only indicate that when the premise is false, the conclusion is also false ( $r_{inv} = \neg p_i \Rightarrow \neg C$ ). Remember that a rule defines the way in which one or more premises support a conclusion.

Note that the use of these two rules at the same time is exactly an equivalence here broken into two pieces ( $(p \equiv C) \equiv ([p \Rightarrow C] \wedge [\neg p \Rightarrow \neg C])$ ).

In the following types, we are going to assign a simple support mass function to each rule.

**Type 1 (Conjunction) :**

- $r_{dir} = (\bigwedge_{i=1}^n p_i) \Rightarrow C :$   
 $m_{dir}([\bigwedge_{i=1}^n p_i] \Rightarrow C) \text{ and } m_{dir}(\Omega) = 1 - m_{dir}([\bigwedge_{i=1}^n p_i] \Rightarrow C)$  (1)

- $r_{inv} = \neg(\bigwedge_{i=1}^n p_i) \Rightarrow \neg C :$   
 $m_{inv}(\neg(\bigwedge_{i=1}^n p_i) \Rightarrow \neg C) \text{ and } m_{inv}(\Omega) = 1 - m_{inv}(\neg(\bigwedge_{i=1}^n p_i) \Rightarrow \neg C)$  (2)

Note that the rule in (2), is equivalent to the conjunction of the inverse rules of the disjunctive type. This equivalence will affect the form of the confidence formula.

$$\begin{aligned} \neg(\bigwedge_{i=1}^n p_i) \Rightarrow \neg C &\equiv (\bigvee_{i=1}^n \neg p_i) \Rightarrow \neg C \\ &\equiv \bigwedge_{i=1}^n (\neg p_i \Rightarrow \neg C) \end{aligned}$$

Notice that :  $[\bigwedge_{i=1}^n (p_i \Rightarrow C)] \equiv [(\bigvee_{i=1}^n p_i) \Rightarrow C]$ .

### Type 2 (Disjunction) :

- $r_{dir} = \bigwedge_{i=1}^n [p_i \Rightarrow C] :$   
 $m_{dir}^i(p_i \Rightarrow C) \text{ and } m_{dir}^i(\Omega) = 1 - m_{dir}^i(p_i \Rightarrow C)$  (3)

- $r_{inv} = \bigwedge_{i=1}^n [\neg p_i \Rightarrow \neg C] :$   
 $m_{inv}^i(\neg p_i \Rightarrow \neg C) \text{ and } m_{inv}^i(\Omega) = 1 - m_{inv}^i([\neg p_i \Rightarrow \neg C])$  (4)

The disjunctive rules that we can be use in this case are :  $([\bigvee_{i=1}^n p_i] \Rightarrow C)$  and  $(\neg[\bigvee_{i=1}^n p_i] \Rightarrow \neg C)$ . However, we choose the form of equations (3) and (4) because it is more easy to grasp.

$$\begin{aligned} \bigwedge_{i=1}^n (p_i \Rightarrow C) &\equiv (\bigvee_{i=1}^n p_i) \Rightarrow C \\ \neg(\bigvee_{i=1}^n p_i) \Rightarrow \neg C &\equiv (\bigwedge_{i=1}^n \neg p_i) \Rightarrow \neg C \end{aligned}$$

### Type 3 (Hybrid) :

The third argument type (hybrid) is a combination of two types (conjunctive and disjunctive). We associate to each proposition of this type a single mass function. To avoid redundancy, we eliminate the conjunctive inverse rule.

- $r_{dir}^1 = (\bigwedge_{i=1}^n p_i) \Rightarrow C.$
- $r_{dir}^2 = \bigwedge_{i=1}^n [p_i \Rightarrow C].$
- $r_{inv} = \bigwedge_{i=1}^n [\neg p_i \Rightarrow \neg C].$

## 2.2 Aggregation formulas

In this section, we use only the Dempster-Shafer rule of combination, without normalisation, to calculate the aggregation formulas. For simplification reasons, we take the example of a conclusion (C) supported by two premises ( $p_1$ ) and ( $p_2$ ) to calculate aggregation formulas in each type. Then, we induce the general formulas.

**Type 1 (C-Arg):** To obtain the combination formula of a C-Arg, we combine the premises (table 1) and then the rules (direct and inverse) related to the premises (table 2). Then, we combine the results ( $m_r$ ) with the focal sets ( $m_p$ ) obtained from the combination of mass function on premises (table 1). Table 3 presents the results of this combination.

Table 1: Combination of the masses on premises

$m_p = m_p^1 \oplus m_p^2$	$m_p^2(p_2)$	$m_p^2(\neg p_2)$	$m_p^2(\top)$
$m_p^1(p_1)$	$p_1 \wedge p_2$	$p_1 \wedge \neg p_2$	$p_1$
$m_p^1(\neg p_1)$	$\neg p_1 \wedge p_2$	$\neg p_1 \wedge \neg p_2$	$\neg p_1$
$m_p^1(\top)$	$p_2$	$\neg p_2$	$\top$

Table 2: Combination the direct and inverse rules

$m_r = m_{dir} \otimes m_{inv}$	$m_{dir}([p_1 \wedge p_2] \Rightarrow C)$	$m_{r_1}(\top)$
$m_{inv}(\neg[p_1 \wedge p_2] \Rightarrow \neg C)$	$(p_1 \wedge p_2) \equiv C$	$\neg(p_1 \wedge p_2) \Rightarrow \neg C$
$m_{inv}(\top)$	$(p_1 \wedge p_2) \Rightarrow C$	$\top$

Table 3: Combination of the premises with the rules

$m = m_p \otimes m_r$	$m_r([(p_1 \wedge p_2) \equiv C])$	$m_r([p_1 \wedge p_2] \Rightarrow C)$	$m_r(\neg[p_1 \wedge p_2] \Rightarrow \neg C)$	$m_r(\top)$
$m_p(p_1 \wedge p_2)$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2$	$p_1 \wedge p_2$
$m_p(\neg p_1 \wedge p_2)$	$\neg p_1 \wedge p_2 \wedge \neg C$	$p_2 \wedge (p_1 \Rightarrow C)$	$\neg p_1 \wedge p_2 \wedge \neg C$	$\neg p_1 \wedge p_2$
$m_p(p_1 \wedge \neg p_2)$	$p_1 \wedge \neg p_2 \wedge \neg C$	$p_1 \wedge (p_2 \Rightarrow C)$	$p_1 \wedge \neg p_2 \wedge \neg C$	$p_1 \wedge \neg p_2$
$m_p(\neg p_1 \wedge \neg p_2)$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg p_2$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg p_2$
$m_p(p_1)$	$p_1 \wedge (p_2 \equiv C)$	$p_1 \wedge (p_2 \Rightarrow C)$	$p_1 \wedge (\neg p_2 \Rightarrow \neg C)$	$p_1$
$m_p(\neg p_1)$	$\neg p_1 \wedge \neg C$	$\neg p_1$	$\neg p_1 \wedge \neg C$	$\neg p_1$
$m_p(p_2)$	$p_2 \wedge (p_1 \equiv C)$	$p_2 \wedge (p_1 \Rightarrow C)$	$p_2 \wedge (\neg p_1 \Rightarrow \neg C)$	$p_2$
$m_p(\neg p_2)$	$\neg p_2 \wedge \neg C$	$\neg p_2$	$\neg p_2 \wedge \neg C$	$\neg p_2$
$m_p(\top)$	$[(p_1 \wedge p_2) \equiv C]$	$[p_1 \wedge p_2] \Rightarrow C$	$\neg[p_1 \wedge p_2] \Rightarrow \neg C$	$\top$

$$\begin{aligned}
 bel_C(C) &= m_p(p_1 \wedge p_2) \times m_r([p_1 \wedge p_2] \equiv C) + m_p(p_1 \wedge p_2) \times m_r([p_1 \wedge p_2] \Rightarrow C) \\
 &= [m_r([p_1 \wedge p_2] \equiv C) + m_r([p_1 \wedge p_2] \Rightarrow C)] \times m_p(p_1 \wedge p_2) \\
 &= [m_{dir}([p_1 \wedge p_2] \Rightarrow C) \times m_{inv}(\neg[p_1 \wedge p_2] \Rightarrow \neg C) + m_{dir}([p_1 \wedge p_2] \Rightarrow C) \times \\
 &\quad (1 - m_{inv}(\neg[p_1 \wedge p_2] \Rightarrow \neg C))] \times m_p(p_1 \wedge p_2) \\
 &= m_p^1(p_1) m_p^2(p_2) m_{dir}([p_1 \wedge p_2] \Rightarrow C) \\
 &= bel_p^1(p_1) bel_p^2(p_2) bel_{\Rightarrow}([p_1 \wedge p_2] \Rightarrow C)
 \end{aligned}$$

$$\begin{aligned}
 disb_C(C) &= m_r([p_1 \wedge p_2] \equiv C) \times [m_p(\neg p_1 \wedge p_2) + m_p(p_1 \wedge \neg p_2) + m_p(\neg p_1 \wedge \neg p_2) + m_p(\neg p_1) + m_p(\neg p_2)] + \\
 &\quad m_r([p_1 \wedge p_2] \Rightarrow C) \times [m_p(\neg p_1 \wedge p_2) + m_p(p_1 \wedge \neg p_2) + m_p(\neg p_1 \wedge \neg p_2) + m_p(\neg p_1) + m_p(\neg p_2)] \\
 &= [m_r([p_1 \wedge p_2] \equiv C) + m_r([p_1 \wedge p_2] \Rightarrow C)] \times [m_p(\neg p_1 \wedge p_2) + m_p(p_1 \wedge \neg p_2) + m_p(\neg p_1 \wedge \neg p_2) \\
 &\quad + m_p(\neg p_1) + m_p(\neg p_2)] \\
 &= \{m_{dir}([p_1 \wedge p_2] \Rightarrow C)\} \times m_{inv}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C) + m_{inv}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C) \times \\
 &\quad [1 - m_{dir}([p_1 \wedge p_2] \Rightarrow C)] \times \{m_p^1(\neg p_1) m_p^2(p_2) + m_p^1(p_1) m_p^2(\neg p_2) + m_p^1(\neg p_1) m_p^2(\neg p_2) \\
 &\quad + m_p^1(\neg p_1) [1 - m_p^2(p_2) - m_p^2(\neg p_2)] + m_p^2(\neg p_2) [1 - m_p^1(p_1) - m_p^1(\neg p_1)]\} \\
 &= m_{inv}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C) \times [m_p^1(\neg p_1) + m_p^2(\neg p_2) - m_p^1(\neg p_1) m_p^2(\neg p_2)] \\
 &= m_{inv}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C) \times [1 - (1 - m_p^1(\neg p_1))(1 - m_p^2(\neg p_2))] \\
 &= bel_{\Rightarrow}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C) \times [1 - (1 - disb_p^1(p_1))(1 - disb_p^2(p_2))]
 \end{aligned}$$

We can generalize the resulting formulas for a C-Arg in the following :

$$bel_C(C) = bel_{\Rightarrow}([\wedge_{i=1}^n p_i] \Rightarrow C) \prod_{i=1}^n bel_p^i(p_i) \quad (5)$$

and

$$disb_C(C) = bel_{\Rightarrow}(\neg[\wedge_{i=1}^n p_i] \Rightarrow \neg C) \times (1 - \prod_{i=1}^n [1 - disb_p^i(p_i)]) \quad (6)$$

Equation (5) expresses a *Multivalued conjunction*. To be equal to one, all the confidence degree in the premises should be equal to one too (assuming that the degree of confidence in the rule is maximal). On the other hand, equation (6) does not express a pure *Multivalued conjunction* of the disbelief case. Instead, it combines the beliefs of a disjunctive form of premises with a conjunctive form of the inverse rule. The nature of this formula is explained above.

**Type 2 (D-Arg) :** To obtain the combination formula of a D-Arg, we are going to combine the rules (direct and inverse) related to each premise (table 4). Then, we are going to combine each premise ( $p_i$ ) with the formulas resulting from the rules combination (table 5). Finally, we are going to combine the previous table for the two premises (tables 6 and 7).

Table 4: Combination the direct and inverse rules of D-Type

$m_r^i = m_{dir}^i \otimes m_{inv}^i$	$m_{dir}^i(p_i \Rightarrow C)$	$m_{r_1}(\top)$
$m_{inv}^i(\neg p_i \Rightarrow \neg C)$	$p_i \equiv C$	$\neg p_i \Rightarrow \neg C$
$m_{inv}^i(\top)$	$p_i \Rightarrow C$	$\top$

From table 4, we can calculate the masses of the four focal set resulting from this combination. For instance,  $m_r^i(p_i \equiv C) = m_{dir}^i(p_i \Rightarrow C) \times m_{inv}^i(\neg p_i \Rightarrow \neg C)$ . And since we use propositional variables for which the frame of discernment has two states :  $\Omega = \{True, False\}$ . The masses of each rule is equal to its belief degree (The same remark goes on premises). So we can also write that :  $m_r^i(p_i \equiv C) = bel_{\equiv}(p_i \equiv C) = bel_{\Rightarrow}(p_i \Rightarrow C) \times bel_{\Rightarrow}(\neg p_i \Rightarrow \neg C)$ .

Table 5: Combination of each premise with its rule

$m_i = m_p^i \otimes m_r^i$	$m_r^i(p_i \equiv C)$	$m_r^i(p_i \Rightarrow C)$	$m_r^i(\neg p_i \Rightarrow \neg C)$	$m_r^i(\top)$
$m_p^i(p_i)$	$p_i \wedge C$	$p_i \wedge C$	$p_i$	$p_i$
$m_p^i(\neg p_i)$	$\neg p_i \wedge \neg C$	$\neg p_i$	$\neg p_i \wedge \neg C$	$\neg p_i$
$m_p^i(\top)$	$p_i \equiv C$	$p_i \Rightarrow C$	$\neg p_i \Rightarrow \neg C$	$\top$

In the same way as in table 4, we can calculate from table 5 the masses of the 8 focal sets resulting from this combination ( $p_i \wedge C, \neg p_i \wedge \neg C, p_i, \neg p_i, p_i \Rightarrow C, \neg p_i \Rightarrow \neg C, p_i \equiv C$  and the tautology ( $\top$ )).

For example the mass or belief degree in the focal set  $p_i \wedge C$  is calculated as the following :

$$\begin{aligned}
bel(p_i \wedge C) &= m_i(p_i \wedge C) \\
&= m_p^i(p_i) \times m_r^i(p_i \equiv C) + m_p^i(p_i) \times m_r^i(p_i \Rightarrow C) \\
&= m_p^i(p_i)[m_r^i(p_i \equiv C) + m_r^i(p_i \Rightarrow C)] \\
&= m_p^i(p_i)[m_{dir}^i(p_i \Rightarrow C) \times m_{inv}^i(\neg p_i \Rightarrow \neg C) + m_{dir}^i(p_i \Rightarrow C) \times (1 - m_{inv}^i(\neg p_i \Rightarrow \neg C))] \\
&= m_p^i(p_i)m_{dir}^i(p_i \Rightarrow C) \\
&= bel_p^i(p_i)bel_{\Rightarrow}^i(p_i \Rightarrow C)
\end{aligned}$$

Notice that the belief degree of equivalence and implication can be deduced from table 4.

In tables 6 and 7, we calculate  $m_1 \oplus m_2$  which represents respectively the combination of each premise with its rules (direct and inverse). Due to lack of space we split the table into two sub-tables.

Table 6: Combination of premises and the rules (part 1)

$m = m_1 \otimes m_2$	$m_2(p_2 \wedge C)$	$m_2(\neg p_2 \wedge \neg C)$	$m_2(p_2)$	$m_2(\neg p_2)$
$m_1(p_1 \wedge C)$	$p_1 \wedge p_2 \wedge C$	$\emptyset$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge (\neg p_2) \wedge C$
$m_1(\neg p_1 \wedge \neg C)$	$\emptyset$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge p_2 \wedge \neg C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$
$m_1(p_1)$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge \neg p_2 \wedge \neg C$	$p_1 \wedge p_2$	$p_1 \wedge \neg p_2$
$m_1(\neg p_1)$	$(\neg p_1) \wedge p_2 \wedge C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge p_2$	$\neg p_1 \wedge \neg p_2$
$m_1(p_1 \Rightarrow C)$	$p_2 \wedge C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$p_2 \wedge (p_1 \Rightarrow C)$	$\neg p_2 \wedge (p_1 \Rightarrow C)$
$m_1(\neg p_1 \Rightarrow \neg C)$	$p_1 \wedge p_2 \wedge C$	$\neg p_2 \wedge \neg C$	$p_2 \wedge (\neg p_1 \Rightarrow \neg C)$	$\neg p_2 \wedge (\neg p_1 \Rightarrow \neg C)$
$m_1(p_1 \equiv C)$	$p_1 \wedge p_2 \wedge C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$p_2 \wedge (p_1 \equiv C)$	$\neg p_2 \wedge (p_1 \equiv C)$
$m_1(\top)$	$p_2 \wedge C$	$\neg p_2 \wedge \neg C$	$p_2$	$\neg p_2$

Table 7: Combination of premises and the rules (part 2)

$m = m_1 \otimes m_2$	$m_2(p_2 \Rightarrow C)$	$m_2(\neg p_2 \Rightarrow \neg C)$	$m_2(p_2 \equiv C)$	$m_2(\top)$
$m_1(p_1 \wedge C)$	$p_1 \wedge C$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge C$
$m_1(\neg p_1 \wedge \neg C)$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg C$
$m_1(p_1)$	$p_1 \wedge (p_2 \Rightarrow C)$	$p_1 \wedge (\neg p_2 \Rightarrow \neg C)$	$p_1 \wedge (p_2 \equiv C)$	$p_1$
$m_1(\neg p_1)$	$\neg p_1 \wedge (p_2 \Rightarrow C)$	$\neg p_1 \wedge (\neg p_2 \Rightarrow \neg C)$	$\neg p_1 \wedge (p_2 \equiv C)$	$\neg p_1$
$m_1(p_1 \Rightarrow C)$	$(p_1 \vee p_2) \Rightarrow C$	$(p_1 \vee \neg p_2) \Rightarrow C$	$(p_1 \Rightarrow C) \wedge (p_2 \equiv C)$	$p_1 \Rightarrow C$
$m_1(\neg p_1 \Rightarrow \neg C)$	$(\neg p_1 \vee p_2) \Rightarrow C$	$(\neg p_1 \vee \neg p_2) \Rightarrow \neg C$	$(\neg p_1 \Rightarrow \neg C) \wedge (p_2 \equiv C)$	$\neg p_1 \Rightarrow \neg C$
$m_1(p_1 \equiv C)$	$(p_2 \Rightarrow C) \wedge (p_1 \equiv C)$	$(\neg p_2 \Rightarrow \neg C) \wedge (p_1 \equiv C)$	$(p_1 \equiv C) \wedge (p_2 \equiv C)$	$p_1 \equiv C$
$m_1(\top)$	$p_2 \Rightarrow C$	$\neg p_2 \wedge \neg C$	$p_2 \equiv C$	$\top$

From tables 6 and 7, we can calculate the belief and disbelief degrees in a conclusion supported by two

premises expressing disjunction relation. And then, we can deduce the general formula.

$$\begin{aligned}
bel_C^*(C) &= m(p_1 \wedge p_2 \wedge C) + m(p_1 \wedge \neg p_2 \wedge C) + m(\neg p_1 \wedge p_2 \wedge C) + m(p_1 \wedge C) + m(p_2 \wedge C) \\
&= m_1(p_1 \wedge C) \times \sum_{\phi_2} m_2(\phi_2) + m_2(p_2 \wedge C) \times \sum_{\phi_1} m_1(\phi_1) - m_1(p_1 \wedge C) \times m_2(p_2 \wedge C) \\
&= m_p^1(p_1)[m_r^1(p_1 \equiv C) + m_r^1(p_1 \Rightarrow C)] + m_p^2(p_2)[m_r^2(p_2 \equiv C) + m_r^2(p_2 \Rightarrow C)] \\
&\quad - m_p^1(p_1)[m_r^1(p_1 \equiv C) + m_r^1(p_1 \Rightarrow C)] \times m_p^2(p_2)[m_r^2(p_2 \equiv C) + m_r^2(p_2 \Rightarrow C)] \\
&= 1 - \{1 - m_p^1(p_1)[m_r^1(p_1 \equiv C) + m_r^1(p_1 \Rightarrow C)]\} \times \{1 - m_p^2(p_2)[m_r^2(p_2 \equiv C) + m_r^2(p_2 \Rightarrow C)]\} \\
&= 1 - \{1 - m_p^1(p_1)m_{dir}^1(p_1 \Rightarrow C)\} \times [1 - m_p^2(p_2)m_{dir}^2(p_2 \Rightarrow C)] \\
&= 1 - [1 - bel_p^1(p_1)bel_{\Rightarrow}^1(p_1 \Rightarrow C)][1 - bel_p^2(p_2)bel_{\Rightarrow}^2(p_2 \Rightarrow C)]
\end{aligned}$$

Note that :  $bel_C^*(C)$  represents the confidence degree before normalizing.

Normally, due to the appearance of conflict between premises, we should normalize by  $(1 - m(\emptyset))$  the conflict coefficient. This conflict is represented by empty intersection between the focal sets. In our case, it appears when one premise supporting the conclusion is verified and the other is not  $([p_i \wedge C] \wedge [\neg p_j \wedge \neg C])$ . At this step, we choose not to normalize.

$$m(\emptyset) = \sum_{i=1, j \neq i}^n [bel_p^i(p_i)bel_{\Rightarrow}^i(p_i \Rightarrow C) \times disb_p^j(p_j)bel_{\Rightarrow}^j(\neg p_j \Rightarrow \neg C)] \quad (7)$$

Noting  $bel_C^i(C)$  as the belief degree of the conclusion supported by one premise. We can generalize this formula for (n) premises.

$$bel_C(C) = 1 - \prod_{i=1}^n [1 - bel_C^i(C)] \quad (8)$$

Following the same reasoning the generalize formula of disbelief degree is :

$$disb_C(C) = 1 - \prod_{i=1}^n [1 - disb_C^i(C)] \quad (9)$$

Where :

$$bel_C^i(C) = bel_p^i(p_i)bel_{\Rightarrow}^i(p_i \Rightarrow C)$$

and

$$disb_C^i(C) = disb_p^i(p_i)bel_{\Rightarrow}^i(\neg p_i \Rightarrow \neg C)$$

To get a total confidence in the conclusion ((dis-)belief degree = 1). It is only required that the degree of confidence (or defiance) in one premise supporting the conclusion ( $bel_C^i(C)$ ) be equal to one. These formulas (equations 8 and 9) express a *Multivalued disjunction* relation because each premise can infer the verification or rejection of the conclusion alone.

**Type 3 (H-Arg) :** To calculate the confidence combination formulas in this argument type, we are going to combine focal sets of tables 6 and 7 (to avoid repetition) with the conjunctive rule of equation (1) in table 8. Then, we are going to deduce the general formulas.

$$m(C) = m_1(C) + m_2(C)$$

where :  $m_1(C)$  represents the confidence from the disjunctive part of the argument type and  $m_2(C)$ , represents the confidence from the conjunctive part.

$$\begin{aligned}
m_2(C) &= m(p_1 \wedge p_2 \wedge C) \\
&= m_d(p_1 \wedge p_2) \times m_c([p_1 \wedge p_2] \Rightarrow C) \\
&= m_c([p_1 \wedge p_2] \Rightarrow C) \times \{m_p^1(p_1)[1 - m_r^1(p_1 \equiv C) \times m_r^1(p_1 \Rightarrow C)] \\
&\quad + m_p^2(p_2) \times [1 - m_r^2(p_2 \equiv C)m_r^2(p_2 \Rightarrow C)]\} \\
&= m_c([p_1 \wedge p_2] \Rightarrow C) \times \{m_p^1(p_1)[1 - m_r^1(p_1 \Rightarrow C)] \times m_p^2(p_2)[1 - m_r^2(p_2 \Rightarrow C)]\} \\
&= bel_{\Rightarrow}([p_1 \wedge p_2] \Rightarrow C) \times bel_p^1(p_1)bel_p^2(p_2)[1 - bel_{\Rightarrow}^1(p_1 \Rightarrow C)][1 - bel_{\Rightarrow}^2(p_2 \Rightarrow C)]
\end{aligned}$$

Table 8: Combination the direct and inverse rules of H-Type

$m = m_d \otimes m_c$	$m_c([\wedge_{i=1}^n p_i] \Rightarrow C)$	$m_c(\top)$
$m_d(p_1)$	$p_1 \wedge [p_2 \Rightarrow C]$	$p_1$
$m_d(\neg p_1)$	$\neg p_1$	$\neg p_1$
$m_d(p_2)$	$p_2 \wedge [p_1 \Rightarrow C]$	$p_2$
$m_d(\neg p_2)$	$\neg p_2$	$\neg p_2$
$m_d(p_1 \wedge p_2)$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2$
$m_d(p_1 \wedge \neg p_2)$	$p_1 \wedge \neg p_2$	$p_1 \wedge \neg p_2$
$m_d(\neg p_1 \wedge p_2)$	$\neg p_1 \wedge p_2$	$\neg p_1 \wedge p_2$
$m_d(\neg p_1 \wedge \neg p_2)$	$\neg p_1 \wedge \neg p_2$	$\neg p_1 \wedge \neg p_2$
$m_d(p_1 \Rightarrow C)$	$p_1 \Rightarrow C$	$p_1 \Rightarrow C$
$m_d(\neg p_1 \Rightarrow \neg C)$	$(\neg p_1 \Rightarrow \neg C) \wedge ([p_1 \wedge p_2] \Rightarrow C)$	$\neg p_1 \Rightarrow \neg C$
$m_d(p_2 \Rightarrow C)$	$p_2 \Rightarrow C$	$p_2 \Rightarrow C$
$m_d(\neg p_2 \Rightarrow \neg C)$	$(\neg p_2 \Rightarrow \neg C) \wedge ([p_1 \wedge p_2] \Rightarrow C)$	$\neg p_2 \Rightarrow \neg C$
$m_d(p_1 \equiv C)$	$p_1 \equiv C$	$p_1 \equiv C$
$m_d(p_2 \equiv C)$	$p_2 \equiv C$	$p_2 \equiv C$
$m_d(p_1 \wedge C)$	$p_1 \wedge C$	$p_1 \wedge C$
$m_d(p_2 \wedge C)$	$p_2 \wedge C$	$p_2 \wedge C$
$m_d(\neg p_1 \wedge \neg C)$	$\neg p_1 \wedge \neg C$	$\neg p_1 \wedge \neg C$
$m_d(\neg p_2 \wedge \neg C)$	$\neg p_2 \wedge \neg C$	$\neg p_2 \wedge \neg C$
$m_d(p_1 \wedge p_2 \wedge C)$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2 \wedge C$
$m_d(\neg p_1 \wedge p_2 \wedge C)$	$\neg p_1 \wedge p_2 \wedge C$	$\neg p_1 \wedge p_2 \wedge C$
$m_d(p_1 \wedge \neg p_2 \wedge C)$	$p_1 \wedge \neg p_2 \wedge C$	$p_1 \wedge \neg p_2 \wedge C$
$m_d(\neg p_1 \wedge \neg p_2 \wedge \neg C)$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$
$m_d(p_1 \wedge \neg p_2 \wedge \neg C)$	$p_1 \wedge \neg p_2 \wedge \neg C$	$p_1 \wedge \neg p_2 \wedge \neg C$
$m_d(\neg p_1 \wedge p_2 \wedge \neg C)$	$\neg p_1 \wedge p_2 \wedge \neg C$	$\neg p_1 \wedge p_2 \wedge \neg C$
$m_d(p_1 \wedge [p_2 \Rightarrow C])$	$p_1 \wedge [p_2 \Rightarrow C]$	$p_1 \wedge [p_2 \Rightarrow C]$
$m_d(p_1 \wedge [p_2 \Rightarrow \neg C])$	$p_1 \wedge [p_2 \Rightarrow \neg C]$	$p_1 \wedge [\neg p_2 \Rightarrow \neg C]$
$m_d(p_1 \wedge [p_2 \equiv C])$	$p_1 \wedge [p_2 \equiv C]$	$p_1 \wedge [p_2 \equiv C]$
$m_d(\neg p_1 \wedge [p_2 \Rightarrow C])$	$\neg p_1 \wedge [p_2 \Rightarrow C]$	$\neg p_1 \wedge [p_2 \Rightarrow C]$
$m_d(\neg p_1 \wedge [\neg p_2 \Rightarrow \neg C])$	$\neg p_1 \wedge [\neg p_2 \Rightarrow \neg C]$	$\neg p_1 \wedge [\neg p_2 \Rightarrow \neg C]$
$m_d(\neg p_1 \wedge [p_2 \equiv C])$	$\neg p_1 \wedge [p_2 \equiv C]$	$\neg p_1 \wedge [p_2 \equiv C]$
$m_d(p_2 \wedge [p_1 \Rightarrow C])$	$p_2 \wedge [p_1 \Rightarrow C]$	$p_2 \wedge [p_1 \Rightarrow C]$
$m_d(p_2 \wedge [\neg p_1 \Rightarrow \neg C])$	$p_2 \wedge [\neg p_1 \Rightarrow \neg C]$	$p_2 \wedge [\neg p_1 \Rightarrow \neg C]$
$m_d(p_2 \wedge [p_1 \equiv C])$	$p_2 \wedge [p_1 \equiv C]$	$p_2 \wedge [p_1 \equiv C]$
$m_d(\neg p_2 \wedge [p_2 \Rightarrow C])$	$\neg p_2 \wedge [p_2 \Rightarrow C]$	$\neg p_2 \wedge [p_2 \Rightarrow C]$
$m_d(\neg p_2 \wedge [\neg p_1 \Rightarrow \neg C])$	$\neg p_2 \wedge (\neg p_1 \Rightarrow \neg C)$	$\neg p_2 \wedge (\neg p_1 \Rightarrow \neg C)$
$m_d(\neg p_2 \wedge [p_1 \equiv C])$	$\neg p_2 \wedge (p_1 \equiv C)$	$\neg p_2 \wedge (p_1 \equiv C)$
$m_d([p_1 \Rightarrow C] \wedge [p_2 \Rightarrow C])$	$(p_1 \Rightarrow C) \wedge (p_2 \Rightarrow C) \wedge ([\wedge_{i=1}^n p_i] \Rightarrow C)$	$(p_1 \Rightarrow C) \wedge (p_2 \Rightarrow C)$
$m_d([\neg p_1 \Rightarrow \neg C] \wedge [p_2 \Rightarrow C])$	$(\neg p_1 \Rightarrow \neg C) \wedge (p_2 \Rightarrow C) \wedge ([\wedge_{i=1}^n p_i] \Rightarrow C)$	$(\neg p_1 \Rightarrow \neg C) \wedge (p_2 \Rightarrow C)$
$m_d([p_1 \Rightarrow C] \wedge [\neg p_2 \Rightarrow \neg C])$	$(p_1 \Rightarrow C) \wedge (\neg p_2 \Rightarrow \neg C) \wedge ([\wedge_{i=1}^n p_i] \Rightarrow C)$	$(p_1 \Rightarrow C) \wedge (\neg p_2 \Rightarrow \neg C)$
$m_d([\neg p_1 \Rightarrow \neg C] \wedge [\neg p_2 \Rightarrow \neg C])$	$[\wedge_{i=1}^n p_i] \equiv C$	$(\neg p_1 \Rightarrow \neg C) \wedge (\neg p_2 \Rightarrow \neg C)$
$m_d([p_1 \equiv C] \wedge [p_2 \Rightarrow C])$	$(p_1 \equiv C) \wedge (p_2 \Rightarrow C) \wedge ([\wedge_{i=1}^n p_i] \Rightarrow C)$	$(p_1 \equiv C) \wedge (p_2 \Rightarrow C)$
$m_d([p_1 \equiv C] \wedge [\neg p_2 \Rightarrow \neg C])$	$(p_1 \equiv C) \wedge (\neg p_2 \Rightarrow \neg C) \wedge ([\wedge_{i=1}^n p_i] \Rightarrow C)$	$(p_1 \equiv C) \wedge (\neg p_2 \Rightarrow \neg C)$
$m_d([p_2 \equiv C] \wedge [p_1 \Rightarrow C])$	$(p_2 \equiv C) \wedge (p_1 \Rightarrow C) \wedge ([\wedge_{i=1}^n p_i] \Rightarrow C)$	$(p_2 \equiv C) \wedge (p_1 \Rightarrow C)$
$m_d([p_2 \equiv C] \wedge [\neg p_1 \Rightarrow \neg C])$	$(p_2 \equiv C) \wedge (\neg p_1 \Rightarrow \neg C) \wedge ([\wedge_{i=1}^n p_i] \Rightarrow C)$	$(p_2 \equiv C) \wedge (\neg p_1 \Rightarrow \neg C)$
$m_d([p_1 \equiv C] \wedge [p_2 \equiv C])$	$(p_1 \equiv C) \wedge [p_2 \equiv C]$	$(p_1 \equiv C) \wedge (p_2 \equiv C)$
$m_d(\emptyset)$	$\emptyset$	$\emptyset$
$m_d(\top)$	$(\wedge_{i=1}^n p_i) \Rightarrow C$	$\top$

From tables 6 and 7, we get :

$$\begin{aligned}
 m_2(C) &= 1 - [1 - m_p^1(p_1)m_r^1(p_1 \Rightarrow C)][1 - m_p^1(p_1)m_r^1(p_1 \Rightarrow C)] \\
 &= 1 - [1 - bel_p^1(p_1)bel_{\Rightarrow}^1(p_1 \Rightarrow C)][1 - bel_p^2(p_1)bel_{\Rightarrow}^2(p_2 \Rightarrow C)]
 \end{aligned}$$

So :

$$\begin{aligned}
 bel_C(C) &= bel_{\Rightarrow}([p_1 \wedge p_2] \Rightarrow C) \times \{bel_p^1(p_1)bel_p^2(p_2)[1 - bel_{\Rightarrow}^1(p_1 \Rightarrow C)][1 - bel_{\Rightarrow}^2(p_2 \Rightarrow C)]\} \\
 &\quad + \{1 - [1 - bel_p^1(p_1)bel_{\Rightarrow}^1(p_1 \Rightarrow C)][1 - bel_p^2(p_1)bel_{\Rightarrow}^2(p_2 \Rightarrow C)]\}
 \end{aligned}$$

We can generalize the resulting formula for an H-Arg in the following :

$$bel_C(C) = bel_{\Rightarrow}([\wedge_{i=1}^n p_i] \Rightarrow C) \times \prod_{i=1}^n bel_p^i(p_i)[1 - bel_{\Rightarrow}^i(p_i \Rightarrow C)] + \{1 - \prod_{i=1}^n [1 - bel_p^i(p_i)bel_{\Rightarrow}^i(p_i \Rightarrow C)]\} \quad (10)$$

We can notice from this equation is mixture of the conjunctive and disjunctive type. If masses on  $(p_i \Rightarrow C)$  are equal to zero, the equation become the formula representing the conjunctive type. In the other hand, if the mass on  $([\wedge_{i=1}^n p_i] \Rightarrow C)$  is null, we get the formula of an disjunctive type.

The formula of disbelief remain the same as a D-Arg, because we used the same inverse rules. We also notice that the formula of  $m(\emptyset)$  does not change from D-Arg to H-Arg.

$$disb_C(C) = 1 - \prod_{i=1}^n [1 - disb_C^i(C)] \quad (11)$$

Where :

$$disb_C^i(C) = disb_p^i(p_i) bel_{\Rightarrow}^i(\neg p_i \Rightarrow \neg C)$$

### 3 Conclusion

This report presents the method of calculation of three argument type using Dempster-Shafer rule of combination. We have chosen to not take into account the conflict in these formulas. However, it can be taken into consideration either by normalizing by the degree of conflict ( $1 - m(\emptyset)$ ) or by subtracting the masses that represent the conflict ( $m(\emptyset)$ ).

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