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Abstract: This paper deals with a sensor fault detection approach using the Optimal Upper Bounded Interval Kalman Filter (OUBIKF) and an adaptive degree of freedom χ^2-statistics method. It is devoted to discrete time linear model subjected to mixed uncertainties in terms of observations and noises. Mixed uncertainties mean both bounded and stochastic uncertainties. The degrees of freedom of this χ^2 hypothesis test method are adaptively chosen thanks to amplifier coefficients improving the detection of the sensor faults. The proposed approach is an extension of a result developped in Lu et al. (2019). Application on a vehicle bicycle model highlights the efficiency of the proposed approach. Comparisons with other efficient estimation and fault detection strategies are provided to discuss the accuracy of the obtained results.

Keywords: FDI for linear system, Filtering and estimation.

1. INTRODUCTION

Fault detection and diagnosis are important for all system engineering problems. Several methods for fault detection in dynamic systems are mentioned in Willsky (1976), including the innovation-based method in which a χ^2-statistic hypothesis testing was used. This last method is applied appropriately with the standard Kalman filter (Kalman (1960)) to process the linear dynamic system with deterministic coefficient matrices. In Sainz et al. (2002), for example, an approach to generate envelopes based on interval techniques of the modal interval analysis is proposed. In Puig (2010), set-membership methods in fault diagnosis (FD) and fault tolerant control (FTC) are reviewed, these methods aim at checking the consistency between observed and predicted behaviour by using simple sets to approximate the exact set of possible behaviour.

In Tran (2017), a fault detection approach is presented and this approach combines a χ^2-statistics hypothesis test with the Upper Bound Interval Kalman Filter (UBIKF). An upper bound for all positive semi-definite matrices belonging to an interval matrix is proposed. This upper bound aims to overcome the singularity of the inverse of interval matrices. Based on the concept of UBIKF, a filter using optimal upper bounds for this class of problems, named OUBIKF, is introduced in Lu et al. (2019). The contribution of the later consists in a rigorous mathematical framework for an optimal upper bound (in the sense of operator norm minimization) providing tight estimate intervals containing all actual states of the dynamic system.

In this paper, an adaptive hypothesis test method is developed to detect sensor faults applied to a linear discrete time dynamic system with assumptions requiring the use of interval computations. The proposed scheme for fault detection combines the developed hypothesis test (based on a χ^2-statistic with degrees of freedom (d.f.) adaptively chosen thanks to amplifier coefficients) and the OUBIKF.

The paper is organized as follows: Section 2 provides some preliminaries including principles and OUBIKF algorithm. In Section 3, the proposed scheme for fault detection is developed and some evaluation indicators are given. An application in simulation of the proposed approach is presented in Section 4. The application model comes from automotive domain. A comparison with other method (see Raka and Combastel (2013)) is given. Conclusions and some ideas for future works are provided in Section 5.

2. PRELIMINARY

2.1 Optimal upper bound of all positive semi-definite matrices belonging to an interval matrix

A real positive semi-definite matrix M is denoted by $M \succeq 0$. Denote also $S(n) \triangleq \{M \in \mathbb{R}^{n\times n} : M = M^T\}$ and $S_+(n) \triangleq \{M \in S(n) : M \succeq 0\}$.
Let M, N be two real squared matrices of the same size. An order between M and N denoted by $N \preceq M$ is defined if and only if $M - N \succeq 0$. M is called an upper bound of N, or says N is dominated by M or M dominates N.
For Hermitian matrices, this order is known as the *Loewner (partial) order* (Pukelsheim (2006); Zhan (2002)).

This partial order is extended to the notion of bounds for a non empty set Ω of real squared matrices as follow. If a real squared matrix K satisfying $M \preceq K$, $\forall M \in \Omega$ then K is an upper bound of Ω, denoted $\Omega \preceq K$. If K and L are two upper bounds of Ω, then K is said better than L if $\|K\| \leq \|L\|$ depending on the choice of matrix norm $\|.\|$.

A real interval matrix, denoted by $[M]$, is a matrix with real interval components (of the type $[a, b]$ with $a, b \in \mathbb{R}$

and $a \leq b$). The components of $[M]$ are denoted $[M]_{ij}$ (i^{th} row and j^{th} column). Write $M \in [M]$ to indicate a punctual matrix M belonging to $[M]$, i.e. $\forall i, j,\ M_{ij} \in [M]_{ij}$. Let $[M]$ be an $n \times n$ real interval matrix. Denote:

a) $S([M]) \triangleq \{M \in [M] : M = M^T\}$,

b) $S_+([M]) \triangleq \{M \in S([M]) : M \succeq 0\}$,

c) $BS_+([M]) \triangleq \{K \in S_+(n) : S_+([M]) \preceq K\}$.

Other standard notations and computations related to interval analysis are referred to Jaulin et al. (2001).

For any $n \times n$ matrix A, we use the notations $\sigma_i(A)$, $\lambda_i(A)$ ($i = 1, ..., n$) to indicate respectively the singular values and eigenvalues of A among which $\sigma_{\max}(A)$ and $\lambda_{\max}(A)$ are the corresponding maximum values. It is referred to Zhan (2002) for vector and matrix norm definitions.

In the following two theorems, let $[M]$ be an $n \times n$ real interval symmetric matrix and assume that $S_+([M]) \neq \emptyset$.

Theorem 1. (**Existence of Optimal upper bounds**). The following properties hold:

i) $\alpha_* \triangleq \sup_{M \in S_+([M])} \{\lambda_{\max}(M)\} < \infty$ and $S_+([M]) \preceq \alpha I$ if and only if $\alpha \geq \alpha_*$.

ii) $\alpha_* I$ is the optimal upper bound of $S_+([M])$ in the set $BS_+([M])$ in the sense of operator norm minimization.

iii) Let $\Omega = \{K \in BS_+([M]) : n^{-1} \sum_{i=1}^{n} \lambda_i(K) \geq \alpha_*\}$. $\alpha_* I$ is the optimal upper bound of $S_+([M])$ in Ω in the sense of nuclear norm minimization.

α_* is said the optimal value of $BS_+([M])$.

Theorem 2. (**Bounds of Optimal value** α_*). The following properties hold:

i) $\mathcal{E} \triangleq \{M \in S_+([M]) : diag(M) = diag(\sup([M]))\}$ is the non empty set of maximal elements of $S_+([M])$.

ii) There exists a matrix $N^* \in \mathcal{E}$ s.t. $\lambda_{\max}(N^*) = \alpha_*$.

iii) Let $\mathrm{Max}([M]) = (\mathrm{Max}_{ij})$ be a matrix determined by

$$\mathrm{Max}_{ij} = \begin{cases} \sup([M])_{ij} \ , & \text{if } \mathrm{mid}([M])_{ij} \geq 0 \\ \inf([M])_{ij} \ , & \text{otherwise} \ . \end{cases} \tag{1}$$

then $\alpha_* \leq \sup_{M \in [M]} \{\|M\|_F\} \leq \|\mathrm{Max}([M])\|_F$. If $\mathrm{Max}([M]) \succeq 0$ then $\lambda_{\max}(\mathrm{Max}([M])) \leq \alpha_* \leq \|\mathrm{Max}([M])\|_F$.

Remark 1. $\alpha_* I$ is the simplest upper bound of $S_+([M])$ to use both in practice and theory. Its existence and optimality are stated in Theorem 1. Theorem 2 provides a simple way to localize the optimal value α_* (via $\mathrm{Max}([M])$).

More details and proofs can be found in Lu et al. (2019).

2.2 Mixed uncertainty linear discrete-time dynamic system for state estimation and fault detection

Consider the following linear discrete time dynamic system

$$\begin{cases} x_k = A_k x_{k-1} + B_k u_k + w_k \ , \\ y_k = C_k x_k + D_k u_k + v_k \ , \end{cases} \qquad k \in \mathbb{N}^*, \tag{2}$$

in which the notations are usual for the standard Kalman filter: $x_k \in \mathbb{R}^{n_x}$ and $y_k \in \mathbb{R}^{n_y}$ represent state variables and measures respectively, $u_k \in \mathbb{R}^{n_u}$ inputs, $w_k \in \mathbb{R}^{n_x}$ state noises, $v_k \in \mathbb{R}^{n_y}$ measurement noises.

Assumptions (H): Matrices A_k, B_k, C_k, D_k are unknown, deterministic and belonging to a given bounded interval matrices $[A], [B], [C], [D]$ respectively. w_k, v_k are centered Gaussian vectors with covariance matrices Q_k and R_k belonging respectively to given interval matrices $[Q]$ and $[R]$. The initial state x_0 is also Gaussian with mean μ_0 and covariance matrix P_0. In addition, x_0, $\{w_1, ..., w_k\}$ and $\{v_1, ..., v_k\}$ are assumed to be mutually independent.

Toward achieving efficient sensor fault detection, OUBIKF has been introduced and Algorithm 1 is the devoted algorithm. The filter focuses on state estimation of system (2) and can be used to get residual intervals $[r_k]$ ($[r_k] = y_k - [\hat{y}_k]$, $[\hat{y}_k] = [C][\hat{x}_{k|k-1}] + [D]u_k$) where $[\hat{x}_{k|k-1}]$ is the predicted state estimate and $[\hat{y}_k]$ the measure estimate.

Algorithm 1 OUBIKF ALGORITHM

Initialization: $[\hat{x}_{0|0}], \mathcal{P}_{0|0}, [A], [B], [C], [D], [Q], [R], p, u_k, y_k, k = 1, 2, ..., N.$
Find n_0 the number of non zero radius of $[C]$
Find γ s.t. $S_+([R]) \preceq \gamma I$
for $k = 1, 2, 3, ...N$ **do**
Prediction step:
$[\hat{x}_{k|k-1}] = [A][\hat{x}_{k-1|k-1}] + [B]u_k$
$[P_{k|k-1}] = [A]\mathcal{P}_{k-1|k-1}[A]^T + [Q]$
Find α_k s.t. $S_+([P_{k|k-1}]) \preceq \alpha_k I$
Correction step:
$\beta_k = 1/(2\alpha_k^p) \quad ; \quad \sigma_k = \beta_k / n_0$
$V_k = \mathrm{mid}([C])\mathrm{mid}([C])^T + \frac{\gamma_k}{\alpha_k(1+n_0/\beta_k)} I$
$\quad + \frac{\beta_k + n_0\sigma_k}{1+n_0/\beta_k} \mathrm{diag}\{\mathrm{rad}([C])\mathrm{rad}([C])^T\}$
$K_k = \mathrm{mid}([C])V_k^{-1}$
$[\hat{x}_{k|k}] = (I - K_k[C])[\hat{x}_{k|k-1}] + K_k(y_k - D_k u_k)$
$\mathcal{P}_{k|k} = (I - K_k \mathrm{mid}([C]))\alpha_k(1 + n_0/\beta_k)$
end for
where $S_+([P_{k|k}]) \preceq \mathcal{P}_{k|k}$, $K_k = \mathrm{argmin}\{\mathrm{tr}(\mathcal{P}_{k|k})\}$.

In this paper, fault detection approach is based on the one hand Algorithm 1 and on the other hand a statistical hypothesis testing. Therefore, it is vital to investigate carefully the stochastic properties of related terms of the system (2), otherwise the fault detection test for the interval case cannot be derived. In this study, it can be proved that state x_k, measure output y_k, estimator $\hat{x}_{k|k}$, estimation error ϵ_k and residual r_k are all Gaussian vectors, in which $r_k \sim \mathcal{N}(0, S_k)$, r_k is a function of $\{x_0, w_{1:k}, v_{1:k}\}$ and S_k is the covariance matrix of estimation error.

Key property (K): Assuming S_k is non singular and let $\eta_k = S_k^{-1/2} r_k = (\eta_{k,1}, ..., \eta_{k,n_y})$. Then $\eta_k \sim \mathcal{N}(0, I)$, that is $\eta_{k,i}$'s are $\mathcal{N}(0,1)$-distributed and independent each other. Furthermore, being a transformation of r_k, η_k is then a function of $\{x_0, w_{1:k}, v_{1:k}\}$, write $\eta_k = \eta_k(x_0, w_{1:k}, v_{1:k})$. Therefore, for any $1 \leq t \leq s$, $\eta_t = \eta_t(x_0, w_{1:t}, v_{1:t})$ and $\eta_s = \eta_s(x_0, w_{1:t}, w_{t+1:s}, v_{1:t}, v_{t+1:s})$ are not independent. The same holds for r_t and r_s.

2.3 Sensor fault detection using the χ^2-statistics in the literature

The measure y_k affected by an additive fault vector f_k^s is given by

$$y_k = C_k x_k + D_k u_k + v_k + f_k^s \ ,$$

where f_k^s is a sensor fault vector belonging to $\mathbb{R}^{n_y}$. The fault vector f_k^s can be composed by multiple or single faults:

-first type: some (or all) sensors are faulty,
-second type: only one sensor is faulty.

In the literature, using the χ^2-statistics test for sensor fault detection is a kind of Innovation approach mentioned in Mehra and Peschon (1971). In Willsky et al. (1974) and Willsky et al. (1975), this method is applied for fault detection problems in which the following statistic is used

$$\nu_k = \sum_{i=k-W+1}^{k} \eta_i^T \eta_i = \sum_{i=k-W+1}^{k} r_i^T S_i^{-1} r_i \ , \qquad (3)$$

where W is a window size ($W \leq k$). This statistic is used as χ^2 -distributed random variable with Wn_y degrees of freedom. A rule for the fault detection test was established: (H_0) $\nu_k \leq \delta$, no error occurred; (H_1) $\nu_k > \delta$, an error occurred, where δ is a threshold determined by $\mathbb{P}(\chi^2(Wn_y) > \delta) = \alpha$ with α a chosen significance level ($\mathbb{P}$ is a probability measure).

Clearly, any $\eta_k^T \eta_k$ follows the distribution $\chi^2(n_y)$ since, by property ***(K)***, $\eta_k^T \eta_k$ is the sum of n_y independent Gaussian components. Also by property ***(K)***, η_t and η_s are not independent for all $t \neq s$, so $\sum_{i=k-W+1}^{k} \eta_i^T \eta_i \sim \chi^2(Wn_y)$ does not hold exactly. Nevertheless, this test worked rather well as an alarm (detection) to faults and was a simple one among others tests. This statistic ν_k might be approximated reasonably by a $\chi^2(Wn_y)$. We distinguish a statistic T following exactly a distribution F and being approximated by another statistic $\tilde{T}$ with distribution F. Any statistic can be used as estimator for a quantity of interest with different accuracies.

3. SENSOR FAULT DETECTION BASED ON OUBIKF AND ADAPTIVE DEGREES OF FREEDOM χ^2-STATISTICS

Considering system (2) and assumptions **(H)**. Measures y_k and interval matrices $[A]$, $[B]$, $[C]$, $[D]$, $[Q]$, $[R]$ are known. The following terms are obtained by computation: measure estimate intervals $[\hat{y}_k] = [C][\hat{x}_{k|k-1}] + [D]u_k$, residual intervals $[r_k] = y_k - [\hat{y}_k]$ and the interval matrix $[S_k] = ([C][A])[P_{k-1|k-1}]([C][A])^T + [C][Q][C]^T + [R]$ which contains all residual covariances S_k.

3.1 Description of the approach

In the literature, to use the χ^2-statistics test, a standard normal distribution form ($\eta_k = S_k^{-1/2} r_k \sim \mathcal{N}(0, I)$) is needed. A similar form but for the interval vector $[r_k]$ is meant to match our goals, and thus the singularity problem of $[S_k]$ is an impediment. To overcome this impact, it is proposed in Tran (2017) to use the upper bound of $S_+([S_k])$ instead of $[S_k]$ and a better choice of this upper bound is applied thanks to properties in Section 2.1.

The following strategy is proposed in the present work:

+ Find Σ_k such that $S_+([S_k]) \preceq \Sigma_k$. This upper bound matrix is of the form $\Sigma_k = a_k I$ ($a_k \in \mathbb{R}^{*+}$) using properties in Section 2.1.
+ Compute:
  $[\tilde{\eta}_k] = \Sigma_k^{-1/2} [r_k] = [r_k]/\sqrt{a_k}$,
  $[\xi_k] = [\tilde{\eta}_k]^T [\tilde{\eta}_k] = [r_k]^T \Sigma_k^{-1} [r_k] = [r_k]^T [r_k]/a_k$.
+ Compute abs($[\xi_k]$), the absolute operator applying for $[\xi_k]$, since $\xi_k = \tilde{\eta}_k^T \tilde{\eta}_k \geq 0$, $\forall \xi_k \in [\xi_k]$ whilst during interval computations, most of the time $0 \in [\xi_k]$.

$$\text{abs}([a\,,b]) = \begin{cases} [\min(|a|,|b|)\,, \max(|a|,|b|)] & ,\ 0 \notin [a\,,b] \\ [0\,, \max(|a|,|b|)] & ,\ 0 \in [a\,,b]. \end{cases}$$

+ Let $U_k = \sup(\text{abs}([\xi_k]))$. The statistic U_k will be used in hypothesis testing for which it is approximated by a $\chi^2(\kappa_k n_y)$ random variable (some details are given in the next paragraph).
  κ_k is called an adaptive amplifier coefficient.

Some remarks can be made immediately as follows: $\forall k \geq 1$,

- $0 \leq \xi_k \leq \eta_k^T \eta_k \sim \chi^2(n_y)$ (since $S_k \preceq \Sigma_k$ for any S_k in $S_+([S_k])$),
- $0 \leq \xi_k \leq U_k$,
- $\mathbb{E}[\chi^2(n_y)] = n_y \ll U_k$ most of the time.

It is reasonable to consider ξ_k as a χ^2-distributed random variable with a d.f. smaller than n_y, but this statistic is actually unknown. The statistic U_k is obtained by computation. Based on previously mentioned remarks, it is proposed to approximate this statistic U_k by a χ^2-distributed random variable with an adaptive d.f. $\kappa_k n_y$ ($\kappa_k > 1$) where κ_k is an adaptive amplifier coefficient (a.a.c.). Thanks to this a.a.c., adaptive thresholds are built and help to detect faults.

The rule for the fault detection test is similar to this one presented previously: (H_0) $U_k \leq \delta_k$, no error occurred; (H_1) $U_k > \delta_k$, an error occurred. δ_k is an adaptive threshold determined by $\mathbb{P}(\chi^2(\kappa_k n_y) > \delta_k) = \alpha$ with α is a chosen significance level.

The proposed approach is summarized in Algorithm 2.

Algorithm 2 FAULT DETECTION ALGORITHM

Initialization: $[\hat{x}_{0|0}], \mathcal{P}_{0|0},\ [A], [B], [C], [D], [Q], [R],\ p,\ \alpha,\ u_k,\ y_k,\ k = 1, 2, ..., N.$
for $k = 1, 2, 3, ...N$ **do**
  Implementation:
    Use Algo.1 to get : $[\hat{x}_{k|k}], \mathcal{P}_{k|k}, [\hat{x}_{k|k-1}], [P_{k|k-1}]$.
    $[r_k] = y_k - [C][\hat{x}_{k|k-1}] - [D]u_k$
    $[S_k] = [C][P_{k|k-1}][C]^T + [R]$
    Find a_k according to theorem 2 s.t.: $S_+([S_k]) \preceq a_k I$.
    $U_k = \sup\{\text{abs}([r_k]^T [r_k]/a_k)\}$
    $\kappa_k = \text{mean}\{\sup([r_k]) - \inf([r_k])\}$
    Find δ_k s.t.: $\mathbb{P}(\chi^2(\kappa_k n_y) > \delta_k) = \alpha$
    Detection signal : $\pi_k = \mathbb{I}(U_k > \delta_k)$
end for
where $\mathbb{I}(x)$ equal 1 if x holds true and null otherwise.

After test running, an adjustment procedure is proposed to obtain detection signals more accurately. That is, in a window of size w, if the number of consecutive error occurrences is smaller than w, we consider that these errors (if exist) don't cause serious effects and will be dismissed. Furthermore, since error is often detected with a delay, all detection signals will be shifted to the left $\lfloor w/2 \rfloor$ steps ($\lfloor . \rfloor$ is the floor function).

3.2 Evaluation indicators

To evaluate the fault detection performance, some indicators are introduced. Assume that system(2) is applied for N iterations among which faults occur in a region $\mathcal{R}$ with length l ($0 \leq l \leq N$). The region $\mathcal{R}$ may be a range or union of ranges. For simplicity, hereafter we call $\mathcal{R}$ an error range. Knowing that the detection signal has value 1 or 0, we call **right detected signal** the 1-value detection signal situated inside the error range and **false detected signal** the 1-value detection signal situated outside the error range. Furthermore,

+ **Detection Rate (DR)** is determined by the number of right detected signals over the length l of error range.
+ **No Detection Rate (NDR)** is determined by $\text{NDR} = 1 - \text{DR}$.
+ **False Alarm Rate (FAR)** is determined by the number of false detected signals over $N - l$, the cardinal of the region outside the error range.
+ The **Efficiency (EFF)** of the detection is determined by $\text{EFF} = \text{DR} - \text{FAR}$.

More details on indicators (with slight differences) can be found in Chen and Patton (1999).

3.3 Choice of the a.a.c. κ_k

A χ^2 distributed random variable has the cumulative distribution function with d.f. k:

$$F(x,k) = \mathbb{P}(\chi_k^2 \leq x) = \frac{\int_0^{x/2} t^{\frac{k}{2}-1} e^{-t} dt}{\int_0^{\infty} t^{\frac{k}{2}-1} e^{-t} dt} . \quad (4)$$

In the literature, k is a positive integer. However, from the analysis point of view, $F(x,k)$ is a continuous function of k ($k > 0$) at any positive value of x (since the Gamma function $\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$ is continuous for all $z > 0$). Consequently a positive real d.f. $\kappa_k n_y$ is used.

For an accurate choice of a.a.c. κ_k, some conditions are required:

- It must be sensitive to the fault occurred.
- It must be large enough to get a small FAR (e.g. $\leq 5\%$) in the fault free case.
- Being a distribution parameter of statistic U_k, it is highly recommended that the chosen κ_k is related to the U_k's construction.

Concretely, by writing residual intervals in the form $[r_k] = \text{mid}([r_k]) + [-\frac{1}{2}, \frac{1}{2}] * \text{width}([r_k])$, the statistic U_k is expressed as

$$U_k = \|\text{width}([r_k]) + 2 * \text{abs}(\text{mid}([r_k]))\|^2/(4a_k) , \quad (5)$$

a function of $\text{mid}([r_k])$ and $\text{width}([r_k])$ where the later is more sensitive to the fault than the former. The residual width is a major factor influencing the U_k's computation and, furthermore, reflects the performance of the model and algorithm. Consequently it is reasonable to chose κ_k as a function of residual width.

Which function of residual width will be chosen is a hard problem due to many impacts, for instance:

- no further information about $\text{width}([r_k])$ and $\text{mid}([r_k])$ is available,
- κ_k and corresponding threshold δ_k are both unknown; it exists only a relation represented via the quite complex function (4) so that $F(\delta_k, \kappa_k n_y) \geq 1 - \alpha$,
- the yielded δ_k (by κ_k) must satisfy the fault detection constraint: $\delta_k \geq U_k$ when no error occurs and $\delta_k < U_k$ otherwise.

Therefore, an additional requirement for the chosen κ_k is that it must (while being sufficiently large in the fault free case as aforementioned) not increase as fast as U_k when an error occurs and affects on the $\text{width}([r_k])$.

Combining all constraints and noticing that, in general, identifying analytically degrees of freedom for a test problem is always not evident, the following a.a.c. is proposed:

$$\kappa_k = \sum_{i=1}^{n_y} \left(\sup([r_k])_i - \inf([r_k])_i\right) / n_y . \quad (6)$$

Simulation results in section 4.2 favored this choice by showing that it provides a small FAR and also satisfies all other requirements aforementioned.

Being not unique, the a.a.c. can be chosen differently, e.g. by a scale of (6) so that it becomes $\tilde{\kappa}_k = \lambda_k \kappa_k$, where $\lambda_k > 0$ is a scale parameter. This choice would be a scope in our perspective research.

4. APPLICATION

In this section, the strategy developed previously is applied in simulation to a model taken from automotive domain (Fergani (2014)). This model is a nonlinear continuous-time model which has been discretized/linearized and thus given under the form (2). The results are compared with those obtained by the method proposed in Raka and Combastel (2013) (called method B in the next).

4.1 Simulation procedure

A discretization phase with a sampling time $T = 0.05s$ is applied to the considered continuous model to get matrices A_d, B_d, C_d, D_d (non interval and independent of time instant k) according to equations in (2). Then, interval matrices $[A]$, $[B]$, $[C]$, $[D]$ are generated as follow: for $M \in \{A_d, B_d, C_d, D_d\}$, let $M = \text{mid}([M])$ and choose the radii $\text{rad}([M])$ at random in $[0 , \text{max\_rad}]$ with $\text{max\_rad} = 0.5$. The covariance matrices $[Q]$ and $[R]$ are generated in the same way, their diagonal elements being intervals of positive real numbers. Inputs u_k are simulated according to a dynamic change for $N = 864$ iterations. The initial state is chosen at $x_0 = (0,0)^T$. At each step k, generate A_k, B_k, C_k, D_k, Q_k, R_k according to uniform distribution in corresponding interval matrices and so that $Q_k \in S_+(n_x)$ and $R_k \in S_+(n_y)$. Then w_k and v_k are simulated. Finally, $\{x_k, y_k\}_{k \in 1:N}$ are computed according to system (2).

Sensor faults are generated in term of bias vector $b_k \in \mathbb{R}^{n_y}$ added to y_k for all k in a range $\mathcal{R}$ with length l. Each sequence of y_k's components, e.g. $\{y_k(i)\}_{k \in 1:N}$ for some $i = 1, ..., n_y$, is called a chain. In this simulation, $b_k = b.\mathbf{1}$ where $\mathbf{1}$ is the all-ones vector in $\mathbb{R}^{n_y}$.

Apply Algorithm 2 for N steps. The following choices are applied inside the algorithm: starting point $[\hat{x}_0] = ([-0.5, 0.5], [-0.5, 0.5])^T$, initial error covariance bound

$\mathcal{P}_{0|0} = \max\{\text{diag}(\sup([Q]))\}I$, $p = 3$, upper bounds $\omega_k I$ of any set $S_+([M])$ identified by $\omega_k = \|\text{Max}([M])\|_F$.

The window size is fixed at $w = 5$ (see 3.1).

Using the previously defined sampling period T, the following discrete matrices are obtained:

$$A_d = \begin{pmatrix} 0.9126 & 0.0446 \\ -0.17 & 0.9081 \end{pmatrix}, \quad C_d = \begin{pmatrix} -1.737 & 0.9768 \\ 1 & 0 \\ -3.723 & -1.837 \\ 0 & 1 \end{pmatrix},$$

$$B_d = \begin{pmatrix} 0.05652 & 0.0005351 & 0.001124 & -0.001124 \\ 0.6177 & 0.0222 & 0.04661 & -0.04661 \end{pmatrix},$$

$$D_d = \begin{pmatrix} 0.8686 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 13.03 & 0.4653 & 0.9772 & -0.9772 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

4.2 Simulation results

The following simulations are to be considered as first results and a preliminary study to highlight the consistency and the efficiency of the proposed method by simulating two scenarios. Other insights are being investigated in the authors ongoing studies.

General implementation: Let $b_k = b.\mathbf{1}$ be identical for all k in an error range $\mathcal{R}$ with length $l = 50$. Let b vary in $0:5:30$. According to each scenario and for each value of b, do $L = 100$ times of fault detection. Indicators are computed for each of L simulation times and their means are yielded afterward.

Remark 2. Let $\tau = b/\text{Max\_width}$ where Max_width is the maximum width of the diagonal elements of $[Q]$ and $[R]$. This quantity gives an idea of how large is the actual fault value b w.r.t. some known quantity causing the fault and propagating according to the dynamic system, that is the maximum covariance of noises.

Scenario 1: Fix variable simulation $\{y_k\}_{k\in 1:N}$. For each fault value b, choose randomly error range $\mathcal{R}$ and do L times error generations. This scenario helps us to consider the method performance in term of fault values b and the positions at which errors occur (in $\mathcal{R}$) w.r.t. a given measurement sample $\{y_k\}_{k\in 1:N}$.

Table 1. Fault detection table for scenario 1 with a fixed data $\{y_k\}_{k=1:N}$.

b	τ	DR%	NDR%	FAR%	EFF%
0	0	1.26	98.74	1.03	0.23
5	13.8	5.34	94.66	1.04	4.30
10	27.5	21.12	78.88	1.04	20.08
15	41.3	67.28	32.72	1.17	66.11
20	55.0	94.94	5.06	1.26	93.68
25	68.8	98.64	1.36	1.36	97.28
30	82.5	99.84	0.16	1.42	98.42

Table 1 shows that DR has ascending trend as well as b increases while FAR is rather stable in $[1.0\,,\,1.5](\%)$. This means that the larger the fault value b, the better the fault detection procedure is performed and, conversely, the b change hardly affects the false alarm rate FAR. This also means that the current choice of a.a.c. κ_k is appropriate for a fault detection eliminating well false alarms and dismissing almost all non clear signs of error existence (a prudent fault detection). For different purposes of fault detection, κ_k can be adjusted.

Seeing more, EFF represents the effectiveness of the fault detection procedure taking into account both DR and FAR. It has also ascending trend according to b. Starting at $b = 15$ ($\approx 41\times$Max_width) EFF begins to achieve remarkable value (66.11%).

Table 2. Adjusted fault detection table for scenario 1 with a fixed data $\{y_k\}_{k\in 1:N}$.

b	τ	DR%	NDR%	FAR%	EFF%
0	0	0	100	0	0
5	13.8	3.30	96.70	0.01	3.29
10	27.5	17.70	82.30	0.02	17.68
15	41.3	63.54	36.46	0.05	63.49
20	55.0	96.36	3.64	0.06	96.30
25	68.8	99.96	0.04	0.15	99.81
30	82.5	100	0	0.38	99.62

Table 2 shows that the adjustment procedure eliminates almost all FAR indexes (at least 73% comparing to those in Table 1). Additionally, this procedure yields a positive effect with large fault value ($b > 15$) and a negative effect otherwise for EFF indexes.

Scenario 2: Fix error range $\mathcal{R}$. For each fault value b, do L times variable simulations to get measurements y_k. This scenario aims to show the effects of different measurement samples $\{y_k\}_s$ ($k = 1:N, s = 1:L$) on the fault detection procedure for a given error range $\mathcal{R}$. Specifically, these effects come from random noises existing inside of y_k since the later is a function of $\{x_0, w_1 : w_k, v_k\}$.

Table 3. Fault detection table for scenario 2 with a fixed error range.

b	τ	DR%	NDR%	FAR%	EFF%
0	0	3.34	96.66	1.90	1.44
5	13.8	3.08	96.92	2.36	0.72
10	27.5	19.24	80.76	2.41	16.83
15	41.3	82.48	17.52	2.18	80.30
20	55.0	94.74	5.26	2.48	92.26
25	68.8	98.66	1.34	2.28	96.38
30	82.5	99.88	0.12	2.37	97.51

In Table 3, DR and EFF indexes are not necessarily increasing functions w.r.t. b but their main trends are always ascending. The FAR index is also stable in $[1.9\,,\,2.5](\%)$. In comparison with the one in Table 1 (FAR $\in [1.0,\,1.5](\%)$), we see that FAR is rather greater in scenario 2 than in scenario 1. This means that FAR is more affected by random noises than by the position of error range. The adjustment procedure eliminates more than 18% of FAR indexes comparing Table 4 and Table 3. It has also positive effect or negative effect for EFF index according to the fault value b being greater or smaller than 15.

Comparison with method B: The method B consists in applying interval observer for linear continuous time dynamic system with additive and multiplicative disturbances to compute adaptively upper bounds (ub_t) and lower bounds (lb_t) of residuals r_t, and the fault detection rule is that a fault is detected if $0 \notin [\text{lb}_t, \text{ub}_t]$. Fig.1 presents the simulation of this method applying to bicycle model with an as similar as possible setting with that

Table 4. Adjusted fault detection table for scenario 2 with a fixed error range.

b	τ	DR%	NDR%	FAR%	EFF%
0	0	2.8	97.20	1.44	1.36
5	13.8	2.16	97.84	1.84	0.32
10	27.5	14.06	85.94	1.97	12.09
15	41.3	82.42	17.58	1.62	80.80
20	55.0	96.96	3.04	1.82	95.14
25	68.8	99.8	0.20	1.71	98.10
30	82.5	100	0	1.89	98.11

used for our method. The setting is that: 1-dimension multiplicative and additive disturbances ($q = 1, \delta_t = d_t = \sin(2\pi t)$) are used, a bias sensor fault with value $b = 20$ is added to all chains of measurements y_t in a time range $[\mathrm{te1}, \mathrm{te2}]$. This error range is determined correspondingly to the discrete error range $[300 : 350]$ used in the simulation of our method. Some remarks can be pointed out:

(i) method B takes the fault detection chain by chain.
(ii) to compare with the method presented in this paper, detection signals of method B in all chains are combined in one, i.e. new detection signal is 1 if there is any detection signal in any chain getting value 1.
(iii) Table 5 shows that the proposed method obtains much better indexes vis-a-vis method B in the context of the implementation simulation.

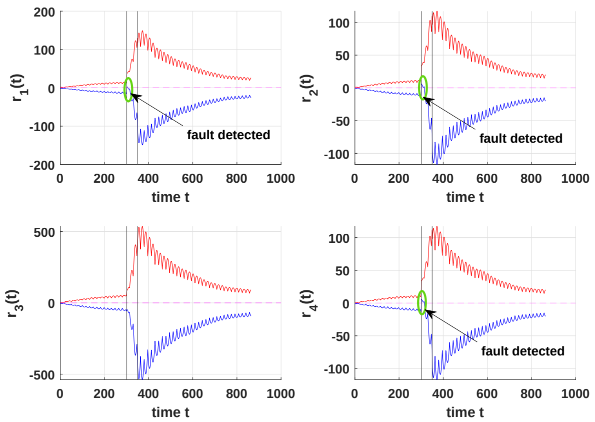

Fig. 1. Method B - Residual for the system with disturbances and bias sensor fault $b = 20$.

Table 5. Method B of Raka and Combastel (2013) versus the Proposed FD scheme.

	DR%	NDR%	FAR%	EFF%
Method B	8.14	91.86	5.64	2.50
Proposed method	98	2	0.12	97.88

5. CONCLUSION AND PERSPECTIVE

A new scheme for fault detection combining OUBIKF and an hypothesis test method using χ^2-statistics with adaptive degrees of freedom is proposed in this paper. Theoretical framework is developed. The proposed approach is then applied to a bicycle vehicle model and it is compared to two others methods. The results obtained in simulations highlight the potential of this approach. Simulation results show that the current choice of a.a.c. κ_k, in (6), is appropriate for fault detection eliminating well false alarms and dismissing almost all non clear signs of error existence (a prudent fault detection). The possibility of adjusting the a.a.c. according to different purposes of fault detection is a scope of future works. Furthermore, the modified EFF index proposed by $\mathrm{EFF} = c_1 \times \mathrm{DR} - c_2 \times \mathrm{FAR}$ with c_1, c_2 two constants in $[0, 1]$ can be applied to control the importance of the two indexes DR and FAR so that a compromise between them is achieved. This index, in turn, might affect the choice of the a.a.c.

Also, the great flexibility of this method by adjusting tuning factors makes it suitable to multiple applications.

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