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Abstract: We propose a hybrid controller for reference tracking for the output current of a solenoid. The solenoid can be modeled as a first-order linear plant with unknown parameters. The proposed hybrid controller comprises 1) a feedforward action, based on an estimate of the unknown plant parameters exploiting a hybrid formulation of the recursive least-squares method, and 2) a feedback action, exploiting a reset control scheme based on a first-order reset element. We prove stability properties for the closed-loop system and we show through simulations that the current output converges to the reference for sufficiently exciting reference signals.

1. INTRODUCTION

Solenoid-based actuators are ubiquitous in industrial applications such as hydraulic and pneumatic systems. For these actuators, manufacturing tolerances require addressing a large variation of unknown parameters. To control the solenoid current, classical output control strategies such as PID control, internal model control (IMC) and sliding mode control (SMC) have been employed Krimpmann et al. (2015); Zhao et al. (2016). IMC and PID controllers result in an equivalent controller structure and are commonly applied in the industry. These two strategies, similar to SMC, feature a high robustness to parameter variations and allow for an easy tuning. However, this robustness comes at the expense of tracking performance. This drawback can be addressed by adaptive control, as the parameter variation due to manufacturing tolerances and fatigue are explicitly incorporated in the control scheme. Recently, batchwise identification based on a least-squares approach was found to have preferable properties for adaptive control over the classical recursive formulation, see, e.g., Pan et al. (2019); Karafyllis et al. (2020). One possible way to cast the dynamics of batchwise identification, combined with feedback laws is to use a hybrid dynamics description wherein the identified parameters remain constant during flowing solutions and then jump to the next estimate whenever reasonable, based on the data collected during flow.

Foloowing this idea, we propose here a hybrid adaptive reset-control scheme achieving current tracking for a solenoid with unknown parameters. As in the works in Panni et al. (2014), Cordioli et al. (2015) and Cocetti et al. (2019), our controller is composed of a feedback stabilizer and a feedforward action. The feedback term of our controller is a first-order reset element (FORE), a control strategy that was first proposed in Clegg (1958) and that has been thoroughly analyzed during the last decade, see Nešić et al. (2011); Prieur et al. (2018); Baños and Barreiro (2011) and references therein. The FORE applies aggressive and diverging control actions, and resets the controller state to zero at appropriate instants, thus generating hybrid trajectories that are overall converging to zero. This type of controller performs well in terms of disturbance rejection (see, e.g., Panni et al. (2014)), and in view of the results in Nešić et al. (2011) it guarantees stability for any choice of the tuning parameters. Our feedforward term is a hybrid formulation of the well-known recursive least squares (RLS) estimation method, with the addition of a directional forgetting law (see Parkum et al. (1992) and Cao and Schwartz (2000)) to ensure boundedness of the information matrix. Similar to Cocetti et al. (2019), the estimate update is performed at jumps of the hybrid solutions, using the information gathered during the preceding flowing interval. The information is collected through non-exponentially-stable filters connected to the plant, which are reset to zero at the adaptation instants. The choice of non-exponentially-stable filters, as opposed to the stable ones in Cocetti et al. (2019) guarantees that no part of the gathered information is forgotten before updating the parameter estimates.

The paper is organized as follows. In Section 2, the physical system introduced and modeled. In Section 3, the structure of the proposed feedforward-feedback controller is explained Section 4 contains the closed-loop stability analysis and our main result. Simulation results are finally reported in Section 5.

Notation: We denote $(x_1,\ x_2) := \left[x_1^T, x_2^T\right]^T$; $|v|$ denotes the Euclidean norm of vector v. Following the hybrid notation of Goebel et al. (2012), a hybrid solution $\phi(t,j)$ is parametrized by continuous-time t and discrete-time j. $\dot{\phi}$ characterizes its derivative with respect to t and ϕ^+ characterizes its next value with respect to j.

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2. PROBLEM STATEMENT

The current dynamics of a solenoid according to Faraday's law, corresponds to

$$\frac{\partial \Psi}{\partial i}\frac{\mathrm{d}i}{\mathrm{d}t} = u - R_L i - \frac{\partial \Psi}{\partial s}\dot{s}, \tag{1}$$

where Ψ is the magnetic flux linkage, i is the solenoid current, u is the voltage across the solenoid terminals, R_L is the terminal resistance, and s is the plunger position. Assume that the current i is measured, whereas the plunger position cannot be accessed by measurement. Then, the resulting change in the magnetic flux linkage $\frac{\partial\Psi}{\partial s}\dot{s}$ may be considered as an unmodeled dynamic effect. For the design of a controller, a simplified model of the solenoid (1)

$$L\frac{\mathrm{d}i(t)}{\mathrm{d}t} = u(t) - R_L i(t)\ , \tag{2}$$

is used in this paper, parametrized by the unknown solenoid inductance L and the unknown resistance R_L.

In standard control notation, the dynamics between the voltage input u and the output $y = i$ of system (2) can be written as

$$\dot{y} = a_p y + b_p u + \overline{d}, \tag{3}$$

where $\overline{d}$ is a constant input bias, and $a_p = -\frac{R_L}{L}, b_p = \frac{1}{L}$ are unknown parameters.

In this paper, we propose an adaptive reset control scheme for current reference tracking with unknown parameters (a_p, b_p) of plant (3), under the following assumption, stemming from (2).

Assumption 1. The unknown scalars a_p and b_p are such that $a_p < 0$ and $b_p > 0$.

We also require some assumption on the reference current $t \mapsto r(t)$ to be tracked. In particular, we assume that both the reference r and its derivative $\dot{r}$ are available to the controller. Moreover, we assume that these signals satisfy the following mild boundedness assumption, without necessarily knowing the values of the bounds.

Assumption 2. The reference input $t \mapsto r(t)$ is a differentiable signal. Both the reference $r(t)$ and its derivative $\dot{r}(t)$ are available for the control scheme at time t. Moreover, both r and $\dot{r}$ are uniformly bounded.

In practical situations, the signal $\dot{r}$ can be obtained by linearly filtering the reference r, by means of strictly proper filters, as explained, for example, in (Forni et al., 2010, Remark 4).

3. CONTROL SCHEME

For tracking the reference input r, following the works in Cordioli et al. (2015); Nešić et al. (2011); Panni et al. (2014); Cocetti et al. (2019), we propose in this paper the control scheme of Figure 1 comprising a feedforward action and a feedback stabilizer. More specifically, our control architecture resembles the one of Cordioli et al. (2015), but the adaptation proposed here follows a radically different paradigm. We describe below the feedback and feedfoward blocks represented in Figure 1, the two key components of our control scheme.

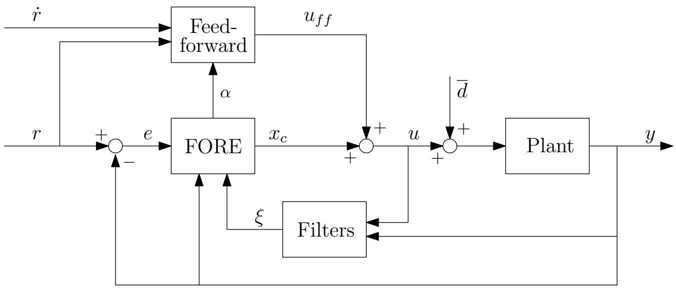

Fig. 1. The closed-loop system block diagram.

3.1 Reset feedback

The feedback part of our controller is performed by a first-order reset element (FORE), whose dynamics is given by

$$\begin{cases} \dot{x}_c = a_c x_c + b_c e, \\ \dot{\tau}_r = 1, \end{cases} \qquad (e, x_c, \tau_r) \in \mathcal{F}_r, \tag{4a}$$

$$\begin{cases} x_c^+ = 0, \\ \tau_r^+ = 0, \end{cases} \qquad (e, x_c, \tau_r) \in \mathcal{J}_r \tag{4b}$$

with the feedback state $x_c \in \mathbb{R}$ and the additional state τ_r having the role of a timer, with "r" standing for "reset". The jump and flow sets are defined as

$$\begin{aligned} \mathcal{F}_r &:= \big\{(e, x_c, \tau_r) : \varepsilon e^2 + 2 e x_c \geq 0 \text{ or } \tau_r \leq \rho_{\min}\big\}, \\ \mathcal{J}_r &:= \big\{(e, x_c, \tau_r) : \varepsilon e^2 + 2 e x_c \leq 0 \text{ and } \tau_r \geq \rho_{\min}\big\}, \end{aligned} \tag{4c}$$

with ε and $\rho_{\min}$ being small positive constants, as in Nešić et al. (2011). Their choice has little effect on the closed-loop solutions as long as they are selected sufficiently small (by "hierarchically" first fixing ε small and then choosing $\rho_{\min}$ small enough – see Nešić et al. (2011) for details). The controller-plant interconnection corresponds to

$$u := x_c + u_{\text{ff}}, \quad e := r - y. \tag{5}$$

Using the interconnection equations (5), we can explicitly write the continuous evolution of the tracking error

$$\begin{aligned} \dot{e} &= \dot{r} - \dot{y} \\ &= \dot{r} - a_p y - b_p u - \overline{d} \\ &= \dot{r} - a_p(r - e) - b_p u_{\text{ff}} - b_p x_c - \overline{d} \\ &= a_p e - b_p x_c - (b_p u_{\text{ff}} - \dot{r} + a_p r + \overline{d}) \\ &= a_p e - b_p x_c - d, \end{aligned} \tag{6}$$

where $d = b_p u_{\text{ff}} - \dot{r} + a_p r + \overline{d}$ is a disturbance term that should be canceled by the feeforward action u_{ff}. Since neither the reference nor the plant output perform jumps, we always have $e^+ = e$. Thus, we can write the complete hybrid dynamics of the feedback subsystem as

$$\begin{aligned} &\begin{cases} \dot{x}_{\text{fb}} = A_F x_{\text{fb}} + Bd, \\ \dot{\tau}_r = 1, \end{cases} && x_{\text{fb}}^T M x_{\text{fb}} \geq 0 \text{ or } \tau_r \leq \rho_{\min}, \\ &\begin{cases} x_{\text{fb}}^+ = A_J x_{\text{fb}}, \\ \tau_r^+ = 0, \end{cases} && x_{\text{fb}}^T M x_{\text{fb}} \leq 0 \text{ and } \tau_r \geq \rho_{\min} \end{aligned} \tag{7a}$$

where $x_{\text{fb}} = (-e, x_c)$ and

$$\left[\begin{array}{c|c} A_F & B \\ \hline A_J & M \end{array}\right] = \left[\begin{array}{cc|cc} a_p & b_p & 1 & \\ -b_c & a_c & 0 & \\ \hline 1 & 0 & \varepsilon & -1 \\ 0 & 0 & -1 & 0 \end{array}\right]. \tag{7b}$$

The following proposition, emerging from Nešić et al. (2011), characterizes the degrees of freedom available in the choice of the controller parameters a_c, b_c, ensuring global exponential stability of the feedback interconnection (7). A useful feature stemming from the proposition is that even though the plant parameters are unknown, a stabilizing feedback is obtained for any positive selection of a_c and b_c in (4), which provides useful degrees of freedom for the tuning of (a_c, b_c).

Proposition 1. Under Assumption 1, for any positive selections of the controller gains $(a_c, b_c) \in \mathbb{R}_{>0} \times \mathbb{R}_{>0}$, the point $(e, x_c) = (0, 0)$ is globally exponentially stable (conditionally to hierarchically small ε and $\rho_{\min}$) for the reset feedback (4), and it is finite gain exponentially input-to-state stable from the disturbance d to the state x_{fb}.

Proof. The closed-loop system (7) coincides with the one in (Nešić et al., 2011, Equations (14)-(15)). Since $a_p < 0$, and the closed-loop gain $b_p b_c$ is positive, we have, with $a_c > 0$, that

$$2\sqrt{b_p b_c} + a_c - a_p > 0.$$

Consequently, item 4 of (Nešić et al., 2011, Theorem 3) is satisfied and the system is proven to be globally exponentially stable. Additionally, applying (Nešić et al., 2011, Theorem 7), global exponential stability also implies finite gain exponential input-to-state stability from d to x_{fb}, as to be proven. ■

3.2 Hybrid feedforward adaptation

As shown in Figure 1 and in equation (5), we want to design a feedforward term u_{ff}. The ideal feedforward selection u^*_{ff} corresponds to perfect cancelation of the disturbance term d in (6), namely

$$u^*_{\text{ff}} = \frac{1}{b_p}(\dot{r} - a_p r - \overline{d}) = \chi^T(r, \dot{r})\theta^* := [1\ r\ \dot{r}]\, \frac{1}{b_p} \begin{bmatrix} -\overline{d} \\ -a_p \\ 1 \end{bmatrix}. \quad (8)$$

Here, χ is a known function of the reference r and its derivative $\dot{r}$, and θ^* comprises the unknown parameters

$$\theta^* := [\theta^*_1\ \theta^*_2\ \theta^*_3]^T = \frac{1}{b_p}\left[-\overline{d}\ -a_p\ 1\right]^T. \quad (9)$$

In view of the linear dependence of u^*_{ff} on θ^*, we introduce a controller state θ representing an estimate of θ^*, to be adapted online via a discrete update law. The feedforward term of the controller is then selected as

$$u_{\text{ff}} := \chi^T(r, \dot{r})\theta \quad (10)$$

ensuring that $u_{\text{ff}} \to u^*_{\text{ff}}$ whenever $\theta \to \theta^*$.

Following the previous adaptive reset control laws in works like Cocetti et al. (2019), Cordioli et al. (2015) and Panni et al. (2014), we freeze the estimate θ during flows and we update it at jumps. However, since the reference is time-varying, we need to perform these updates according to some quantity that depends on the evolution of the system between two consecutive jumps. To this end, similar to Cocetti et al. (2019), we augment the scheme with non-stable filters (two integrators and a memory element), introducing a new state $\xi = (\xi_y, \xi_u, \xi_s) \in \mathbb{R}^3$. The hybrid dynamics of the resulting controller is therefore given by

$$\begin{cases} \dot{\theta} = 0, \\ \dot{\xi} = (y,\ u,\ 0), \\ \dot{\tau}_a = 1, \quad \dot{R} = 0, \end{cases} \qquad \tau_a \in [0, \rho_{\max}], \quad (11a)$$

$$\begin{cases} \theta^+ = g_\theta(\theta, R, \xi, \tau_a, y) \\ \xi^+ = (0,\ 0,\ y), \\ \tau_a^+ = 0, \\ R^+ = g_R(R, \xi, \tau_a, y), \end{cases} \qquad \tau_a \in [\rho_{\min}, \rho_{\max}], \quad (11b)$$

where τ_a is an additional timer, with "a" standing for "adaptation", and $R \in \mathbb{R}^{3\times 3}$ is a uniformly upper and lower bounded symmetric matrix, typically called *information matrix*, which is kept constant during flows and is updated at jumps, just like the parameter vector estimate θ. While scheme (11) shares similarities with the solution proposed in Cocetti et al. (2019), an interesting new feature emerges from the fact that the filters used in Cocetti et al. (2019) are exponentially stable, and cannot be reset to zero at jumps without invalidating that stability proof. The approach that we follow here is more desirable because no forgetting effect is introduced until the update of θ is performed, and once used in the update, the information in the filters is completely reset to zero.

The update of θ and R depends on the information about the system evolution between the jumps, which is collected in the vector

$$\varphi := [\tau_a\ \xi_y\ y - \xi_s]^T. \quad (12)$$

Taking inspiration from the recursive least-squares (RLS) adaptation method with directional forgetting (DF), well summarized in Cao and Schwartz (2000), we use φ to define the update functions g_θ and g_R in (11b) as follows

$$g_\theta(\theta, R, \xi, \tau_a, y) := \theta - P^+ \frac{\varphi}{\varphi^T\varphi}(\varphi^T\theta - \xi_u), \quad (13a)$$

$$\begin{aligned} g_R(R, \xi, \tau_a, y) &:= R - \eta\Delta R + \Phi \quad (13b) \\ &:= R - \eta\frac{R\varphi\varphi^T R}{\varphi^T R\varphi} + \frac{\varphi\varphi^T}{\varphi^T\varphi}. \end{aligned}$$

Here we denote by $P := R^{-1}$ the inverse of matrix R, usually known as the *covariance matrix*. Due to the properties of R, also P is positive definite and uniformly lower and upper bounded. We also denote by $\Phi := \frac{\varphi\varphi^T}{\varphi^T\varphi}$ the projection matrix induced by the available direction φ and we introduce a forgetting factor $\eta \in (0, 1)$, with $\Delta R := \frac{R\varphi\varphi^T R}{\varphi^T R\varphi}$ being the part of the information matrix to be (partially, as per the tunable scalar η) forgotten.

Remark 1. According to Cao and Schwartz (2000), the matrix ΔR in (13b) is the only one satisfying

$$\text{rank}(\Delta R) = 1, \quad \Delta R\varphi = R\varphi. \quad (14)$$

In view of this fact, the rationale behind the update law (13b) is to introduce some forgetting of the old data only in the direction where the new data provides information. This both prevents the covariance matrix $P = R^{-1}$ from losing rank and avoids the so called estimator windup, which happens in the simpler exponential forgetting algorithm when the signal φ is not sufficiently exciting in some directions. See Parkum et al. (1992) for a detailed explanation on the topic, noting that in that work the notation "forgetting factor" refers to the quantity $1 - \eta$, therein denoted μ. ◦

Remark 2. Note that the adaptation laws in (13) are well defined; indeed φ is never 0, since its first element is the timer state τ_a, which satisfies $\tau_a \geq \rho_{\min}$ due to the constraint in (11b). Also note that, as proven in Parkum et al. (1992), R (and therefore P) is positive definite, while ΔR is positive semi-definite. ◦

Following a typical approach in adaptive control theory, we assume the information matrix R to be both lower- and upper-bounded. In particular, given a lower bound $\alpha_m \in (0,1)$ for matrix $R(0,0)$, extensive simulations (see e.g. Figure 5) suggest that the following conjecture is true. Note that in equation (15), below, solution R has a domain dom R comprising two times (continuous and discrete), as customary with hybrid solutions.

Conjecture 1. Given $\alpha_m \in (0,1)$, consider the compact set

$$X_R := \left\{ R \in \mathbb{R}^{3\times 3} : \alpha_m I \le R \le \frac{I}{\eta} \right\}.$$

In view of the update law (13b), the information matrix R satisfies

$$R(0,0) \in X_R \implies R(t,j) \in X_R \tag{15}$$

for all $(t,j) \in \text{dom } R$.

Conjecture 1, whose proof is subject of future work, is in line with the rank loss and estimator windup observations discussed in Remark 1.

4. STABILITY ANALYSIS

Our main result discussed below establishes stability of the error dynamics, exploiting a few technical lemmas.

4.1 A few preliminary lemmas

Based on the nominal value of the parameter vector θ^* in (9), define the parameter estimation error

$$\tilde{\theta} := \theta - \theta^*. \tag{16}$$

In order to derive the dynamics of $\tilde{\theta}$, consider the following scalar error variable

$$\tilde{\xi}_a(t,j) := \frac{1}{b_p}\Bigg[\big(y(t_j,j) - \xi_s(t_j,j)\big) - a_p\bigg(\xi_y - \int_{t_j}^{t} y(s,j)ds\bigg) - b_p\bigg(\xi_u - \int_{t_j}^{t} u(s,j)ds\bigg) - \overline{d}\Big(\tau_a - (t-t_j)\Big)\Bigg], \tag{17}$$

whose rationale follows from the fact that its continuous dynamics satisfies $\dot{\tilde{\xi}}_a = 0$, and its discrete dynamics is $\tilde{\xi}_a^+ = \tilde{\xi}_a$ when the feedback controller is reset and $\tilde{\xi}_a^+ = 0$ at the adaptation instants. The following holds.

Lemma 1. Given φ as in (12), $\tilde{\theta}$ in (16) and $\tilde{\xi}_a$ in (17), along any flowing solution of (11)-(13) it holds that

$$\varphi^T\theta - \xi_u = \varphi^T\tilde{\theta} + \tilde{\xi}_a. \tag{18}$$

Proof. Integrating the plant dynamics (3) starting from the jump (adaptation) time (t_j, j), we obtain

$$\begin{aligned}\int_{t_j}^{t} \dot{y}(s,j)\, ds &= y(t,j) - y(t_j,j)\\ &= a_p\int_{t_j}^{t} y(s,j)\, ds + b_p\int_{t_j}^{t} u(s,j)\, ds + \int_{t_j}^{t} \overline{d}\, ds.\end{aligned} \tag{19}$$

Substituting (19) in (17), and rearranging, we get for all $t \in [t_j; t_{j+1}]$ (the dependence on (t,j) is dropped)

$$b_p\xi_u = (y - \xi_s) - a_p\xi_y - \overline{d}\tau_a - b_p\tilde{\xi}_a.$$

Recalling (9), (12) and (16), this can be rewritten as

$$\begin{aligned}\xi_u &= \frac{1}{b_p}\begin{bmatrix}-\overline{d} & -a_p & 1\end{bmatrix}\begin{bmatrix}\tau_a\\ \xi_y\\ y-\xi_s\end{bmatrix} - \tilde{\xi}_a\\ &= \left(\theta^*\right)^T\varphi - \tilde{\xi}_a = \varphi^T\theta^* - \tilde{\xi}_a,\end{aligned}$$

and therefore we obtain

$$\varphi^T\tilde{\theta} + \tilde{\xi}_a = \varphi^T\theta - \varphi^T\theta^* + \tilde{\xi}_a = \varphi^T\theta - \xi_u,$$

as to be proven. ■

From Lemma 1 we may obtain a convenient dynamics for the parameter estimation error $\tilde{\theta}$. Conversely to what we did for the tracking error e, in (6) and (7), we consider only the jump dynamics of $\tilde{\theta}$, since the estimate θ is constant along flows, namely $\dot{\tilde{\theta}} = 0$. Using (11b) and (13a), we get

$$\begin{aligned}\tilde{\theta}^+ &= \theta^+ - \theta^* = \theta - \theta^* - P^+\frac{\varphi}{\varphi^T\varphi}(\varphi^T\tilde{\theta} + \tilde{\xi}_a)\\ &:= \left(I - P^+\frac{\varphi\varphi^T}{\varphi^T\varphi}\right)\tilde{\theta} - P^+\frac{\varphi}{\varphi^T\varphi}\tilde{\xi}_a.\end{aligned} \tag{20}$$

Notice that the filter states do not appear explicitly in (20), but only as arguments of φ. Thus, we can directly consider φ as a state when writing the closed-loop error dynamics. In particular, introducing the transformed input $\chi_\text{i}(t,j) := \int_{t_j}^{t} \chi(s,j)ds$, which is uniformly bounded due to the boundedness assumed in Assumption 2, we introduce the following "incremental" version of φ:

$$\tilde{\varphi} := \varphi - \chi_\text{i} := \varphi - \int_{t_j}^{t} \chi(s,j)ds, \tag{21}$$

which does not change across resets of the FORE and is reset to 0 across adaptations of the feedforward controller. The flow dynamics of $\tilde{\varphi}$ is characterized next.

Lemma 2. Function $\tilde{\varphi}$ in (21) satisfies

$$\begin{aligned}\dot{\tilde{\varphi}} &= Cx_\text{fb} + Fb_p\chi^T\tilde{\theta}\\ &:= \begin{bmatrix}0 & 0\\ 1 & 0\\ a_p & b_p\end{bmatrix}x_\text{fb} + \begin{bmatrix}0\\0\\1\end{bmatrix}b_p\chi^T\tilde{\theta}.\end{aligned} \tag{22}$$

Proof. Using the definitions of φ and χ in (12) and (8), and recalling equation (5), we obtain

$$\begin{aligned}\tilde{\varphi} &= \begin{bmatrix}\tau_a\\ \int_{t_j}^{t} r(s,j) - e(s,j)ds\\ r - e - r(t_j,j) + e_s\end{bmatrix} - \begin{bmatrix}\int_{t_j}^{t} 1ds\\ \int_{t_j}^{t} r(s,j)ds\\ \int_{t_j}^{t} \dot{r}(s,j)ds\end{bmatrix}\\ &= \begin{bmatrix}\tau_a - (t-t_j)\\ -\int_{t_j}^{t} e(s,j)ds\\ -[e(t,j) - e(t_j,j)]\end{bmatrix}.\end{aligned}$$

Differentiating the expression obtained above and using the feedback error dynamics in (7) we obtain (22). ■

4.2 Error dynamics and main stability result

Using the results of the previous section and (7), we may now describe the dynamics of the error coordinates associated with the complete system. The state is given by

$$x := \big(x_\text{fb},\, \tilde{\theta},\, \tilde{\varphi},\, \tilde{\xi}_a,\, R,\, \tau_r,\, \tau_a\big),$$

which evolves in the state space

$$X := \mathbb{R}^9 \times X_R \times [0, \rho_\text{max}]^2. \tag{23}$$

The ensuing error dynamics is obtained by combining the relations in (7), (11)-(13), (18), (22) and corresponds to

$$\begin{cases} \dot{x}_{\text{fb}} = A_F x_{\text{fb}} + B b_p \chi^T \tilde{\theta} \\ \dot{\tilde{\theta}} = 0 \\ \dot{\tilde{\varphi}} = C x_{\text{fb}} + F b_p \chi^T \tilde{\theta} \\ \dot{\tilde{\xi}}_a = 0, \quad \dot{R} = 0, \quad \dot{\tau}_r = 1, \quad \dot{\tau}_a = 1 \end{cases} \quad x \in C, \quad (24a)$$

$$\begin{cases} x_{\text{fb}}^+ = x_{\text{fb}} \\ \tilde{\theta}^+ = \big(I - P^+ \Phi(\tilde{\varphi} + \chi_{\text{i}})\big)\tilde{\theta} - P^+ \dfrac{(\tilde{\varphi} + \chi_i)}{(\tilde{\varphi} + \chi_i)^T(\tilde{\varphi} + \chi_i)} \tilde{\xi}_a \\ \tilde{\varphi}^+ = 0, \quad \tilde{\xi}_a^+ = 0 \\ R^+ = R - \eta \Delta R(\tilde{\varphi} + \chi_{\text{i}}) + \Phi(\tilde{\varphi} + \chi_{\text{i}}) \\ \tau_r^+ = \tau_r, \quad \tau_a^+ = 0 \end{cases} \quad x \in D_a, \quad (24b)$$

$$\begin{cases} x_{\text{fb}}^+ = A_J x_{\text{fb}} \\ \tilde{\theta}^+ = \tilde{\theta} \\ \tilde{\varphi}^+ = \tilde{\varphi}, \quad \tilde{\xi}_a^+ = \tilde{\xi}_a \\ R^+ = R \\ \tau_r^+ = 0, \quad \tau_a^+ = \tau_a \end{cases} \quad x \in D_r, \quad (24c)$$

where combining the original definitions of Φ and ΔR with the definition in (21), we denoted

$$\Phi(\tilde{\varphi} + \chi_{\text{i}}) = \frac{(\tilde{\varphi} + \chi_{\text{i}})(\tilde{\varphi} + \chi_{\text{i}})^T}{(\tilde{\varphi} + \chi_{\text{i}})^T(\tilde{\varphi} + \chi_{\text{i}})} = \frac{\varphi\varphi^T}{\varphi^T\varphi},$$

and similarly for ΔR. Moreover, we introduced the jump and flow sets as

$$C := \{x \in X : x_{\text{fb}}^T M x_{\text{fb}} \geq 0 \text{ or } \tau_r \leq \rho_{\min}\}, \quad (25a)$$

$$D_a := \{x \in X : \tau_a \geq \rho_{\min}\}, \quad (25b)$$

$$D_r := \{x \in X : x_{\text{fb}}^T M x_{\text{fb}} \leq 0 \text{ and } \tau_r \geq \rho_{\min}\}. \quad (25c)$$

The complete hybrid error dynamics (24) is written in a convenient form because it corresponds to an autonomous dynamics evolving under the action of the external inputs χ and χ_{i}, which are uniformly bounded by assumption. Our main result is given below. Its proof, omitted due to space constraints, exploits the techniques in Cocetti et al. (2019); Cao and Schwartz (2000); Parkum et al. (1992).

Theorem 1. The attractor

$$\mathcal{A} := \big\{x \in X : \big(x_{\text{fb}},\, \tilde{\theta},\, \tilde{\varphi},\, \tilde{\xi}_a\big) = 0\big\} \quad (26)$$

is uniformly globally stable (UGS) for the error dynamics (24)-(25). Moreover, if $\tilde{\theta} \to 0$ then $x_{\text{fb}} \to 0$.

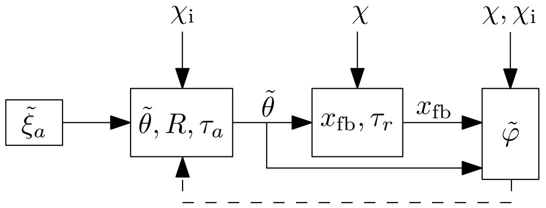

Fig. 2. The pseudo-cascaded interconnection of the error dynamics (24).

The error dynamics (24) enjoys an interesting pseudo-cascade structure, represented in Fig. 2. In particular, the dynamics comprises a first (upper) subsystem with the states $\tilde{\theta}$, R and τ_a, associated with the adaptation. Even though this subsystem is perturbed by the variable $\tilde{\varphi}$ (see Fig. 2), we can show that this perturbation provides zero gain in the stability bound. The second subsystem involves the feedback states x_{fb}, τ_r and is perturbed by $\tilde{\theta}$. Finally, the third subsystem, with state $\tilde{\varphi}$, is perturbed by the two previous ones. The subsystems are also perturbed by the external inputs χ, χ_{i}, which are bounded by assumption.

Remark 3. Theorem 1 ensures that the estimation error and the tracking error do not diverge, for any choice of the controller gains and of the external signals χ and χ_{i}. The convergence of the errors to the origin could be ensured by adding some persistence of excitation requirement to the reference input, similar to the one introduced in Parkum et al. (1992). This requirement could also suggest a way to optimally choose the adaptation instants. ∘

5. SIMULATIONS

We consider a solenoid in the form (3) with parameters $a_p = 3$ and $b_p = 1$ in closed loop with the proposed adaptive reset control strategy of Fig. 1, corresponding to equations (4), (8), (11)-(13). To represent a reasonable experimental scenario, the solenoid input voltage u is augmented with band limited white noise with a standard deviation of 1 V.

From Proposition 1, any positive selection of the feedback control parameters a_c and b_c induces closed-loop stability. The unstable pole a_c of the controller can be associated with a controller time constant, whereas b_c determines the error-dependent control action, comparable to an integral controller gain. In contrast to tuning stable control systems, the time constant of the unstable controller can be tuned faster, to fully leverage the benefits of the resetting action in the hybrid control scheme. After the tuning procedure, the reset feedback controller was used with the parameters $a_c = 6.3$ and $b_c = 28$. For the tuning of the feedforward adaptation, the forgetting factor $\eta \in (0, 1)$ is selected as $\eta = 0.5$ to induce a suitable trade-off between parameter convergence and robustness to noise. Moreover, we select $\alpha_m = 10^{-3}$ so that we may pick a small initial value of $R(0, 0)$, to induce a fast initial transient on the adaptation. The reference r is a gear shifting cycle of an automatic transmission solenoid valve (see, e.g., Cordioli et al. (2015)). This trajectory includes an impulsive rise of the reference current and a second plateau. In contrast to these rapid changes of the reference current, the signal features periods of constant current (low excitation).

The simulated plant output current is depicted in Fig. 3 (blue curve). It is clearly visible that the system output converges quickly to the reference trajectory (red curve). However, at $t = 2$ s an external disturbance signal of $\bar{d} = 2$ V is applied to the input. The control input is quickly adjusted and the parameters are rapidly updated. The time evolution of the identified parameters is reported in Fig. 4, which shows their piecewise constant nature. From Fig. 4 it is clear that the adaptation every 0.25 s achieves rapid convergence to zero of the estimation error $\tilde{\theta}$ and excellent robustness to the white noise affecting input u. To perform a worst-case test on the adaptation we select initial values of the estimate θ very far from the actual parameters θ^*. At the same time, we initialize $R(0, 0) = \alpha_m I = 10^{-3} I$, thus leading to a large adaptation gain in the initial transient. As a result, Fig. 4 shows that after the second adaptation step, the parameter estimation errors are negligible.

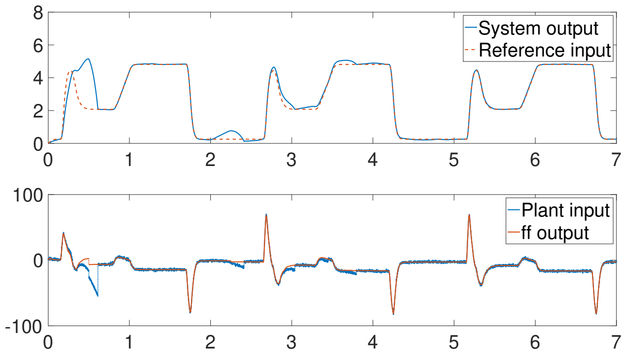

Fig. 3. Top: plant output (blue) and current reference (red). Bottom: plant input (blue), and feedforward input (red).

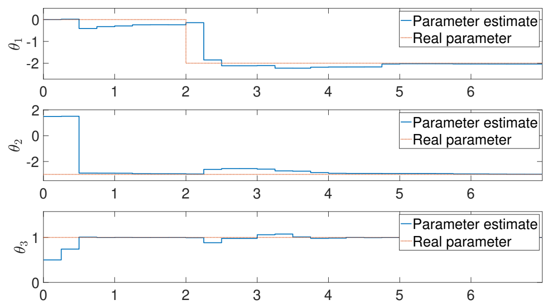

Fig. 4. Parameter estimates θ (blue) and real values θ^*.

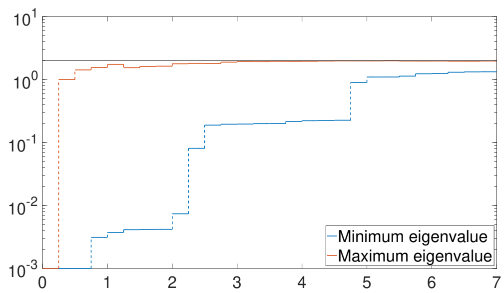

Fig. 5. Minimum and maximum eigenvalues of R.

After the external disturbance occurs at $t = 2\,\text{s}$, the parameters are quickly adjusted without too much overshoot. This desirable behavior can be associated to the stability property proven in Theorem 1. The fact that θ_1 jumps to a nonzero value is also clearly visible in the eigenvalues of R, see Fig. 5 where the smallest eigenvalue starts growing after $t = 2\,\text{s}$. Due to the initial strong excitation in one of the directions of the parameter space, one of the eigenvalues of R increases within the first second. As visible in the logarithmic scale, the other eigenvalue remains above the lower limit α_m, thus confirming Conjecture 1. This effect, leading to a slower adaptation of the corresponding parameter direction, mitigates possible drift caused by the noisy signals.

6. CONCLUSIONS

A hybrid reset-based control scheme has been proposed for current tracking in a solenoid with unknown parameters. After introducing the feedback/feedforward controller architecture, we derived an autonomous dynamics for the error system, perturbed by uniformly bounded external inputs. We proved closed-loop stability for any choice of the feedback controller gains and for any reference signal satisfying some mild boundedness assumptions. We also proved convergence when the parameter estimates converge to their nominal values. Simulations confirm that this happens when the reference signal is sufficiently rich to guarantee persistence of excitation. Future work will address conditions on the reference signal to guarantee convergence to zero of the tracking error, together with optimal choices of the adaptation instants.

REFERENCES

Baños, A. and Barreiro, A. (2011). *Reset control systems.* Springer.

Cao, L. and Schwartz, H. (2000). Directional forgetting algorithm based on the decomposition of the information matrix. *Automatica*, 36, 1725–1731.

Clegg, J.C. (1958). A nonlinear integrator for servomechanisms. *Transactions of the American Institute of Electrical Engineers*, 77(1), 41–42.

Cocetti, M., Tarbouriech, S., Zaccarian, L., and Ragni, M. (2019). A hybrid adaptive inverse for uncertain siso linear plants with full relative degree*. In *2019 American Control Conference (ACC)*, 2315–2320.

Cordioli, M., Mueller, M., Panizzolo, F., Biral, F., and Zaccarian, L. (2015). An adaptive reset control scheme for valve current tracking in a power-split transmission system. In *European Control Conference*, 1878–1883. Linz, Austria.

Forni, F., Galeani, S., and Zaccarian, L. (2010). An almost anti-windup scheme for plants with magnitude, rate and curvature saturation. In *Proceedings of the 2010 American Control Conference*, 6769–6774.

Goebel, R., Sanfelice, R., and Teel, A. (2012). *Hybrid Dynamical Systems: modeling, stability, and robustness.* Princeton University Press.

Karafyllis, I., Kontorinaki, M., and Krstic, M. (2020). Adaptive control by regulation-triggered batch least squares. *IEEE Trans. Aut. Contr.*, 65(7), 2842–2855.

Krimpmann, C., Schoppel, G., Glowatzky, I., and Bertram, T. (2015). Performance evaluation of nonlinear surfaces for sliding mode control of a hydraulic valve. In *2015 IEEE Conf. on Control Applications*, 822–827.

Nešić, D., Teel, A., and Zaccarian, L. (2011). Stability and performance of SISO control systems with first-order reset elements. *IEEE Transactions on Automatic Control*, 56(11), 2567–2582.

Pan, Y., Sun, T., and Yu, H. (2019). On parameter convergence in least squares identification and adaptive control. *International Journal of Robust and Nonlinear Control*, 29(10), 2898–2911.

Panni, F.S., Waschl, H., Alberer, D., and Zaccarian, L. (2014). Position regulation of an egr valve using reset control with adaptive feedforward. *IEEE Transactions on Control Systems Technology*, 22(6), 2424–2431.

Parkum, J.E., Poulsen, N.K., and Holst, J. (1992). Recursive forgetting algorithms. *International Journal of Control*, 55(1), 109–128.

Prieur, C., Queinnec, I., Tarbouriech, S., and Zaccarian, L. (2018). Analysis and synthesis of reset control systems. *Found. & Trends in Syst. and Contr.*, 6(2-3), 117–338.

Zhao, X., Li, L., Song, J., Li, C., and Gao, X. (2016). Linear control of switching valve in vehicle hydraulic control unit based on sensorless solenoid position estimation. *IEEE Transactions on Industrial Electronics*, 63(7), 4073–4085.