A qualitative counterpart of belief functions with application to uncertainty propagation in safety cases

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Abstract. Critical systems such as those developed in the aerospace, railway or automotive industries need official documents to certify their safety via convincing arguments. However, informal tools used in certification documents seldom cover the uncertainty that pervades safety cases. Several works use quantitative approaches based on belief functions to model and propagate confidence/uncertainty in the argument structures (particularly those using goal structuring notation). However the numerical uncertainty information is often a naive encoding of qualitative expert inputs. In this paper, we outline a qualitative substitute to Dempster-Shafer theory and suggest new qualitative confidence propagation models. We also propose a more faithful encoding of expert inputs.

Keywords: Goal Structuring Notation · argument structures · confidence elicitation · Dempster-Shafer Theory · qualitative capacities.

1 Introduction

As the use of artificial intelligence (AI) in systems increases, the need for safety assessment methods in the latter is also increasing. However, the lack of confidence can jeopardize the social acceptance of these systems and therefore their existence. Several approaches are used to assess confidence/uncertainty in such systems (especially, the safety critical ones).

Many papers addressing the assessment of safety of systems rely on the graphical representation of an argument structure like GSN (Goal Structuring Notation), plus quantitative representations of uncertainty. Typically, probability theory is often used in Bayesian network models of GSNs. In order to address the issue of incomplete information, Dempster-Shafer theory (DST) is also proposed. In the latter case, argument trees can be modelled in classical logic using if-then rules [6, 7].

However the quantification of uncertainty is often problematic, when it relies on expert assessments. In many cases, experts supply qualitative assessments using linguistic values like *probable, very probable, unlikely*, etc., which are then translated into numbers on the $[0,1]$ scale. This translation is somewhat arbitrary. So, a legitimate question is whether a purely qualitative approach to uncertainty, that would be a counterpart to the belief function approach, could be promising. The idea is to avoid the quantitative encoding of qualitative estimates. It makes all the more sense as numerical degrees of belief obtained via uncertainty propagation are often translated back to the qualitative scale, so as to make the results more palatable. So it is legitimate to investigate a qualitative approach.

This paper is a first step in this direction. It is structured as follows. Section 2 presents theoretical background on qualitative capacities that can be viewed as a qualitative counterpart of belief functions, based on [5]. Section 3 deals with the elicitation of qualitative capacities, based on an existing method where linguistic term scales were mapped to belief functions. Section 4 use qualitative belief measures on classical inference patterns. Section 5 recalls the a graphical representation called Goal Structuring Notation (GSN), dedicated to argument structures for safety cases. This section applies the qualitative uncertainty propagation method from premises to conclusions of several types arguments. In Section 6, a preliminary comparison of qualitative and quantitative uncertainty propagation is proposed via an example.

2 From belief functions to qualitative capacities

As a generalization of probability theory, Dempster-Shafer theory [8](DST) offers tools to model and propagate both aleatory (due to random events) and epistemic (due to incomplete information) uncertainty.

A mass function, or basic belief assignment (BBA), is a probability distribution over the power set of the universe of possibilities (W), known as the *frame of discernment*. Formally, a mass function $m : 2^W \to [0,1]$ is such that $\sum_{E \subseteq W} m(E) = 1$, and $m(\emptyset) = 0$. Any subset E of W such as $m(E) > 0$ is called a focal set of m. $m(E)$ quantifies the probability that we only know that the truth lies in E; in particular $m(W)$ quantifies the amount of ignorance.

A mass assignment induces a so-called belief function $Bel : 2^W \to [0,1]$, defined by: $Bel(A) = \sum_{E \subseteq A} m(E)$. It represents the sum of all the masses supporting a statement A. The degree of belief in the negation $\neg A$ of the statement A is called *disbelief*: $Disb(A) = Bel(\neg A)$; the value $Uncer(A) = 1 - Bel(A) - Disb(A)$ quantifies the lack of information about A.

The *conjunctive rule of combination* combines multiple pieces of evidence (represented by mass functions m_i, with $i = 1, 2$) coming from independent sources of information: $m_\cap = m_1 \otimes m_2$ such that: $m_\cap(A) = \sum_{E_1 \cap E_2 = A} m_1(E_1) \cdot m_2(E_2)$. In DST, an additional step eliminates conflict that may exist by means of a normalization factor (dividing $m_\cap$ by $1 - m_\cap(\emptyset)$). This is Dempster rule of combination [8], which is associative.

In contrast, we outline the qualitative approach in [3–5]. Let L be a finite totally ordered set representing certainty levels. A qualitative capacity (q-capacity, for short) is a function $\gamma : 2^W \to L$ such that:

$\gamma(\emptyset) = 0$; $\gamma(W) = 1$; $A \subseteq B \Rightarrow \gamma(A) \leq \gamma(B)$. *Any* q-capacity can be put in the form:

$$\gamma(A) = \max_{\emptyset \neq B \subseteq A} \rho(B), \forall A \subseteq W, \quad (1)$$

where ρ is formally a basic possibility assignment (BΠA) [3], namely, a possibility distribution $\rho : 2^W \to L$ on the power set of W, such that $\max_{B \subseteq W} \rho(B) = 1$ and $\rho(\emptyset) = 0$. The value $\rho(B)$ is the strength of piece of evidence B. Several BΠA's can generate the same γ, the least of which is the qualitative Moebius transform (QMT) of γ such that:

$$\gamma_{\#}(A) = \begin{cases} \gamma(A) \text{ if } \gamma(A) > \gamma(A \setminus \{w\}), \forall w \in A; \\ 0 \text{ otherwise.} \end{cases} \quad (2)$$

The value $\gamma(A)$ (resp. $\gamma(\neg A)$) qualifies the support in favor of (resp. against) A, i.e. belief (resp. disbelief) in A using an element in the qualitative scale L. The pair $(\gamma(A), \gamma(\neg A))$ thus describes our epistemic stance with respect to A in terms of belief and disbelief, ranging from no information (i.e., (0, 0)), to full conflicting information (i.e., (1, 1)), from full belief (i.e., (1, 0)) to full disbelief (i.e., (0, 1)). This is more general than possibility theory where the case $(1, 1)$ is not allowed.

Figure 1 presents the credibility and information orderings on pairs (belief, disbelief) including extreme cases [5]. A proposition A is at least as credible as B if $\gamma(A) \geq \gamma(B)$ and $\gamma(\neg A) \leq \gamma(\neg B)$ (solid arrows from B to A), thus ranging from certainty of falsity $(0, 1)$ up to certainty of truth $(1, 0)$. A proposition A is at least as informed as B if $\gamma(A) \geq \gamma(B)$ and $\gamma(\neg A) \geq \gamma(\neg B)$ (dotted arrows from B to A), thus ranging from ignorance ($(0, 0)$, no information) up to conflict ($(1, 1)$, full contradictory information). In this situation, the amount of evidence supporting the conclusion is equal to the one rejecting it. The set $L \times L$ is then equipped with a bilattice structure. In order to qualitatively combine pieces of

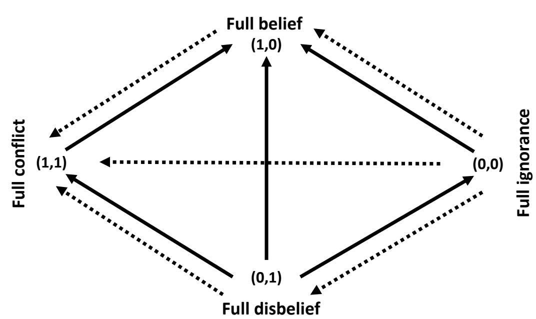

Fig. 1. Evolution of certainty and information in pairs (belief, disbelief)

evidence represented by possibilistic mass functions, i.e., BΠA's ρ_i, coming from several sources of information, the qualitative counterpart of the conjunctive rule of combination for belief functions is: $\rho_\cap = \rho_1 \oplus \rho_2$ such that:

$$\rho_\cap(A) = \max_{E_1 \cap E_2 = A} \{\min[\rho_1(E_1), \rho_2(E_2)]\} \tag{3}$$

Due to the use of the (idempotent) minimum operation, the combined pieces of evidence are not supposed to be independent. The result is not always a BΠA, strictly speaking. First we may have that $\rho_\cap(A) < 1$ for all A. So we must add the condition $\rho_\cap(W) = 1$. This will not occur if we restrict to non-dogmatic BΠA's such that $\rho_i(W) = 1$, which we assume in this paper. Besides, we may have that $\rho_\cap(\emptyset) > 0$, indicating conflict between the pieces of evidence.

3 Expert elicitation approach

In order to elicit qualitative capacities, we borrow from a methodology by Cyra and Gorski [2]. Two types of information are collected from experts about a statement A: A so-called *decision* and a level of *confidence* associated to it. Then, these pieces of information are numerically encoded, and transformed to belief and disbelief degrees in the sense of Shafer (see also [7]). More precisely:

- The decision index denoted by $Dec(A)$, describes which side the assessor leans towards, i.e., acceptance or rejection of A. It is associated with a bipolar scale $D = \{0_D = d_{-k}, d_{k-1}, \dots, d_0 = e, d_1, \dots d_k = 1_D\}$ with $2k+1$ values, the bottom of which (0_D) expressing rejection, the top (1_D) acceptance, and the midpoint (e) a neutral position. Here we assume $k = 2$.
- The confidence index denoted by $Conf(A)$ reflects the amount of information an assessor possesses to support the decision. It uses a positive uni-polar scale K with $k+1$ values (the top 1_K expresses full confidence, the bottom 0_K is neutral - no information). For $k = 2$: these levels mean: lack of confidence ($C_0 = 0_K$), moderate confidence (C_1) and full confidence ($C_2 = 1_K$).

The bipolar scale D is equipped with an order-reversing map ν_D such that $\nu_D(d_{-i}) = d_i$. Especially we have that $\nu_D(Dec(A)) = Dec(\neg A)$. The unipolar scale K is isomorphic to the positive part of D. This assumption makes K and D commensurate. K is equipped with an order-reversing map ν_K such that: $\nu_K(C_i) = C_{k-i}$.

In order to switch from a ($Dec(A)$, $Conf(A)$) pair to ($\gamma(A)$, $\gamma(\neg A)$), we use a transformation that maps $D \times K$ to the belief-disbelief scale $L \times L$ containing pairs $(\gamma(A), \gamma(\neg A))$. The scale L has the same number of elements as K (i.e., 3 here). The mapping $f : D \times K \to L \times L$: must satisfy some conditions [5]:

- If the expert declares lack of confidence, the result is $f(Dec(A), 0) = (0, 0)$, whatever the trend expressed on the decision scale.
- If the expert is fully confident, then $f(1, 1) = (\gamma(A), \gamma(\neg A)) = (1, 0)$, $f(0, 1) = (0, 1)$, $f(e, 1) = (1, 1)$. Indeed, for the latter, there is a total conflict: the expert is maximally informed ($Conf(A) = 1$), and cannot decide between A and its negation ($Dec(A) = e$).
- $\max(\gamma(A), \gamma(\neg A)) = Conf(A)$: the belief in A or its negation cannot be stronger than the confidence.

- if $Dec(A)$ is the midpoint of D, then $\gamma(A) = \gamma(\neg A)(= Conf(A))$ (no reason to take side).
- if $Dec(A)$ is less than the midpoint of D, then $\gamma(A) < \gamma(\neg A) = Conf(A)$, and the smaller $Dec(A)$, the smaller $\gamma(A)$.
- if $Dec(A)$ is greater than the midpoint of D, then $\gamma(A) = Conf(A) > \gamma(\neg A)$, and the greater $Dec(A)$, the smaller $\gamma(\neg A)$.

These conditions lead to propose the following translation formulas [5]:

$$\text{if } Dec(A) < e,\ \gamma(A) = \min[\nu_K(Dec(\neg A)), Conf(A)] \text{ and } \gamma(\neg A) = Conf(A);$$
$$\text{if } Dec(A) > e,\ \gamma(A) = Conf(A) \text{ and } \gamma(\neg A) = \min[\nu_K(Dec(A)), Conf(A)];$$
$$\text{if } Dec(A) = Dec(\neg A) = e,\quad \gamma(A) = \gamma(\neg A) = Conf(A).$$

In Table 1, we grouped all possible (Dec, $Conf$) pairs on premises with their appropriate counterparts ($\gamma(A)$, $\gamma(\neg A)$)$\in L \times L$, using the formulas above. We can notice an anti-symmetry between belief and disbelief degrees regarding the central column ($D_0 = e$: no decision). We also notice that when no information is available (C_0: Lack of confidence), no matter what choice is made the degrees of belief and disbelief take a minimal value. On the other hand, in the case of a fully informed expert (C_2: Full confidence) the decision value varies from rejection to acceptance.

Table 1. Values from (Dec, $Conf$) to (Bel,$Disb$) pairs on premises

$Conf$ \ Dec	D_{-2} (Rej)	D_{-1} (Opp)	$D_0(ND)$	D_1 (Tol)	D_2 (Acc)
C_0 (Lack of confidence)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
C_1 (Moderate confidence)	(0,λ)	(λ,λ)	(λ,λ)	(λ,λ)	(λ,0)
C_2 (Full confidence)	(0,1)	(λ,1)	(1,1)	(1,λ)	(1,0)

4 Logical inference for qualitative capacities

Logical reasoning and numerical belief functions are not often put together. An approach to reasoning with Dempster rule of combination was proposed in [1]. In this approach each formula in a knowledge base is viewed as a simple support function and combined with other formulas in the knowledge base. Besides, the application of belief functions to argument structures has been studied in [2, 10, 7] to build models for uncertainty propagation. For instance in [7], we assign mass functions to logical expressions such as facts p_i, $\neg p_i$, and rules $p_i \Rightarrow C$, $\neg p_i \Rightarrow \neg C$, $(\wedge_{i=1}^n p_i) \Rightarrow C$ and $(\wedge_{i=1}^n \neg p_i) \Rightarrow \neg C$, in order to deduce the belief on the conclusions C and $\neg C$. Here we develop the same approach, albeit using qualitative capacities.

The simplest pattern is modus ponens, i.e. inferring C from p and $p \Rightarrow C$. We assume two BΠA's ρ_p on $\{p, \neg p\}$ with values in L, say $\rho_p(p) = \alpha_p, \rho_p(\neg p) = \overline{\alpha}_p, \rho_p(\top) = 1$, and a simple support function $\rho_\Rightarrow$ with $\rho_\Rightarrow(p \Rightarrow C) = \beta_\Rightarrow$,

$\rho_{\Rightarrow}(\top) = 1$, where $\top$ stands for the tautology. The capacity γ_C is obtained via projection for the conclusion C by the q-conjunctive rule has a BΠA ρ_C such that: $\gamma_C(C) = \rho_C(p \wedge C) = \min(\rho_p(p), \rho_{\Rightarrow}(p \Rightarrow C)) = \min(\alpha_p, \beta_{\Rightarrow}), \gamma_C(\neg C) = 0$.

If $p = \bigwedge_{i=1}^{n} p_i$, then the above formula holds with $\rho_p(p) = \min_{i=1}^{n} \rho_p^i(p_i)$. If there is also a B$\Pi$A $\rho_{\Leftarrow}$ assigning a weight $\beta_{\Leftarrow}$ to the reversed implication $\neg p \Rightarrow \neg C$ there is an additional weight on $\neg C$ via the combination $\rho_p \oplus \rho_{\Rightarrow} \oplus \rho_{\Leftarrow}$ using equation (3) and projection on C's universe.

$$\gamma_C(\neg C) = \rho_C(\neg p \wedge \neg C) = \min(\rho_p(\neg p), \rho_{\Leftarrow}(p \Leftarrow C)) = \min(\overline{\alpha}_p, \beta_{\Leftarrow}).$$

Consider the case with more than one premise. Suppose we have to merge BΠA's ρ_p^i on p_i, $\rho_{\Rightarrow}^i$, $\rho_{\Leftarrow}^i, i = 1, \dots n$. As in its quantitative counterpart, the BΠA pertaining to the conclusion C obtained from this fusion may assign a mass to the contradiction. Conflict always appears when four items are merged of the form: p_i and $p_i \Rightarrow C$ with $\neg p_j$ and $\neg p_j \Rightarrow \neg C, j \neq i$, whose conjunction is a contradiction $\emptyset$ with mass:

$$\begin{aligned} \rho_C^{ij}(\emptyset) &= \min[\rho_C^i(p_i \wedge C), \rho_C^j(\neg p_j \wedge \neg C)] \\ &= \min[\rho_p^i(p_i), \rho_{\Rightarrow}^i(p_i \Rightarrow C), \rho_p^j(\neg p_j), \rho_{\Rightarrow}^j(\neg p_j \Rightarrow \neg C)]] \end{aligned}$$

For two premises, the final mass on contradiction is $\rho_C(\emptyset) = \max(\rho_C^{12}(\emptyset), \rho_C^{21}(\emptyset))$. Besides, using (1) we get: $\gamma_C(C) = \max[\rho_C(p_1 \wedge C), \rho_C(p_2 \wedge C)] \geq \rho_C(\emptyset)$ and $\gamma_C(\neg C) = \max[\rho_C(\neg p_1 \wedge \neg C), \rho_C(\neg p_2 \wedge \neg C)] \geq \rho_C(\emptyset)$.

5 Application to safety cases

Goal structuring notation (GSN) is a graphical notation/language which represents argument structures (i.e., safety and assurance cases) in the form of directed acyclic graphs (directed trees or arborescences). It breaks down a top claim, called "goal", into elementary sub-goals following a specific strategy and in accordance with a particular context. Each sub-goal is associated with pieces of evidence, called solutions, which support the conclusion. Despite the fact that it presents all the evidence supporting the safety of the system, GSN fails to show how premises support the conclusion and the confidence that can be given to them. Both questions bring uncertainty to arguments, which may affect their merits. To address this issue, confidence propagation schemes were proposed to complement GSN patterns.

Some approaches use DST to model and propagate confidence in GSN patterns in the literature [2, 7, 9]. These papers consider a number of argument types and associate confidence propagation formulas to each of them. In practice, they also devise transformation formulas that turn uncertainty assessments of experts (on a qualitative scale) about premises to numerical belief and disbelief degrees. This transformation is a source of uncertainty. Indeed, qualitative inputs are often naively translated into equidistant values in the unit interval. Therefore, the qualitative approach to uncertainty developed in [3–5], and the elicitation and the inference methods of Sections 3 and 4 may lead to more robust confidence assessment approaches.

Here, we use the argument types defined in [7]. An argument type describes the interaction between premises to support a conclusion. This type of interaction is either a conjunction (C-Arg), a disjunction (D-Arg), or a combination of both (H-Arg, a hybrid type). We can translate each argument type into logical expressions often called rules. Since we use only implication to describe links between the universe of premises ($W_p = \{p, \neg p\}$) and that of the conclusion ($W_C = \{C, \neg C\}$), two kinds of rules are used: *direct rules*, which model the acceptance of the conclusion (C), and *reverse rules* which model its rejection ($\neg C$). Then, to each rule, we assigned a simple support function (a mass on the rule, and another on the tautology). We also assigned masses to the premises and their negation. Finally the propagation formulas, for each type, are obtained using the qualitative combination rule (3). Below, we recall our argument types and associate to each of them to qualitative uncertainty propagation formula.

Simple argument (S-Arg): This argument describes the case of a conclusion (C) supported by a single premise (p), hence the name "simple". If the premise is true, then so is the conclusion: $p \Rightarrow C$. Note that only the information about the acceptance of the conclusion can be inferred in this situation. Since we work on a three-state paradigm (belief, disbelief and uncertainty), the reverse rule $\neg p \Rightarrow \neg C$ is introduced to add conditions for the possible denial of the conclusion. Then, we associate to the direct and reverse rules simple BΠA's (resp., $\rho_{\Rightarrow}$ and $\rho_{\Leftarrow}$), and a BΠA on the premise space, as done above. We can prove:

$$\text{S-Arg} : \begin{cases} \gamma_C(C) &= \min[\gamma_p(p), \gamma_{\Rightarrow}(p \Rightarrow C)] \\ \gamma_C(\neg C) &= \min[\gamma_p(\neg p), \gamma_{\Leftarrow}(\neg p \Rightarrow \neg C)] \end{cases} \tag{4}$$

We can notice that the belief $\gamma_C(C)$ depends only on the direct rule and the acceptance of the premise, while the disbelief $\gamma_C(\neg C)$ only depends on the reverse rule and the disbelief of the premise.

Conjunctive argument (C-Arg): This argument type describes the situation when two premises or more are jointly needed to support a conclusion. We formally defined its direct and reverse rules (resp.) by: $(\wedge_{i=1}^n p_i) \Rightarrow C$ and $\wedge_{i=1}^n(\neg p_i \Rightarrow \neg C)$. Following the same reasoning of the previous argument type, we put a simple BΠA on each rule ($\rho_{\Rightarrow}$ and $\rho_{\Leftarrow}^i$), and another BΠA on each premise (ρ_p^i). Then we combine them with the rule of combination ($\rho = \rho_r \oplus \rho_p$, with $\rho_p = \rho_p^1 \oplus ... \oplus \rho_p^n$ and $\rho_r = \rho_{\Rightarrow} \oplus (\oplus_{i=1}^n \rho_{\Leftarrow}^i)$) and get:

$$\text{C-Arg} : \begin{cases} \gamma_C(C) &= \min\{\min_{i=1}^n \gamma_p^i(p_i), \gamma_{\Rightarrow}([\wedge_{i=1}^n p_i] \Rightarrow C)\} \\ \gamma_C(\neg C) &= \max_{i=1}^n\{\min[\gamma_p^i(\neg p_i), \gamma_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)]\} \end{cases} \tag{5}$$

In the formulas of the quantitative approach [7] operations $a + b - ab$ and ab replace $\max, \min$, respectively, thus highlighting the similarity between the results obtained from each model. Indeed, we can notice that the C-Arg, like its quantitative counterpart, favors the propagation of the premise with the least strength (minimal belief, with a maximal disbelief degree).

Disjunctive argument (D-Arg): In this situation, each premise can support alone the whole conclusion. Formally, the direct and reverse rules are defined as follows:

$\wedge_{i=1}^{n}(p_i \Rightarrow C)$ and $(\wedge_{i=1}^{n}\neg p_i) \Rightarrow \neg C$. The calculation of $\gamma_C(C)$ and $\gamma_C(\neg C)$ is identical to the one above, swapping the two expressions:

$$\text{D-Arg}: \begin{cases} \gamma_C(C) \;\;\; = \max_{i=1}^{n}\{\min[\gamma_p^i(p_i), \gamma_{\Rightarrow}^i(p_i \Rightarrow C)]\} \\ \gamma_C(\neg C) = \min\{\min_{i=1}^{n}\gamma_p^i(\neg p_i), \gamma_{\Leftarrow}([\wedge_{i=1}^{n}\neg p_i] \Rightarrow \neg C)\} \end{cases} \quad (6)$$

We can notice that this model, as its quantitative counterpart [7], favors the propagation of the premise with the greatest strength (maximal belief and minimal disbelief degree).

Hybrid argument (H-Arg): This argument type describes the situation where each premise supports the conclusion to some degree, but their conjunction does it to a larger one. Therefore, all conjunctive and disjunctive rules will be used in this argument type. Thus, we obtain:

$$\text{H-Arg}: \begin{cases} \gamma_C(C) = \;\; \max\{\min[\min_{i=1}^{n}\gamma_p^i(p_i), \gamma_{\Rightarrow}([\wedge_{i=1}^{n}p_i] \Rightarrow C)], \\ \qquad\qquad\quad \max_{i=1}^{n}(\min[\gamma_p^i(p_i), \gamma_{\Rightarrow}^i(p_i \Rightarrow C)]\} \\ \gamma_C(\neg C) = \max\{\min[\min_{i=1}^{n}\gamma_p^i(\neg p_i), \gamma_{\Leftarrow}([\wedge_{i=1}^{n}\neg p_i] \Rightarrow \neg C)], \\ \qquad\qquad\quad \max_{i=1}^{n}\min[\gamma_p^i(\neg p_i), \gamma_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)]\} \end{cases} \quad (7)$$

We can notice that eq.(7), presents a combination between C-Arg formulas (5), and D-Arg (6). Assuming a maximal belief (= 1) (resp. disbelief) on premises, it is enough that the simple direct rules take a null value (resp. the reversed conjunctive one) to get the conjunctive argument type. And conversely, to get the disjunctive argument type, put null values on direct conjunctive and simple reversed rules. The S-Arg, represent a special case when only one premise is available ($n = 1$). In the following, only the H-Arg will be used since it covers the four types.

6 Application example

On an artificial example (Figure 3) that displays three argument types (C-Arg, D-Arg and H-Arg), we apply our approach in order to see how each type affects the propagation of uncertainty from premises to the overall goal (conclusion). We also apply the quantitative approach presented in [7] on the same example. To compare results from both approaches, we will use the same decision and confidence scales (see Figure 2).

Regarding elicitation, we use the evaluation matrix in Figure 2 to collect expert opinions, and transform them using formulas in Section 3 to get belief and disbelief on premises. Regarding the elicitation of belief weight on rules, we benefit from an observation made on the quantitative models [7]. Indeed, we notice that under some assumptions for the premises, the value of the conclusion is the value of the rule. For instance, assuming full support (resp. positive or negative) on all premises gives the value of the conjunctive rule (resp. direct and reversed): $\gamma_C(C) = \gamma_{\Rightarrow}([\wedge_{i=1}^{n}p_i] \Rightarrow C)$ or $\gamma_C(\neg C) = \gamma_{\Leftarrow}([\wedge_{i=1}^{n}\neg p_i] \Rightarrow \neg C)$. On the other hand, assuming a total support (resp. positive or negative) on one premise (p_i) and total ignorance on the other gives the value of the appropriate

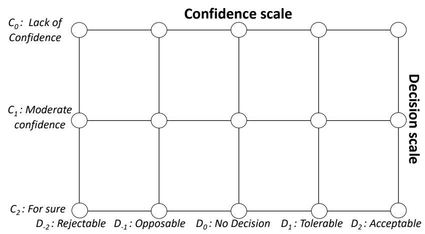

Fig. 2. Evaluation matrix

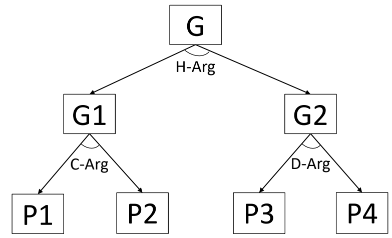

Fig. 3. GSN artificial example

disjunctive rule: $\gamma_C(C) = \gamma^i_{\Rightarrow}(p_i \Rightarrow C)$ or $\gamma_C(\neg C) = \gamma^i_{\Leftarrow}(\neg p_i \Rightarrow \neg C)$. So, we will use the same Table 1 to transform the assessment to rules. However, to avoid the negation of rules, the assessor can only choose between the positive decision (from *"no decision"* to *"acceptable"*) for direct rules; only negative decisions (from *"rejectable"* to *"no decision"*) for the reversed ones. Indeed, rules can only infer uncertainty on one side of the decision scale.

The example in Figure 3, presents a top-goal (G) supported by two sub-goals (G1) and (G2) through a hybrid argument type (H-Arg). Each one of them is also supported, respectively, by two premises. Goal (G1) is supported by the premises (P1) and (P2) related by a conjunctive argument type (C-Arg). On the other hand, goal (G2) is supported by the premises (P3) and (P4) related by a disjunctive argument type (D-Arg). For simplicity, we set all masses on rules to their maximal values (= 1). Then, we use four settings with different premise values and compute the confidence in the top goal.

Table 2. Pairs (decision, confidence) according to both qualitative (Qual.) and quantitative [7] (Quant.) methods for the example (see Fig. 2 for the meaning of symbols)

	Meth.	1^{st}	2^{nd}	3^{th}	4^{th}
P_1	-	$(Opp;C_2)$	$(Tol;C_2)$	$(Tol;C_2)$	$(Opp;C_2)$
P_2	-	$(Tol;C_2)$	$(Tol;C_2)$	$(Tol;C_2)$	$(Opp;C_2)$
P_3	-	$(Tol;C_2)$	$(Tol;C_2)$	$(Tol;C_2)$	$(Opp;C_2)$
P_4	-	$(Tol;C_2)$	$(Opp;C_2)$	$(Tol;C_2)$	$(Opp;C_2)$
G	Quant.	$(ND;C_0)$	$(Tol;C_1)$	$(Tol;C_1)$	$(Opp;C_1)$
	Qual.	$(ND;C_2)$	$(Tol;C_2)$	$(Tol;C_2)$	$(Opp;C_2)$

In general, we can see from Table 2 that both approaches give close results which fit well with our expectations. The only difference is in the confidence values. We can say that, in this case the qualitative approach gives results with higher levels of confidence than the quantitative one.

We notice from Table 2 that the first case gives a "*no decision*". This result is explained by the fact that we end up with two opposite judgments in the H-Arg (conflict situation) due to C-Arg that propagates the premise with least strength (opposable) to G_1 (G_2: tolerable). On the contrary, in the 2^{nd} column, we get a "*tolerable*" decision, because the D-Arg favors the propagation of the premise with the greatest weight (tolerable) to G_2 (G_1: tolerable). In the 3^{th}

and 4^{th} columns we can notice, as expected, that the top goal keeps the same decision as premises respectively: "*tolerable*" and "*opposable*".

The difference in the degree of confidence between qualitative and quantitative approaches is due to the nature of the operations used. For example, the C-Arg favors the propagation of the weakest premise (weaker belief and stronger disbelief). In the quantitative setting, we use the product and the probabilistic sum. And in the qualitative case, we use min and max, which does not model attenuation or reinforcement effects in case of independent pieces of information. This is one limitation of the qualitative approach.

7 Conclusion

In this paper, we propose a qualitative confidence assessment approach. We provide formulas to propagate confidence in GSN from the premises to the top-goal using qualitative mass functions. Each of these functions is collected from experts in the form of a decision and the associated confidence degree, and then transformed into a q-capacity. By sticking to qualitative values, the possible arbitrariness of the transformation of expert opinions into quantitative values (used in some previous approaches) is eliminated. Furthermore, it seems that the qualitative approach gives results similar to the quantitative one in [7]. However, more experiments are needed to confirm this conclusion.

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