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Fault diagnosability utilizing quasi-static and structural modelling

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Abstract

This paper presents a diagnosis model-based method to analyse fault discriminability and assess diagnosability. The technique is based on the state space representation of quasi-static models. Fault diagnosability characterises the faults that can be discriminated using the available sensors in a system. The method can be used to select the minimum set of sensors that guarantee discriminability of an anticipated set of faults. The approach is applied on a two-tanks system benchmark and is compared to a diagnosability analysis method based on structural analysis.

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1. Introduction

Safety, availability, and reliability of processes are among the main objectives in system automatization. These characteristics can be greatly enhanced by the very early diagnosis of changes in component efficiencies. For this reason, diagnosability analysis and methods for locating the required sensors in a plant have gained much industrial interest. There is significant amount of work dealing with this topic in the scientific community [1–5].

To analyse system diagnosability, techniques arising from model-based fault diagnosis can be used. The term *model-based* fault diagnosis refers to the fact that the knowledge about the system is represented in an explicit model.

Two research fields have developed model-based diagnosis independently: the DX community rooted in artificial intelligence (AI) and the FDI community rooted in control. The work of this article is influenced by ideas from both fields. A comparative study of the DX and FDI approaches to model-based diagnosis has been presented in [6].

In the FDI community, faults are modelled as deviations of parameter values or unknown signals and the diagnostic models are often brought back to a residual form. Residual quantities are zero in the absence of faults and each residual acts as an alarm that is expected to trigger to a non-zero value upon the occurrence of some faults, in which case the residual is said to be sensitive to these faults. The expected triggering pattern(s) of a set of residuals under some fault is interpreted as the signature of the fault. Fault isolation is performed by checking the observed residual pattern against

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the different fault signatures [9]. The main approaches to construct residuals are the parity space approach based on Analytical Redundancy Relations (ARRs) [7], and the observer-based approach [8].

In the DX community a plant is assumed to be composed of a set of components that may fail. A diagnostic model describes the behaviour of each component and its interconnections. This model can be used to make predictions about the system behaviour. An inconsistency between the predictions and the observations can be interpreted as a conflict among the set of components whose behavioural model is involved in the inconsistent prediction. All the components in a conflict cannot behave normally. Diagnoses can be computed from conflicts [6].

In this article, a new model-based method to evaluate the degree of diagnosability of a system or equivalently the number of faults that can be discriminated is presented. The method can be used for proposing the (minimal) set of sensors that result in maximal discriminability for the system. The approach gives a new formulation of the FDI problem for systems that can be represented by a quasi-static model (QSM) in state space form. The method is applied to the two-tanks benchmark and compared to the structural model-based diagnosability analysis method of [14].

The paper is organised as follows. Section 2 deals with the definitions of diagnosability. Section 3 introduces diagnosability analysis using QSMs. Diagnosability analysis following a structural approach is considered in Section 4. Section 5 presents the QSM method applied to the two-tanks system and the comparison to the structural analysis method. Finally, some conclusions end the paper in Section 6.

2. Diagnosability and sensor location concept

Following the definitions given by [7], a dynamical system, defined by a system model with a set of measured variables (Y), input variables (U) and a set of disturbances (N) is subjected to some faults (F) (see Fig. 1).

The set of observed variables is given by: $Z = \{U, Y\}$, which consists of the temporal sequences of input and output values at discrete time points k within a given time horizon k_h :

$$U = \{u(0), u(1), \dots, u(k_h)\}$$

$$Y = \{y(0), y(1), \dots, y(k_h)\}$$

The unknown variables considered are internal variables, X , and perturbations, N .

A fault in a dynamic system is an alteration of the system structure or the system parameter values from the nominal situation [1]. A fault may manifest as an unexpected deviation from the normal behaviour of one or more components of the system. Two kinds of faults are considered depending on how they affect the model behaviour:

- Multiplicative faults: system parameters take values different from nominal ones;
- Additive faults: unknown variables act in an additive way on the sensors and actuators.

For a given system model that describes the normal behaviour of the system subjected to faults $F = \{f_1, f_2, \dots, f_{nf}\}$, the objective of diagnosis algorithms is to identify the set(s) of faults $F_j \subseteq F$ that explain(s) the unexpected behaviour of the system. Diagnosis algorithms for continuous variable systems generally consist of two steps:

- Fault detection, which decides whether or not a fault has occurred.
- Fault isolation, which localises the faulty component(s).

This is conditioned by the diagnosability properties of the system, which define whether or not faults are discriminable. Given the set of observed variables $Z = \{z_1, \dots, z_{nz}\}$ the set of all the possible value tuples for Z under the fault f_i is defined as OBS_{f_i} . Then two faults f_i and f_j , $f_i \neq f_j$, are said to be (weakly) discriminable¹ if and only if $\text{OBS}_{f_i} \neq \text{OBS}_{f_j}$.

In [11,14] the *Diagnosability Degree* of an instrumented system is characterized by the quotient of the number of discriminable faults by the number of faults in F , $\text{Card}(F)$.

Definitions 1 and 2, already present in the literature, will be useful through the paper.

Definition 1 (Full Diagnosability). A system is fully diagnosable if all possible hypothesized single faults F are discriminable.

¹ Strong discriminability requires $\text{OBS}_{f_i} \cap \text{OBS}_{f_j} = \emptyset$.

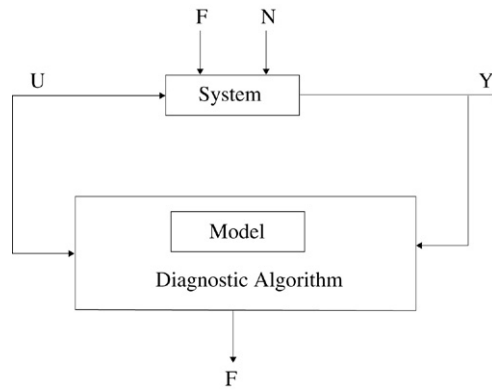


Fig. 1. Fault diagnostic.

In other words, for a given set of observed variables Z , a system is fully diagnosable if $OBS_{f_i} \neq OBS_{f_j}, f_i \neq f_j$. The sensor placement problem searches for the subset of unknown variables, $X^* \subset X$, such that $X^* \cup Z_a$, makes the system fully diagnosable. Z_a is the set of current known variables.

Definition 2 (Minimum Sensor Placement). A Minimal Additional Sensor Set (MASS) is defined as a minimum set of variables X^* whose observation turns the system fully diagnosable.

Since X^* has not necessarily only one solution, a minimum cost function system-sensor can be included in order to select the best MASS X^* .

3. Diagnosability analysis using quasi-static modelling

3.1. Quasi-static model (QSM)

Let us have a linear state space representation of a discrete system. Difference equations for such systems are:

$$\begin{aligned} X(k + 1) &= AX(k) + BU(k) \\ Y(k) &= CX(k) + DU(k) \end{aligned} \tag{3.1}$$

In control theory, a complex system can be modelled by mixed dynamic and algebraic equations system. Quasi-static models are a state space representation that includes both types of models, dynamic and algebraic. The dynamic one is for slow variables and the algebraic one for fast ones.²

Quasi-static models are often used in water networks [13] where reservoirs have a time constant much higher than the sampling period and are modelled with discrete dynamic system. On the other hand, pipes, pumps and valves are considered through algebraic equations.

The general representation of a quasi-static model is an extension of state space model (3.1). The set of internal variables $X \in \mathfrak{R}^n$ is partitioned into $X_1 \in \mathfrak{R}^{n_1}$, the set of dynamic variables, and $X_2 \in \mathfrak{R}^{n_2}$, the set of static variables. The set of observed variables is composed of $U \in \mathfrak{R}^{m_u}$, the set of inputs and $Y \in \mathfrak{R}^{m_y}$, the set of outputs. The set of observed variables Y is partitioned into dynamic observed variables $Y_1 \in \mathfrak{R}^{m_{y_1}}$ and static observed ones $Y_2 \in \mathfrak{R}^{m_{y_2}}$, $Y = Y_1 \cup Y_2$. QSM is then represented by:

$$\begin{aligned} X_1(k + 1) &= \begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + B_1 U(k) \\ \begin{bmatrix} 0_{(n_2 \times 1)} \\ Y_1(k) \\ Y_2(k) \end{bmatrix} &= \begin{bmatrix} A_{21} & A_{22} \\ C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + \begin{bmatrix} B_2 \\ D_1 \\ D_2 \end{bmatrix} U(k) \end{aligned} \tag{3.2}$$

² Slow and fast variables are defined by comparing time constants and sampling period.

where $A_{11} \in \mathfrak{R}^{n_1 \times n_1}$, $A_{12} \in \mathfrak{R}^{n_1 \times n_2}$ and $B_1 \in \mathfrak{R}^{n_1 \times nu}$ are the matrices corresponding to discrete dynamic equations, $A_{21} \in \mathfrak{R}^{n_2 \times n_1}$, $A_{22} \in \mathfrak{R}^{n_2 \times n_2}$ and $B_2 \in \mathfrak{R}^{n_2 \times nu}$ are the matrices of algebraic equations. $C_{11} \in \mathfrak{R}^{ny_1 \times n_1}$, $C_{21} \in \mathfrak{R}^{ny_2 \times n_1}$, $C_{12} \in \mathfrak{R}^{ny_1 \times n_2}$, $C_{22} \in \mathfrak{R}^{ny_2 \times n_2}$, $D_1 \in \mathfrak{R}^{ny_1 \times nu}$ and $D_2 \in \mathfrak{R}^{ny_2 \times nu}$ correspond to observation equations. The vector $0_{(n_2 \times 1)}$ represents the residual of algebraic equations.

When the whole set of observed variables $Z = Y \cup U$ is used the new formulation of the QSM (3.2) is:

$$\begin{aligned} X_1(k+1) &= \tilde{A} \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + \tilde{B}U(k) \\ \begin{bmatrix} 0_{(n_2 \times 1)} \\ Z(k) \end{bmatrix} &= \tilde{C} \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + \tilde{D}U(k) \end{aligned} \tag{3.3}$$

where $\tilde{A} = \begin{bmatrix} A_{11} & A_{12} \end{bmatrix}$, $\tilde{B} = B_1$, $\tilde{C} = \begin{bmatrix} A_{21} & A_{22} \\ C_{11} & C_{12} \\ C_{21} & C_{22} \\ 0_{(nu \times n_1)} & 0_{(nu \times n_2)} \end{bmatrix}$ and $\tilde{D} = \begin{bmatrix} B_2 \\ D_1 \\ D_2 \\ Inu \end{bmatrix}$.

The extended system realisation matrix is defined as:

$$M = \begin{bmatrix} \tilde{A} & \tilde{B} & -A_{11} \\ \tilde{C} & \tilde{D} & 0 \\ 0_{(ny_1 \times n)} & 0_{(ny_1 \times nu)} & C_{11} \end{bmatrix} \tag{3.4}$$

where M links the extended observations vector, $\mathcal{Z}^T = \begin{bmatrix} 0_{(nx_1)}^T & Z(k)^T & Y_1(k+1)^T \end{bmatrix}$ with the extended variables vector $\mathcal{V}^T = \begin{bmatrix} X_1(k)^T & X_2(k)^T & U(k)^T & X_1(k+1)^T \end{bmatrix}$. The system of equations represented by M will be determined if and only if M is full-rank. If the system is determined matrix M can be inverted and hence the system can be solved:

$$\mathcal{V} = M^{-1} \mathcal{Z}. \tag{3.5}$$

This condition is equivalent to:

$$\text{rank}(M) = \text{card}(\mathcal{V}). \tag{3.6}$$

3.2. Diagnosability analysis using quasi-static models

Using the QSM of a system, as given by Eq. (3.4), fault diagnosability analysis is a classical linear algebraic problem.

In general faults are considered as new state variables. Additive faults are unknown variables ($F \in \mathfrak{R}^{nf}$) that are not constant but their dynamics are unknown. The faults are assumed to be static variables. Model (3.4) can be generalized to dynamic faults if some knowledge about their dynamics is known. Multiplicative faults result in a non-linear model that has to be linearised.

$$\begin{aligned} X_1(k+1) &= \begin{bmatrix} \tilde{A} & \tilde{A}_F \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \\ F(k) \end{bmatrix} + \tilde{B}U(k) \\ \begin{bmatrix} 0_{(n_2 \times 1)} \\ Z(k) \end{bmatrix} &= \begin{bmatrix} \tilde{C} & \tilde{C}_F \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \\ F(k) \end{bmatrix} + \tilde{D}U(k). \end{aligned} \tag{3.7}$$

The new matrix $\tilde{A}_F \in \mathfrak{R}^{n_1 \times nf}$ represents the effect of faults on dynamic variables, $\tilde{C}_F \in \mathfrak{R}^{(n_2+ny+nu) \times nf}$ is the matrix corresponding to the effect of faults on static variables and observations. Matrix \tilde{C}_F can be decomposed in $\tilde{C}_F^T = \begin{bmatrix} A_{2F}^T & C_{1F}^T & C_{2F}^T & C_{UF}^T \end{bmatrix}$ where $A_{2F} \in \mathfrak{R}^{n_2 \times nf}$ is the matrix corresponding to the faults in the algebraic equations. $C_{1F} \in \mathfrak{R}^{n_1 \times nf}$, $C_{2F} \in \mathfrak{R}^{n_2 \times nf}$ and $C_{UF} \in \mathfrak{R}^{nu \times nf}$ represents the effect of faults on observations Y_1 , Y_2 and inputs U respectively.

Defining M_F as $M_F^T = [\tilde{A}_F^T \quad \tilde{C}_F^T \quad C_{1F}^T]$, the extended system realisation matrix where faults are included is given by:

$$\mathcal{M} = [M \quad M_F]. \quad (3.8)$$

The equation system represented by \mathcal{M} is determined if and only if \mathcal{M} is full-rank. If the system is determined, matrix \mathcal{M} can be inverted and hence the system can be solved:

$$\begin{bmatrix} \mathcal{V} \\ F \end{bmatrix} = \mathcal{M}^{-1} \mathcal{Z}. \quad (3.9)$$

Hence all variables can be estimated and consequently the faults can be discriminated and identified. This condition is equivalent to:

$$\text{rank}(\mathcal{M}) = \text{card}(\mathcal{V}) + \text{card}(F) \quad (3.10)$$

where $F = \{f_i\}, i = 1, \dots, n_f$, is the set of faults.

Note that this condition guarantees not only full diagnosability for single faults but also full diagnosability for multiple faults.

Assuming that M in (3.4) is full-rank and considering only single faults diagnosability, the analysis of Eq. (3.10) can be carried out for each pair of possible faults. If \mathcal{M} in (3.8) is full-rank for all pairs of faults, the diagnosis system is able to decide which fault is present.

\mathcal{M}_{ij} denotes the extended system realisation for the pair of faults f_i and f_j . If \mathcal{M}_{ij} is full-rank for all $\{f_i, f_j\} \in F$, that means that if a single fault f_k is present, the system equations corresponding to \mathcal{M}_{ik} :

$$\mathcal{Z}_{ik}(k) = \mathcal{M}_{ik} \begin{bmatrix} \mathcal{V}(k) \\ F_{ik}(k) \end{bmatrix} \quad (3.11)$$

gives a unique solution where $f_i = 0$ and $f_k \neq 0$. For all remaining \mathcal{M}_{ij} where $i, j \neq k$ the possible solutions can be:

1. both $f_i \neq 0$ and $f_j \neq 0$ and it is impossible, hence faults are assumed to be single.
2. $f_i = 0$ and $f_j \neq 0$ that means that $f_j = f_k$ because all \mathcal{M}_{ij} are full-rank and a fault that implies the same changes in all state variables is the same fault.

3.3. Sensor placement using quasi-static models

In this section, singular value decomposition of matrix \mathcal{M} is used to solve the sensor placement problem [13]. When matrix \mathcal{M} loses rank, a zero singular value is present. Singular value decomposition gives a base in the variable space $X \cup U \cup F$ in which one direction is related with the minimal singular value.

$$\mathcal{M} = \Psi \Sigma \Upsilon^T. \quad (3.12)$$

Eq. (3.12) corresponds to singular value decomposition, where Σ is a diagonal matrix of the same size as \mathcal{M} . Its diagonal contains singular values, σ_l , in diminishing order, Υ and Ψ are square matrices and their columns form a base of the input and output vectorial space respectively. Each σ_l is associated to an input direction, γ_l , and an output direction, ψ_l . σ_l is the gain between γ_l and ψ_l corresponding to matrix M .

The direction in the input space associated to a minimal singular value shows which variables are poorly related with the output space. It indicates which measurements should be introduced to improve diagnosability. Each pair of faults (f_i, f_j) gives a subset $X^{i,j} \subset X$. Once this study has been done for all pairs of possible faults, sensors that have to be introduced are given by the union of all sets obtained for the different pairs:

$$X^* = \cup_{i,j} \{X^{i,j}\}. \quad (3.13)$$

Then the set of observed variables is given by:

$$Z = X^* \cup Z_a. \quad (3.14)$$

This method allows one to place sensors in an optimal way based on the numerical behaviour showed by minimal singular values.

4. Diagnosability analysis using a structural model

4.1. Structural model

A structural model is an abstraction of the behaviour model in which every structural relation only captures information about which variables are involved in the relation [10].

The behaviour model of a system can be defined as a pair (E, V) where V is a set of variables, and E is a set of equations or relations. The relations E may be expressed in several different forms as algebraic and differential equations, difference equations, rules etc. The set of variables V can be partitioned as $V = X \cup Z$, where Z is the set of observed variables and X is the set of unknown variables.

In a component-oriented-model, these relations are associated to the system's physical components, including sensors.

The abstracted structural model can be represented by an *Incidence Matrix* which crosses model relations in rows and model variables in columns: an entry e_{ij} of the matrix is 1 when variable $v_j \in V$ appears in relation $e_i \in E$, and 0 otherwise.

4.2. Diagnosability using structural analysis (SA)

Once the structural model of a system is derived, it can be used to search for the *analytical redundancies*, concretized by the so-called Analytical Redundancy Relations (ARRs). ARRs are relations that only contain observed variables and can hence be evaluated from the observations [10].

An ARR is obtained from a redundant relation of the model, and from the relations used to solve for the unknown variables involved in the redundant relation. Each ARR can be put in the form $r = 0$ and gives rise to a residual r . Residual quantities are zero in the absence of faults and each residual acts as an alarm that is expected to trigger to a non-zero value upon the occurrence of some faults, in which case the residual is said to be sensitive to these faults (this may depend on the amplitude of the fault). The set of faults that affect an ARR is called the *ARR sensitivity fault set*. The expected triggering pattern(s) of a set of residuals $\{r_1, \dots, r_{nr}\}$ under some fault f hence provides an abstracted observation tuple OBS_f^r , or a set of possible abstracted observation tuples in the case of a multiple mode system. The signature of the fault f can be defined as $\text{Sig}(f) = \text{OBS}_f^r$ while OBS_f^r equivalently represents the original observation subspace under the fault f , OBS_f [14]. The fact that a residual r is expected or not to be sensitive to a fault can be determined from the structure of the model, and so the fault signatures can be determined.

Fault signatures are summarized in the so-called *Fault Signature Matrix* (FSM) which crosses ARRs in rows and anticipated faults in columns. An entry e_{ij} to "1" represents that ARR_i is sensitive to fault f_j and it is expected to be triggered to a non-zero value under the occurrence of this fault.

The diagnosability properties of a system depend on fault signatures [14]. In particular, a system is strongly diagnosable if and only if for all $f_i, f_j, f_i \neq f_j$, we have $\text{Sig}(f_i) \cap \text{Sig}(f_j) = \emptyset$ [14,15].

Ref. [14] presents an algorithm to generate the set of all possible structural ARRs in all different conditions of available sensors and in the case when the modelled system has multiple operational modes. Each so-called Hypothetic (structural) ARR, the so-called H-ARR, is labelled by the set of sensors required to actually obtain this H-ARR. This set of sensors is also called the *sensor-support* of the H-ARR. The starting point of the method hypothesizes that all the variables are sensed and generates the corresponding set of H-ARRs that are in this case directly issued from the structural model relations. The H-ARR generation algorithm then removes sensors one by one and combines H-ARRs consequently (removing one sensor comes back to eliminating the H-ARRs involving the corresponding observed variable and adding the combined H-ARRs obtained by substituting this variable). First a sensor $S(x_i)$ corresponding to the observation relation $z_i = e_i(x_i)$ is selected, where $x_i \in X$ and $z_i \in Z$, and a set J of H-ARR's indexes is defined as the set of H-ARR in which z_i is involved. Then the pairs of H-ARRs to be combined are taken in J . Obviously, another sensor is chosen if the cardinality of J is equal to 1.

Two H-ARR selected in the set J can be combined under several conditions exhibited in [14]. If all the conditions are fulfilled, then a new H-ARR with its corresponding sensor-support can be generated. The whole set of H-ARRs is

used to fill the *Hypothetical Fault Signature Matrix* (HFSM) which crosses H-ARRs in rows and anticipated faults in columns.

4.3. Sensor placement using structural analysis

The problem of determining the minimal set of sensors that guarantee a specified level of diagnosability is approached in [14], based on the HFSM. A procedure for determining the MASSs (Minimal Additional Sensor Sets) is provided. It is based on an exhaustive search of all the alternative fault signature matrices, i.e. submatrices of the HFSM that correspond to all the alternative possible sensor sets. Some results providing bounds for the minimum number of ARR's needed to discriminate a set of faults F and the minimum number of sensors required are proposed to restrict the search.

Following this work, [16] formulates the optimisation problem of determining the MASS that achieves a given diagnosability level and optimises a cost criterion. It proposes a method to solve this problem using an evolutionary approach. Different combinations of sensors are codified in the chromosomes of a first population, and then a genetic algorithm searches for the most advantageous ones in terms of diagnosability degree over cost ratio.

5. Application

The QSM and SA methodologies are applied to a real laboratory system, PCS4, provided by Festo [12].

5.1. Process description

The system is made up of two tanks interconnected by a pump and a valve (Fig. 2). There is only one level sensor in the top tank (LT).

Flow in the pump, q_p , depends on both levels, h_u and h_l , and control input to the pump, u_1 . Flow in the valve, q_v , depends on top level, h_u , and control input to the valve, u_2 .

Eq. (5.1) shows the non-linear mathematical equations (E) describing the process behaviour.

$$\begin{aligned}
 e_1 : \dot{h}_u &= \frac{1}{S_u}(q_p - q_v) \\
 e_2 : \dot{h}_l &= \frac{1}{S_l}(q_v - q_p) \\
 e_3 : q_p &= \begin{cases} c_1 f(u_1) \cdot \sqrt{|h_u - h_l|} & \text{if } |h_u - h_l| > 0 \text{ and } f(u_1) > 0 \\ 0 & \text{else,} \end{cases} \\
 e_4 : q_v &= \begin{cases} c_2 \cdot f(u_2) \sqrt{h_u} & \text{if } h_u > 0 \text{ and } f(u_2) > 0 \\ 0 & \text{else,} \end{cases}
 \end{aligned} \tag{5.1}$$

where S_u and S_l are the sections of the upper and lower tank, respectively. The equations given above include differential and algebraic equations.

Hypothetical faults considered are: the sensor gives a wrong reading of the level; mis-function of the pump or the valve (wrong flow); leaks in any of the tanks. Table 1 summarises these hypothetical faults.

Table 1
Hypothetical faults considered

Fault	Symbol
Sensor level fault	f_1
Pump fault	f_2
Valve fault	f_3
Up tank leak	f_4
Low tank leak	f_5

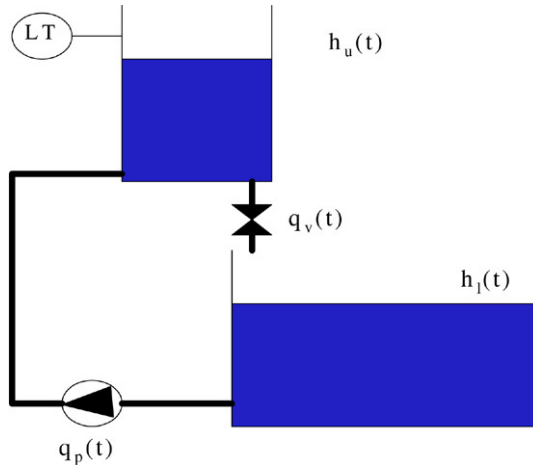


Fig. 2. Two-tanks system.

5.2. Quasi-static model for the two-tanks system

For the two-tanks system we have the following discrete time equations:

$$\begin{aligned}
 h_u(k+1) &= h_u(k) + \frac{(q_p(k) - q_v(k)) \Delta t}{S_u} \\
 h_l(k+1) &= h_l(k) + \frac{(q_v(k) - q_p(k)) \Delta t}{S_l} \\
 q_p(k) &= a_1 h_u(k) + a_2 h_l(k) + a_3 u_1(k) \\
 q_v(k) &= b_1 h_u(k) + b_2 u_2(k)
 \end{aligned}
 \tag{5.2}$$

where slow variables are $X_1 = \{h_u, h_l\}$ and fast variables are $X_2 = \{q_p, q_v\}$. (5.2) is obtained by linearising (5.1), where $a_i \{i = 1, \dots, 3\}$ and $b_j, \{j = 1, 2\}$ are known parameters and Δt represents the sampling period. Discretisation induces the presence of two instances of the same slow variables X_1 in the same snapshot.

The set of Eq. (5.2) is modified in order to introduce the effect of faults (Table 1). Four new equations are included, corresponding to measurements and inputs that are known. Since the level of the upper tank variable, which is the only one measured, appears twice in the same snapshot, it generates two observation equations in which faults are assumed to be static variables.

$$\begin{aligned}
 h_u(k+1) &= h_u(k) + \frac{(q_p(k) - q_v(k) - f_4(k)) \Delta t}{S_u} \\
 h_l(k+1) &= h_l(k) + \frac{(q_v(k) - q_p(k) - f_5(k)) \Delta t}{S_l} \\
 q_p(k) &= a_1 h_u(k) + a_2 h_l(k) + a_3 u_1(k) + f_2(k) \\
 q_v(k) &= b_1 h_u(k) + b_2 u_2(k) + f_3(k) \\
 h_{um}(k) &= h_u(k) + f_1(k) \\
 u_{1m}(k) &= u_1(k) \\
 u_{2m}(k) &= u_1(k) \\
 h_{um}(k+1) &= h_u(k+1) + f_1(k).
 \end{aligned}
 \tag{5.3}$$

In Eq. (5.4), the extended system realisation matrix is used in the case of two possible faults: f_1 and f_2 . It is equivalent to Eq. (5.3) restricted to f_1 and f_2 .

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ h_{um}(k) \\ u_{1m}(k) \\ u_{2m}(k) \\ h_{um}(k+1) \end{bmatrix} = \begin{bmatrix} S_u & 0 & \Delta t & -\Delta t & 0 & 0 & -S_u & 0 & 0 & 0 \\ 0 & S_l & -\Delta t & \Delta t & 0 & 0 & 0 & -S_l & 0 & 0 \\ a_1 & a_2 & -1 & 0 & a_3 & 0 & 0 & 0 & 0 & 1 \\ b_1 & 0 & 0 & -1 & 0 & b_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} h_u(k) \\ h_l(k) \\ q_p(k) \\ q_v(k) \\ u_1(k) \\ u_2(k) \\ h_u(k+1) \\ h_l(k+1) \\ f_1(k) \\ f_2(k) \end{bmatrix}. \tag{5.4}$$

In this case, the extended system realisation matrix \mathcal{M}_{12} is of rank 8 because there are only 8 equations.

Applying the methodology described in Section 3.3, Σ has two null singular values. These null singular values correspond to the last two vectors of the input vectorial space base, \mathcal{Y} .

$$\gamma_{9,10}^T = \begin{bmatrix} 0.57 & 0.00 & 0.06 & 0.06 & -0.57 & -0.06 & 0.57 & 0.00 & 0.00 & -0.00 \\ 0.00 & 0.71 & 0.00 & 0.00 & -0.00 & -0.03 & 0.00 & 0.71 & -0.00 & 0.00 \end{bmatrix}.$$

Each value of $\gamma_{9,10}$ corresponds to one of the variables in $[\mathcal{Y}(k)^T \ F(k)^T]^T$. Considering only unobserved variables, $\{h_l, q_p, q_v\}$, the maximum values are present in the second and eighth components of the vector γ_{10} . These components correspond to h_l .

If such sensor is introduced the system improves diagnosability but it is not full-rank yet, with a minimal singular value of:

$$\sigma = 4.47e^{-15}.$$

The new direction related to this minimal singular value is:

$$\gamma_{10}^T = [-0.57 \quad -0.00 \quad -0.06 \quad -0.06 \quad 0.57 \quad -0.06 \quad -0.57 \quad 0.00 \quad -0.00 \quad 0.00].$$

The new maximal components, third and fourth, point out exactly the only unobserved variables left, q_p and q_v . Introducing a sensor for one of these variables makes the system diagnosable. From the numerical point of view, placing a sensor in q_v is better. This can be seen by comparing minimal singular values for each configuration:

$$\sigma_{h_l \wedge q_v} = 0.0071; \quad \sigma_{h_l \wedge q_p} = 0.0006.$$

Thus the variables subset for this pair of faults can be:

$$X^{1,2} = \{h_l, q_v\} \quad \text{or} \quad X^{1,2} = \{h_l, q_p\}.$$

The same method is applied for each pair of faults. Table 2 presents the results with all combinations related to the pump fault, f_2 .

Variables are ordered by numerical influence and bold symbols indicate the selected sensors. The same analysis has been performed for the six combinations left, leading to the conclusion that two sensors are enough to make all faults discriminable.

In the example, two possible sets are obtained for X^* :

$$\begin{aligned} X_1^* &= \{h_l, q_v\} \\ X_2^* &= \{h_l, q_p\}. \end{aligned}$$

Table 2
Results of system diagnosability using QSM

Faults to discern	Logical formula indicating sensors to add
$f_1 \ f_2$	$\mathbf{h_l} \wedge (q_p \vee \mathbf{q_v})$
$f_3 \ f_2$	$\mathbf{h_l} \wedge (q_p \vee \mathbf{q_v})$
$f_4 \ f_2$	$\mathbf{h_l}$
$f_5 \ f_2$	$\mathbf{h_l}$

Table 3
Extended incidence matrix

	Known variables			Unknown variables						Faults				
	u_{1m}	u_{2m}	h_{um}	h_l	h_u	q_v	q_p	u_1	u_2	f_1	f_2	f_3	f_4	f_5
e_1					1	1	1						1	
e_2				1		1	1							1
e_3				1	1		1	1			1			
e_4					1	1			1			1		
e_5			1		1					1				
e_6	1							1						
e_7		1							1					

Table 4
Hypothetical analytical redundant relations with h_l and q_p as known variables

	Known variables			Hypothetically known		Faults HFSM				
	u_{1m}	u_{2m}	h_{um}	h_l	q_p	f_1	f_2	f_3	f_4	f_5
ARR₉	1			1	1		1		1	1
ARR₁₃		1	1		1	1		1	1	
ARR₁₄	1	1	1	1	1	1	1	1	1	
ARR₁₅		1		1	1			1	1	1
ARR₁₆		1	1	1		1		1	1	1
ARR₁₇	1	1	1			1	1	1	1	1

The second combination X_2^* is worse from a numerical point of view. However, the decision about which sensors to introduce may also depend on the cost of each sensor.

5.3. Structural model for the two-tanks system

The methodology described in [14] is now used with the objective to check the results.

For the case studied, the set of variables and relations are: $V = \{h_u, h_l, q_v, q_p, u_1, u_2, h_{um}, u_{1m}, u_{2m}\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$. The equations e_5, e_6 and e_7 correspond to sensor relations:

$$\begin{aligned}
 e_5 : h_{um} &= h_u \\
 e_6 : u_{1m} &= u_1 \\
 e_7 : u_{2m} &= u_2
 \end{aligned}
 \tag{5.5}$$

where h_{um}, u_{1m} and u_{2m} are measured variables.

The extended incidence matrix, including the set of hypothetical faults considered above $F = \{f_1, f_2, f_3, f_4, f_5\}$, is given in Table 3.

The procedure described in 4.2 is applied to obtain the whole set of H-ARRs and fills the *Hypothetical Fault Signature Matrix* (HFSM). Table 4 presents a sub-matrix of the HFSM which includes $\{h_l, q_p\}$ as hypothetical known variables.

The FSM is given by the fault columns (grey colour in Table 4). Faults are discriminable if the rank of FSM is equal to the number of faults [11,14]:

$$\text{rank}(\text{FSM}) = \text{card}(F).
 \tag{5.6}$$

We consider the incorporation of two sensors only: pump flow q_p , and level of the lower tank h_l . In this case $\text{rank}(\text{FSM}) = 5$ so the five faults are discriminable.

Applying the same procedure for all possible combinations of hypothetical variables, the minimal sets of sensors that guarantee full diagnosability are:

$$\begin{aligned}
 X_1^* &= \{h_l, q_v\} \\
 X_2^* &= \{h_l, q_p\}.
 \end{aligned}$$

6. Conclusion

In this paper a model-based approach for fault diagnosability has been presented. The method is based on a state space representation of quasi-static models and is compared to a structural model-based approach. The QSM methodology has been illustrated through its application on a two-tanks system. The case of single fault has been considered and the approach gives the same results as the SA method.

In QSM, an iterative algorithm is needed to achieve the optimal sensor location for full diagnosability. The QSM approach needs more information than the SA approach. It is based on a linearised model and the knowledge of how the considered anticipated faults affects this model.

The QSM method, on the other hand, provides more information. Different faults on the same component can be discriminated and it also indicates which subset of variables is most numerically relevant for fault diagnosability.

The comparative study developed in this article and the obtained results open perspectives based on the integration of both approaches which may advantageously reduce computational complexity.

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