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On Graphs, Geometry, Motion and Turbulences in Leonardo's Virgin and Child with Saint Anne

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Abstract: In this study, we focus on the composition of the Virgin and Child with Saint Anne by Leonardo da Vinci. The composition is considered first from the point of view of graphs and the genealogy of the Christ is analyzed accordingly. Then, the composition of the masterpiece is studied from the perspective of scientific knowledges in geometry. The many ellipses or part of ellipses in the composition are detailed. This study permits us to revisit the movement of the figures. Knowledges on celestial mechanics at the time of Leonardo are presented and themes like destiny and resurrection are examined consequently. Finally, the representation of instabilities and turbulences is considered in the painting and the renewal of forms is examined.

Keywords: Virgin and Child with Saint Anne; Saint Anne Trinitarian; Leonardo da Vinci; Renaissance painting; Graphs; Geometry; Conic sections; Celestial mechanics; Turbulences.

I Introduction

Many questions remain unanswered about Leonardo's Virgin and Child with Saint Anne in Le Louvre (Fig. 1) (da Vinci, ca. 1503-1519). So far, no document has been discovered attesting to a given commission for that specific painting and one does not know precisely when Leonardo started to paint the 168.4 cm high and 113 cm wide oil on poplar that was later enlarged to 126.3 cm.

On April 3, 1501, Fra Pietro da Novellara wrote a letter from Florence to Isabella d'Este in which he makes a description of Leonardo's sketch in a cartoon on the Saint Anne Trinitarian

theme including a lamb: “A facto solo dopoi che è ad Firenci un schizo in uno cartone; finge uno Christo bambino de età cerca uno anno, que uscindo quasi de bracci al la mamma piglia uno agnello et pare che lo stringa”.

The panel in Le Louvre is placed in the period from around 1501 to 1519 by scholars and experts. It is dated to circa 1501–1517 by Villata (2015) who claims that it is derived from the sketch described by Fra Pietro da Novellara. Nevertheless, Fra Pietro’s description of the cartoon does not match exactly the painting, nor the “spolvero” dots discovered in 2011-2012 by a French scientific team (Delieuvin, 2012b; Delieuvin, 2019, pp. 281-282; Barone, 2013, p. 747). In particular, Saint Anne is slightly more static in the masterpiece than in Fra Pietro’s description. In 1968, Pedretti dated the painting to ca. 1508-1509. Delieuvin dates the painting to circa 1503 – 1519 (2012d, pp. 205-208). He positions the work in the political context that prevailed in Florence by that time; the masterpiece could be Leonardo’s personal tribute to the recently renovated Republic (Delieuvin, 2019, pp. 272-273). The grandmother of Christ was venerated in the city; Florence having gained its freedom in 1343 on Saint Anne’s day. An unfinished painting on the Saint Anne Trinitarian theme was seen in Leonardo’s workshop by Agostino Vespucci, clerk of the Republic of Florence, as attested in a personal note, dated October 1503, in his copy of Cicero’s “*Epistulae ad Familiares*” incunabula. This note tends to prove that Leonardo started to paint a work on this theme in 1503 or even a little earlier. Leonardo da Vinci made many sketches for preparing one or several paintings on Saint Anne Trinitarian theme (Arasse, 1997, p. 496; Delieuvin, 2012a).

The purpose of the paper is to analyze the composition of Le Louvre painting from the point of view of mathematics. Graphs, geometry, instabilities and turbulences are considered. We show how the treatment of themes such as motion, destiny and resurrection can be revisited accordingly.

The composition of Le Louvre painting is investigated from the point of view of graphs in Section II; the ancestry of the Christ is considered accordingly. In section III, the panel is studied from the perspective of geometry. We focus on conic sections and we detail ellipses or part of ellipses in the composition. Subsequently, we analyze the motion of characters. We discuss also scientific knowledge of celestial mechanics at the time of Leonardo and examine accordingly the possible cause for the omnipresent rotations in the painting. Section IV deals with hydrology, motion, instabilities and turbulences and their impact on the painting. Conclusions are presented in Section V.

II Graph, Tree and Ancestry

The composition of Virgin and Child with Saint Anne in Le Louvre is complex (Fig. 1). It has been studied for a long time (Arasse, 1997, pp. 455-456; Kemp 2019, p. 496). It combines a pyramidal or triangular construction whose right side concentrates the action with many ellipses. The landscape is also an important element of the painting and more particularly the chasm in the foreground, the tree in the middle ground and mountain range, waterfalls and rivers in the background.



Fig. 1. Da Vinci, Leonardo. *Virgin and Child with Saint Anne*, oil on wood, Le Louvre, Paris.

The characters are organized in two distinct couples, namely the Saint Anne and Virgin couple, in the center of the panel and the Child and lamb couple, to the right. This process introduces some elements of symmetry that contribute to structuring the composition, especially with regard to the arrangement of the heads of the characters. To the symmetry of roles of the two mothers with miraculous succession according to the tradition, responds that of the two creatures called to sacrifice. These elements of symmetry greatly participate to the harmony of the panel.

Beyond symmetry, Leonardo's painting features superposition of figures. In each couple, the bodies of the characters are somewhat entangled, e.g., the right arm of the Virgin seems to be an extension of Saint Anne's right shoulder just as the right front paw of the lamb seems to be an extension of the right thigh of the Child. The natural superposition of characters is uncommon and virtuoso.

The movements of the figures seem opposed. The Virgin, who is seated on Saint Anne's lap, leans as the Child prepares to straddle the lamb. She gently tries to draw the Child towards her in order to prevent him from riding the lamb. It highlights the movement of the figures. Nevertheless, the scene is imbued with life, spontaneity, simplicity and grace. It could be the representation of a moment of family rejoicing, if it was not known that Saint Anne had died at the birth of Christ, according to tradition.

The Child apparently wants to be one with the lamb. He grasps the lamb with dynamic and elegant elliptical gestures. Saint Anne appears to enjoy the Child's game. The Virgin pays more attention to the Child. She looks at him as if foreseeing his fate and holds him carefully with elliptical gestures similar to that of the Child in order to prevent him from falling. Leonardo depicted the idealized and graceful faces of the two women by softly rendering their loving and tenderness expressions.

The composition places the eyes of the figures almost on the same diagonal line that goes from heaven to the Child. The composition, organized along the linear structure, corresponds to a simple directed graph that displays the part of the genealogical line of the Christ with divine ascendance according to the tradition, i.e., the "*Humanissima Trinitas*": Saint Anne, the Virgin and the Child. The diagonal line and the vertical line formed by the trunk of the tree on the right side of the picture converge above the lamb.

The representation of branches and leaves of the tree is somewhat similar to that of the juniper tree in Leonardo's portrait of Ginevra de' Benci, ca. 1475-1476. It is a very old juniper tree.

The symbolism of the juniper tree is particularly important in the Christianity, where it is associated with purification. The tree could evocate also another graph, i.e., the genealogical tree of the Christ: the Jesse tree. We recall that according to the tradition, the Jesse tree is a representation of the genealogy of the Christ that is based on the prophecy of Isaiah (Lepape, 2009). The root of this tree is David's father: Jesse. Le Louvre masterpiece can appear as the synthesis of the genealogical line of the Christ including an allusion to its Passion and a message of purification.

Nevertheless, the panel is not limited to a representation of the genealogy of Christ which is based on graph depiction. Thanks to his talent, Leonardo also emphasizes several patterns that rely on mathematical concepts and more or less complex physical phenomena. In the next section, we study ubiquitous movement in Le Louvre painting in light of scientific results in geometry and models of celestial mechanics in use in Leonardo's time.

III Geometry, Motion and Celestial Mechanics

In this section, we focus on the many ellipses in the composition of Le Louvre painting and examine how geometry influenced Leonardo da Vinci in the depiction of movement of the characters. We also analyze the link with the scientific theory in celestial mechanics used at the time of Leonardo, *i.e.*, the Ptolemaic system.

The letter of Fra Pietro da Novellara to Isabella d'Este (1501) about an original sketch in a cartoon related to the Saint Anne Trinitarian theme points out that all characters are in motion. The swirling motion of the Virgin and the Child was noticed in the Burlington House Cartoon as well as in preparatory studies (Arasse, 1997, p. 310). Paul Klee underlines also that figures in Le Louvre painting can only be perceived in motion; this can be rotation or more complex motion (Arasse, 1997, p. 459; Delieuvin, 2012a, p. 64). One might resume the situation by saying that from the very beginning, the Saint Anne Trinitarian theme was in Leonardo's mind,

the pretext to deal with figures in motion. This represented an important innovation compared to its predecessors and contemporaries in Southern and Northern Europe, for whom the Trinitarian representations were essentially static and many of which were still close to the Gothic tradition (Delieuvin, 2012c; Hand, 1982).

The Virgin gently tries to draw the Child towards her with elliptical gestures. She exerts an attraction on the Child. The Child not only attracts the lamb to him with the same elliptical gestures but also tries to ride the reluctant animal that collapses in a clockwise motion as if following the arrow of time towards the Passion. The Child does not want to part with the lamb and the Virgin does not want to let him go.

The presence of some ellipses in the composition of the picture was reported by several authors. We claim that the number of ellipses or parts of ellipses is exceptionally large to be fortuitous. Reference is made to da Novellara's letter to Isabella d'Este, dated to 3 April 1501, which concludes by saying that Leonardo sketch is not finished and that Leonardo devotes himself to geometry and does not paint: "Et questo schizo ancora non e finite... Da opra forte a la geometria impacientissimo al pennello."

Ellipses or part of ellipses can be noticed under the face of Saint Anne, on the bust of the Virgin or in her gestures, as well as the gestures of the Child and the outline of the lamb's body. We represent some ellipses in light shade via a raster graphics editor (Fig.2). For the clarity of the presentation, we display a limited number of ellipses. Many more ellipses or parts of ellipses can be observed in the folds of the left sleeve of Saint Anne's cloak, the light fabric around the right arm of the Virgin (Fig.3), as well as the Virgin's dress, or in the landscape. These ellipses have been carefully reproduced by Leonardo. They feature various eccentricities. We calculate a few of them. The eccentricity is about 0.90 for the ellipse corresponding to the arms of the Virgin and 0.82 for that relating to the arms of the Child. The ellipse corresponding to the contour of the lamb's body features one of the smallest eccentricities, that is about 0.56. Finally,

we note that the directrices of these conics are not organized according to a given pattern. In particular, the different conics seem to have no relationship with the pyramidal structure that was pointed out previously in the literature.



Fig. 2. Examples of ellipses (in light shade) or portions of ellipses in the composition of Le Louvre painting

We underline that Leonardo's interest in geometry lasted for many years. In particular, in the end of the fifteenth-century, Leonardo collaborated in Milan with the mathematician Luca Pacioli to the illustration of works in geometry. Pacioli's manuscript: "De Divina Proportione",

that dates to 14 December 1498, was copied and finally printed in 1509. It contains several representations of solids or polyhedra that are associated to Platonic mathematical ideals exposed in “Timaeus” which were designed by Leonardo, as quoted by Pacioli in fol. LXXXIII v. In the printed version of “De Divina Proportione”, Pacioli notes also that Leonardo and him left together Milan and shared a home in Florence, i.e., in 1500 (Kemp, 2019, p. 180). We note that the departure from Milan probably took place after December 14, 1499. Mathematics was surely a major field of interest for Leonardo in the very beginning of the sixteenth-century when Da Novellara relates to Isabella d’Este having seen Leonardo’s sketch of Saint Anne Trinitarian. The interest of Leonardo in Mathematics is confirmed in the new letter of da Novellara to Isabella d’Este that is dated to 14 April 1501 (Delieuvin, p. 2019, p. 258). Leonardo continued to study geometry in the years that followed. In particular, some representations of conics can be seen on the Manuscripts of the Bibliothèque de l’Institut de France, Paris, MS 2178 61v 62r, ca. 1510-1515 and Codex Atlanticus, fol. 263ra and 115rb, ca. 1510.

We note that beyond the fact that ellipses are aesthetic figures, they are much more complex to reproduce than rectangles, triangles or even circles. We point out that conic compasses were available in Italy at the time of Leonardo in order to carry out the continuous drawing of conic sections (Raynaud, 2007). During the Renaissance, conic compasses were primarily used for perspective drawings, although they were useful for many other applications, such as mechanics and architecture. Leonardo represented compasses that are designed to continuously draw some conic sections in several folios such as Codex Atlanticus, fol. 32ra, ca. 1480, Codex Arundel, fol. 73 r, ca. 1509 and Codex Atlanticus, fol. 394ra, ca. 1515. Paternity of this compass is even attributed to Leonardo by Pedretti. Reference is made to (Raynaud, 2007) for detailed analysis of the origins and transmission of the conic compass and study of the continuous drawing of conic sections during the Renaissance. It is very likely that Leonardo used several techniques

and tools to reproduce ellipses in his works and in particular the conic compass in the special case of precise representations of circles seen in perspective.

One cannot consider that the numerous ellipses or part of ellipses of the composition are only marks of virtuosity. We believe that they participate strongly to the composition. The introduction of ellipses or parts of ellipses in the composition contributes to the reinforcement of the illusion of movement. More precisely, we can quote the movement of the characters, such as the Virgin, the Child and the lamb, as well as movement of drapery, rock stratifications and rivers.

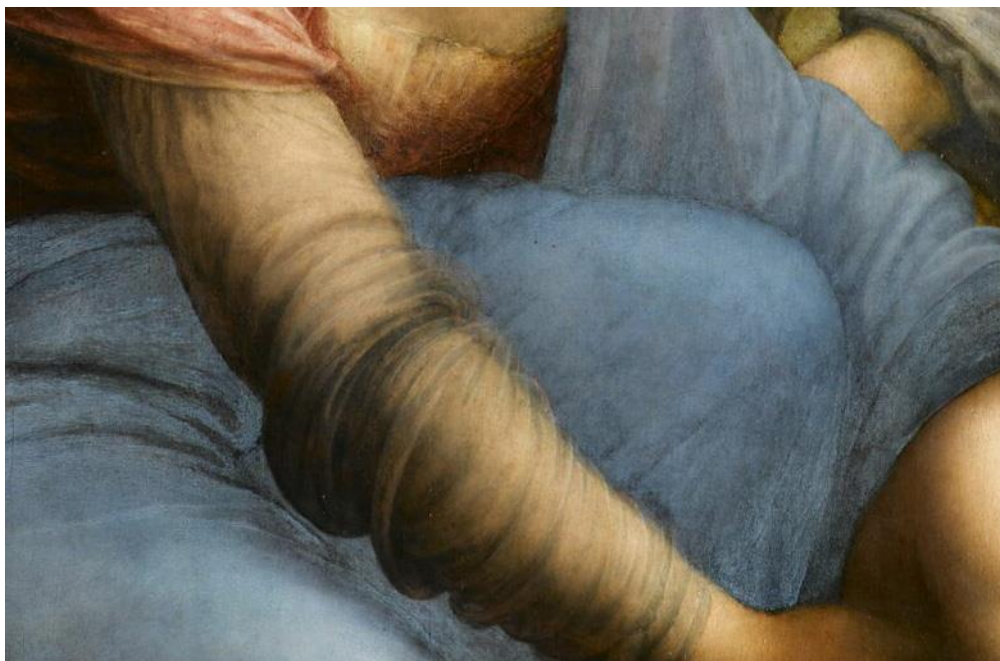


Fig. 3. Detail of the light fabric around the right arm of the Virgin in Le Louvre painting

We note also that the light fabric around the right arm of the Virgin seems to vibrate (Fig. 3). The shape of the right sleeve of the Virgin in Le Louvre painting is more regular and displays more ellipses than the one in Leonardo's study RCIN 912532 at Windsor Castle, ca. 1510-1515. The study is dated to ca. 1507-1510 in (Delieuvin, 2012d, p. 142). The lost Resta-Esterházy Cartoon, ca. 1503-1506, that is attributed to the Workshop of Leonardo da Vinci, did not even

contain this pattern. Le Louvre masterpiece and its cartoon (also missing) were copied or reused several times by Leonardo's assistants and followers but it seems that only a limited number of artists (most of which were Leonardo's close assistants) reproduced exactly the Virgin's right sleeve in their paintings. In particular, one of the most accurate representations of the right sleeve is found in the painting at Hammer Museum (Workshop of da Vinci, ca. 1508-1513). This painting is attributed to Gian Giacomo Caprotti, *i.e.*, Salaì, by several experts (Alganon, 2020; Barone, 2013, p. 749). Moreover, this type of sleeve was rarely represented in Italian paintings of the fifteenth and sixteenth-century; the reader is mainly referred to the painting of Francesco Melzi, on the antique theme of Vertumnus and Pomona; Delieuvin dates Melzi's Vertumnus and Pomona to ca. 1508-1513 (Delieuvin 2012d, p. 60). Delieuvin dates also the Panel at Hammer Museum to ca. 1508-1513 (Delieuvin, 2012d, pp. 168-169); so, does Barone (2013, p. 749). It can be assumed that the representation of such a subtle fabric was finally introduced around 1507-1513 in order to precise Leonardo's intentions. Nevertheless, this point is still a conjecture since there is no consensus on the date of Melzi's painting (the painting at Gemäldegalerie Berlin, is also placed in the period from 1518 to around 1528 by some experts). The theme of Melzi's painting is linked to that of the seasons. The god of seasons and change, Vertumnus, tries to seduce Pomona; for that, he disguises himself as an old woman in order to enter Pomona's orchard and talk to her. Pomona's elliptical gestures can also be compared to the gestures of the Virgin in the Virgin and Child with Saint Anne.

The right sleeve of the Virgin occupies a central position in Le Louvre painting. It is a very important component since it contributes to launch the series of elliptical gestures of the Virgin and the Child and to the illusion of movement. The repetition of elliptical gestures of the Virgin and the Child is a noticeable characteristic of the panel.

Proximity between Vertumnus and Pomona by Melzi and Leonardo's Saint Anne Trinitarian project was also noted by Delieuvin. The face of Pomona appears to be taken from the head of

the Virgin in the Burlington House Cartoon, ca. 1499-1500, which displays the Virgin and Child with Saint Anne and the Infant Saint John the Baptist (Delieuvin, 2019, p.280).

We remark that ellipses appear in another Italian masterpiece of the Renaissance. Louis Frank highlights the eccentricity of the figures, the obliquity of the planes and the presence of ellipses in Christ and Saint Thomas, a bronze statue related to the theme of death and resurrection of Christ, that Verrocchio designed and casted from ca. 1467 to ca. 1483 for one of the niches on the exterior walls of Orsanmichele in Florence in 1483 (Frank, 2019, pp. 29-30).

We have shown that ellipses in the composition of Le Louvre painting participate strongly to the illusion of movement and more particularly of rotations. In the sequel, we study the reasons for these rotations.

We think the causes for rotations in Le Louvre painting appear when it is placed in the succession of masterpieces produced over three generations of Florentine masters belonging to the same artistic lineage and which present the same elliptic patterns or comparable gestures and fabrics, i.e., Christ and Saint Thomas by Verrocchio, ca. 1467 to ca. 1483, Leonardo's Virgin and Child with Saint Anne, ca. 1503-1519 and Vertumnus and Pomona by Francesco Melzi, ca. 1508-1528. Leonardo stay in Verrocchio's workshop is placed in the period from ca. 1464-1465 to ca. 1478 (Delieuvin, 2019, pp. 19, 58 and 90). Melzi's presence with Leonardo can be placed from ca. 1507-1508 until Leonardo's death in 1519. Each friendly disciple stayed with his master for a long time. In particular, each disciple attended the elaboration of the work of his master and certainly knew what concepts and patterns his masters work was based on and reused it accordingly for a related purpose, in appreciation and tribute.

Melzi's painting and Verrocchio's masterpiece deal with seasons and resurrection, respectively. In particular, Melzi's painting features an anemone, symbol of renewal, in the basket in the hands of Pomona. The commonalities of the works suggest that Leonardo's panel evokes also seasons, past, present and future. We think that Le Louvre painting deals with the

origins, life and resurrection of Christ and recalls the celestial sphere, always in motion, the harmonious ordering of the planets and the perfection of the universe which already forced the admiration of the scholars of the antiquity, such as Ptolemy and philosophers, such as Plato. This last deduction is particularly attractive since it relies on scientific and philosophical concepts that were popular at the time of Leonardo. Leonardo's interest in astronomy is supported by several entries in the inventory of his books in the second Codex of Madrid. Platonism was particularly studied in Florence by philosophers such as Marsilio Ficino and the humanists. Leonardo, who frequented Luca Pacioli and collaborated to his work on "De Divina Proportione" featuring Plato's perfect solids and who was also host of the elites in Florence and Milan, knew the philosophy of Plato and doctrine of the soul that is source of life and motion. In the sequel, we develop this part.

We recall that celestial bodies had circular movement in the Ptolemaic system that was in use from around the second century AD to the middle of the sixteenth-century. Ellipses were already identified as flat sections of cones of revolution in ancient times (Apollonius, ca. second-century BC; Berger, 1978). We underline that any ellipse is the image of a circle by projection on a plane, when the circle does not intersect the plane (Girard & Lentin, 1964; Jullien, 1875; Raynaud 2007). This last point is important for the representation of circles in perspective.

Leonardo's interest in astronomy was well known to his contemporaries. On January 13, 1490, Leonardo da Vinci was the director the Paradise Festival, book of Bernardo Bellincioni. This event occurred at the court in Milan to celebrate the wedding of Jean Galeas Sforza, Duke of Milan and Isabel of Aragon, princess of Naples. Leonardo designed the special effects of the court spectacle and helped to the choreography of the ballet in which dancers were evoking the motion of the planets and the sun. This was the Ballet of the Planets. Leonardo was also the designer of the half egg-shape décor representing the night sky (Kemp, 2019, p. 208).

Though Leonardo was more interested in optics, i.e., celestial bodies and their appearance, than in celestial mechanics, he knew well the Ptolemaic system that describes the trajectories of celestial bodies according to a geocentric model. Several entries in the inventory of Leonardo's books in the second Codex of Madrid, 2v – 3r, ca. 1503 – 1505, refer to Ptolemy and the geocentric model. In particular, Kemp notes that Leonardo was faithful to the geocentric vision (Kemp, 2019, p. 463). This is attested by a note on the geocentric system in the Codex Leicester, 2r ca. 1507–1510. Nevertheless, Kemp underlines also possible filiation with writings of Cardinal Nicholas of Cusa, 1401–1464, theologian, philosopher and astronomer who reached the conclusion that earth is hardly different by nature from other planets (Kemp, 2019, p. 465). Leonardo makes a panegyric of the sun in the folio MS 2177 4v, ca. 1508-1513 of the Bibliothèque de l'Institut de France; in particular, he notes that there is no body in the universe larger than the sun and that its light illuminates all the celestial bodies. Kemp quotes also part of a sentence in the folio MS 2177 56r, ca. 1508-1513, of the Bibliothèque de l'Institut de France, where Leonardo highlights that earth is almost like the moon. We underline that this comparison does not privilege particularly our planet.

With time, Leonardo's conception of the universe evolved. In particular, almost one century before Galileo and Kepler, Leonardo reports that the ashy light of the moon is due to the reverberation of the earth, i.e., the earthshine phenomenon, in the Codex Leicester, 2r ca. 1507–1510. This discovery contradicts the opinion of Aristotle who believed that all planets have their own luminosity. Leonardo already writes in the manuscript MS 2172 64r, ca. 1492, that the moon has no light in itself; it does not shine without the help of the sun.

The Ptolemaic system was made obsolete by the theories of Copernicus and Kepler. In the beginning of the sixteenth-century, Nicolaus Copernicus presented to his friends a hand-written treatise on heliocentric model known as the "*Commentariolus*". This work called into question the Aristotelian physics and the long-established Ptolemaic system. Copernicus model was

finally published in a book in 1543. Later on, Johannes Kepler showed the ellipticity of the orbits of planets. The first two laws of Kepler were published in the “*Astronomia Nova*” in 1609 (we note here that the eccentricities of the orbit of the earth and moon are small, i.e., 0.0167 and 0.0549, respectively).

Previously, circles were used to describe the movement of celestial bodies in the Ptolemaic system that was popular throughout the Late Antiquity, the Medieval Period and at the time of Leonardo (Hayli, 1971). As shown above, several manuscripts of Leonardo show that he had an intense activity related to astronomy around 1507-1513, period he was also working on the *Virgin and Child with Saint Anne*.

Themes of seasons, birth, death and resurrection in nature are closely related to astronomy and celestial mechanics. The analogy between ellipses, circles and celestial mechanics is mainly motivated by the fact that mathematically, ellipses and conics are flat sections of cones of revolution. This result has been known since Antiquity, e.g., see the work of Apollonius, ca. second-century BC, on ellipses and more generally on conical sections and orthogonal projections of circles (Berger, 1978; Lebesgue 1942). Moreover, on what concerns celestial mechanics, we recall that Ptolemy was using circular movements to account for the perfect motion of celestial bodies (Hayli, 1971).

Rotation, twist and bending of figures allowed Leonardo to create original motifs which characterize his work. They have established his fame and situate him at the source of a new current of artists interested in accurate depiction of figures and nature, as well as motion that were prolific in Western Art. In the sixteenth-century, Leonardo’s motifs were extensively copied by painters, including Raphael.

Finally, we note that the revolution of the planets is generally seen as an allegory of changes in political systems and cycles in societies. The values of the republican ideal being traditionally embodied by the Greek and Roman republics.

IV Hydrodynamics, Instabilities and Turbulences

Le Louvre panel is a subtle representation of the Virgin and Child with Saint Anne in a stunning and unfinished landscape that combines accurate atmospheric, hydrodynamic and geological observations. In particular, it displays a precise rendering of geology from the details of the rock stratification in the foreground to the mountains in the background. The importance of water in geophysics is particularly emphasized by the many rivers, lakes and waterfalls on the right and left sides as well as the pebble stones in the foreground. It is a fantastic vision which highlights the grandeur of a primitive nature and opens the way to Romanticism. In the background, the sublime effects of waterfalls and rivers are subtly rendered. The precise depiction of the hydrodynamic phenomena catches the attention.

We underline that Leonardo's scientific approach was mostly based on observation, questioning, analogy and testing and sometimes on mathematical modeling, e.g., his work on tree growth, that are still essential approaches in modern sciences along with the latest important aspects such as numerical simulation. In particular, Leonardo had a very precise perception of light, hydrological phenomena and forces through his observations of nature, experiments on the anatomy, studies of physiology and flight of birds, as well as architecture.

Leonardo's desire to represent or evoke natural phenomena and the mathematical laws which possibly underlie them can be found in some subtle details of the landscape in the background of the painting like hydrodynamic instabilities and turbulences in the river at the bottom of the waterfall to the left of the right shoulder of the Virgin. With this respect, it is noticeable that the motif of hydrodynamic instabilities or turbulences in water is evoked also in the neighboring uncommon shirred ornament on the Virgin red dress over her right shoulder (Fig. 4). Turbulences are also evoked in some subtle details of Saint Anne hairdressing and veils as well as in the mantle of the Virgin and the lamb's fleece (Fig. 5).



Fig. 4. Detail of the ornament of Virgin dress in Le Louvre painting



Fig. 5. Detail of the lamb in Le Louvre painting

Turbulence has been extensively studied by Leonardo. Before the end of the fifteenth-century, Leonardo draw turbulent motion of water around obstacles like in the Manuscript MS 2179 16r (ca. 1493-1494) of the Bibliothèque de l'Institut de France, Paris (Fig. 6).

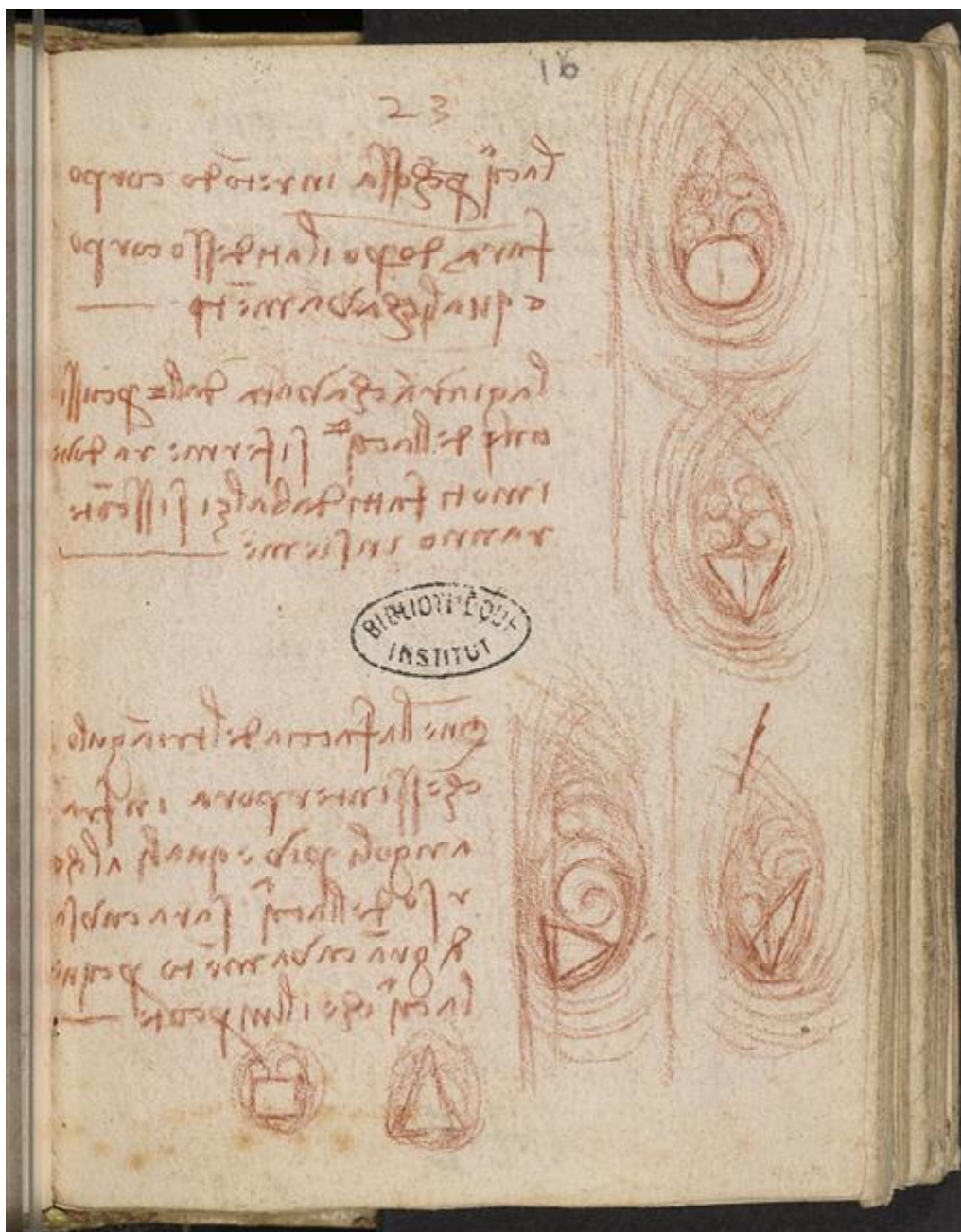


Fig. 6. Manuscript MS 2179 folio 16r (ca. 1493-1494) of the Bibliothèque de l'Institut de France, Paris



Fig. 7. Manuscript W. 912579r of the Royal Collection, Windsor (ca. 1508-1509)



Fig. 8. Leonardo da Vinci, study for the mantle of the Virgin, Le Louvre, Paris

Between 1508 and 1509, Leonardo continued to study turbulences around obstacles and at the bottom of waterfalls. In particular, turbulences are represented in details in the manuscripts on hydrodynamics W. 912579r (Fig. 7) and W. 912660v to W. 912662 of the Royal Collection, Windsor and Manuscript MS 2177 48r of the Bibliothèque de l'Institut de France, Paris. Motifs related to instabilities and turbulences appear also in the ridge folds over the thigh and near the back of the Virgin in the study of the mantle of the Virgin, ca. 1507-1510 by Leonardo (Fig. 8). El Baz has noted that this mantle recalls Greek mantle ("*himatia*") and is similar to the one

displayed in manuscripts W. 912579r (Fig. 7). On the same page, Leonardo notes: “observe the movement of water on its surface as it resembles that of hair, one movement of which depends on the weight of the hair and the other on the orientation of the curls.” Turbulences seem to be at the center of Leonardo concerns during the period from 1508 to 1510. They present an obvious pictorial interest which allows him to renew the form in his paintings by introducing new motifs that increase the impression of movement. In particular, the motif of turbulence is evoked in the hairdressing of the Madonna of the Yarnwinder.

Abid et al. claim that Leonardo was the first to study the formation of vortices in water and air flowing around an obstacle and that he introduced the name turbulence to describe this phenomenon. For studies of hydrodynamic instabilities and turbulent flows the reader is also referred to (Candel, 1990; Leslie, 1973; Lin, 1955; Pope 2001).

Leonardo’s detailed representations of turbulence and instabilities have clearly the quality of scientific contributions. As a consequence, it appears that besides the representation of the Saint Anne Trinitarian theme, the picture contains also part of Leonardo’s questioning on nature, physics and mathematics.

V Conclusions

In this paper, we have considered graphs, geometry and the accurate representation of instabilities and turbulences in the composition of Leonardo’s Virgin and Child with Saint Anne. Clearly, Leonardo’s aesthetic differs from that of previous depictions of the Saint Anne Trinitarian theme; especially static Medieval representations. The panel features symmetries, repetitions of gestures and superposition of figures in a virtuous but natural way. It is precisely scientific questioning that permitted Leonardo to renew the pictorial form, especially by focusing on the motion of figures and movements of drapery, rock stratification and water. In particular, the composition of the Virgin and Child with Saint Anne rests on a solid

mathematical foundation. The many ellipses or parts of ellipses in the composition and the duplication of the elliptical gestures of the Virgin and the Child play a major part in the perception of movement of figures and pay a tribute to the perfection of the universe. It is remarkable that the Virgin and Child with Saint Anne, which expresses movement to the highest degree, displays a series of curves from the simplest one to the most sinuous and intricate. We can summarize by saying that the panel tends to reveal a reality of a higher order: the universe obeys physical laws which are expressed by means of mathematics. The painting covers from the simplest to the most complex forms, ranging from order to chaos. Moreover, the panel

Somehow, the famous triplet of the Greek philosophy: chaos, cosmos, logos is also represented in Le Louvre panel. Several levels of reading are possible in the painting: artistic, religious, scientific and philosophical. Concepts related to destiny, resurrection and liberation can be revisited accordingly in Le Louvre panel.

The painting combines light, strength, love of nature, life, emotions, grace and the illusion of relief and motion. The luminous parts of the painting correspond to portions assigned to the horizon, water and flesh. Above all, the picture emphasizes that light, water and life cannot be dissociated. It is truly a synthesis of all the artistic and scientific knowledges gathered by Leonardo during his life.

The work is unfinished, e.g., the left hand of the Child, and part of the middle ground are not complete. Some experts believe that the process of painting the masterpiece could have lasted more than ten years. Leonardo had certainly enough time to finish his work. Several painters, some of whom were Leonardo's assistants, managed to complete their paintings on the Saint Anne Trinitarian theme. Once completed, their panels were both good representations of the current state of Leonardo's painting at the same time as well as extrapolations, then envisaged in Leonardo's workshop. Leonardo chose to perfect and slowly modify his painting in the process of creating a masterpiece. With time, he preferred to continue his research and consider

his painting as a work in progress than any other solution that could have disrupted the balance of his depiction and possibly diverted viewer's attention to secondary details. Indeed, the idea of having an assistant completing the panel did not impose itself on Leonardo; though his longtime friends and talented assistants, Francesco Melzi and Salaì, had accompanied him to France with the painting (Salaì, nevertheless returned to Italy), as reported somewhat by Antonio de Beatis on 10 October 1517. In the travel diary, de Beatis underlines that Leonardo draws but is no longer able to paint: “M. Lunardo non possa colorire con quella dolceza che solea, pur serve ad fare disegni et insignari ad altri.” The painting is thereby the image of creation and living creatures which are incomplete and perfectible.

Another remarkable point of Leonardo's panel lies in the fact that it revisits by means of new techniques, the description of a primitive nature and a Christian motif. The painting appears as a surprising synthesis between elements linked to the Christian tradition including a representation of the wilderness (which even has a certain romantic character that is frequently opposed to modernism in our time) and a descriptive process that is based in accurate scientific observations and representations. The result of this astonishing synthesis of the ancient and the modern emphasizes Leonardo's open-mindedness and the spiritual dimension of his approach. It also strengthens the sacred character of the painting.

We believe that the material and societal constraints imposed to Leonardo as well as the transformations in society, new scientific approaches, and Leonardo's awareness of the importance of mathematics for understanding nature nurtured his creativity and allowed his awesome talent to express itself at an exceptional level whereby he was able to address the many issues required in order to produce a masterpiece. Leonardo's creativity and talent supported by the spirit of scientific research of the Renaissance and a high spirituality characterize him to the highest degree and constitute his signature.

In future work, we plan to study Greek influence in the Saint Anne Trinitarian project. In particular, we shall concentrate on the attires and postures of the characters. We shall detail also aspects related to the influence of Plato's philosophy on Le Louvre masterpiece.

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