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► To cite this version:

Agustina d'Jorge, Isabelle Queinnec, Sophie Tarbouriech, Luca Zaccarian. Reset control analysis and design for hybrid Lur'e dynamical systems. International Symposium on Mathematical Theory of Networks and Systems, IFAC, Sep 2022, Bayreuth (DE), Germany. hal-04185288

HAL Id: hal-04185288

<https://laas.hal.science/hal-04185288>

Submitted on 22 Aug 2023

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Reset control analysis and design for hybrid Lur'e dynamical systems

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Abstract: We introduce a new class of hybrid Lur'e dynamical systems where a sector nonlinearity may affect both the continuous-time evolution and the reset map acting on suitable closed-loop states, under a time-regularization mechanism ensuring dwell time of solutions. For this class of systems we characterize Lyapunov-based exponential stability conditions exploiting homogeneity of the closed loop. In particular, we show that, with quadratic Lyapunov certificates these conditions can be cast as linear matrix inequalities. We then focus on the control design problem, where both the feedback gains acting on the continuous evolution and the reset action must be designed, in addition to the sets where such resets are triggered, expressed by sign-indefinite quadratic forms. For this control design problem we also show that the synthesis can be performed by solving a set of linear matrix inequalities.

Keywords: Reset control systems, Lur'e systems, Lyapunov theory, exponential stability

1. INTRODUCTION

Reset control systems (see, for example, Goebel et al. (2009, 2012), Prieur et al. (2018), Le and Teel (2021) and references therein) is a specific class of hybrid dynamical systems wherein continuous motion of the plant-controller dynamics is equipped with resetting rules inducing instantaneous re initializations of certain controller states, whenever the so-called reset conditions are satisfied. Beyond the fact that this class of systems allows to deal with a broad range of applications, as automotive systems, power systems and biological systems, they can overcome the limitation of classical continuous control law and achieve desired behavior as for example robustness, performance improvement (see, e.g., Hespanha and Morse (1999); Prieur (2005), Safaei et al. (2010), Aangenent et al. (2010); Beker et al. (2001); Goebel and Teel (2009); Hespanha et al. (2003); Prieur and Astolfi (2003); Nešić et al. (2008, 2011), Prieur et al. (2011, 2013)).

In this paper, a new class of hybrid Lur'e dynamical systems is introduced, where a sector nonlinearity may affect both the continuous-time evolution and the reset map acting on suitable closed-loop states. In other words, we consider reset control systems with Lur'e non-linearity, consisting in the interconnection between a linear plant and a non-linearity through a feedback loop. The nonlinearity verifies a cone bounded sector condition Khalil (2002), Castelan et al. (2008). Lyapunov-based exponential stability conditions exploiting homogeneity of the closed loop are proposed by adapting results issued from Goebel et al. (2012); Nešić et al. (2008); Zaccarian et al. (2011). Following the ideas presented in Fichera et al. (2016a), Fichera et al. (2016b), with quadratic Lyapunov certificates these conditions can be cast as linear matrix inequalities (LMIs). We then focus on the control design problem, where we must design 1) the state feedback gains acting on the continuous evolution and the reset action and 2) the shape of the sets where such resets are triggered, expressed as sign-indefinite quadratic forms. For

this control design problem we also show that the synthesis can be performed by solving a set of linear matrix inequalities. The contribution of the current note can be viewed as complementary to the results developed in Fiacchini et al. (2012) dealing with quadratic stability problem for hybrid systems with nested saturations.

The extended abstract is organized as follows: Section 2 introduces the class of hybrid systems under consideration and states the problems at stake. Section 3 presents theoretical results dealing with the stability analysis. Section 4 then expands the stability analysis results in order to handle the control design problem. Finally, some concluding remarks are given in Section 5.

Notation. The notation is standard. The Euclidean norm of a vector is denoted by $|\cdot|$. If \mathcal{A} is a compact set, the notation $|x|_{\mathcal{A}} = \min\{|x-y| : y \in \mathcal{A}\}$ indicates the distance of the vector x from the set \mathcal{A} . If \mathcal{A} is the origin then $|x|_{\mathcal{A}} = |x|$. For any $s \in \mathbb{R}$, the function $\text{dz} : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $\text{dz}(s) = 0$ if $|s| \leq 1$ and $\text{dz}(s) = \text{sign}(s)(|s| - 1)$ if $|s| \geq 1$. Given a matrix Q , $\text{He}(Q) = Q + Q^T$. Moreover, $\lambda_{\min}(Q)$ (respectively, $\lambda_{\max}(Q)$) denotes the minimum (respectively, the maximum) eigenvalue of Q .

2. PROBLEM STATEMENT

Consider the following hybrid system, including an input nonlinearity ϕ ,

$$\begin{cases} \dot{x} = A_F x + B_F \phi(u_F), \\ \dot{\tau} = 1 - \text{dz}\left(\frac{\tau}{\rho}\right) \end{cases} \quad (x, \tau) \in \mathcal{C} \quad (1a)$$

$$\begin{cases} x^+ = A_J x + B_J \phi(u_J) \\ \tau^+ = 0, \end{cases} \quad (x, \tau) \in \mathcal{D} \quad (1b)$$

where $x \in \mathbb{R}^n$ is the physical state, $\tau \in \mathbb{R}$ is a dwell-time logic (with $\rho > 0$), and $u_F \in \mathbb{R}^m$ and $u_J \in \mathbb{R}^m$ are suitable

control inputs to be designed. The flow and jump sets \mathcal{C} and \mathcal{D} are defined as follows:

$$\begin{aligned}\mathcal{C} &:= \{(x, \tau) : x \in \mathcal{F} \text{ or } \tau \in [0, \rho]\} \\ &= \{(x, \tau) : x \in \mathcal{F}\} \cup \{(x, \tau) : \tau \in [0, \rho]\}\end{aligned}\quad (2)$$

$$\begin{aligned}\mathcal{D} &:= \{(x, \tau) : x \in \mathcal{J} \text{ and } \tau \in [\rho, 2\rho]\} \\ &= \{(x, \tau) : x \in \mathcal{J}\} \cap \{(x, \tau) : \tau \in [\rho, 2\rho]\}\end{aligned}\quad (3)$$

with \mathcal{F} and \mathcal{J} symmetric cones defined by a symmetric (typically not sign definite) matrix $M = M^\top \in \mathbb{R}^{n \times n}$ as

$$\begin{aligned}\mathcal{F} &:= \{x \in \mathbb{R}^n : x^\top M x \leq 0\} \\ \mathcal{J} &:= \{x \in \mathbb{R}^n : x^\top M x \geq 0\}.\end{aligned}\quad (4)$$

The nonlinearity ϕ affecting the system input is a known, continuous, decentralized cone bounded nonlinearity (see, for example, Khalil (2002), Castelan et al. (2008)) as stated in the following assumption.

Assumption 1. The nonlinearity $\phi : \mathbb{R}^m \rightarrow \mathbb{R}^m$ is a known, continuous and decentralized function, which verifies the following generic cone bounded sector condition for any diagonal positive definite matrix $S \in \mathbb{R}^{m \times m}$:

$$\phi^\top(\zeta) S (\phi(\zeta) - \Omega \zeta) \leq 0 \quad \forall \zeta \in \mathbb{R}^m \quad (5)$$

Matrix $\Omega \in \mathbb{R}^{m \times m}$ is a positive definite diagonal matrix defining the sector $[0, \Omega_{i,i}]$, where each component ϕ_i of ϕ lies.

In this note, the inputs of the flow and jump maps are selected as linear static state feedbacks:

$$u_F = K_F x \text{ and } u_J = K_J x, \quad (6)$$

with $K_F \in \mathbb{R}^{m \times n}$, $K_J \in \mathbb{R}^{m \times n}$. Then the closed loop (1)-(6) can be rewritten as

$$\begin{cases} \dot{x} = A_F x + B_F \phi(K_F x) \\ \dot{\tau} = 1 - \text{dz} \left(\frac{\tau}{\rho} \right), \end{cases} \quad (x, \tau) \in \mathcal{C} \quad (7a)$$

$$\begin{cases} x^+ = A_J x + B_J \phi(K_J x) \\ \tau^+ = 0, \end{cases} \quad (x, \tau) \in \mathcal{D}, \quad (7b)$$

where the flow and jumps sets \mathcal{C} and \mathcal{D} are defined in equation (2) and (3), respectively.

Remark 1. Note that in the closed-loop system (7), the two nonlinearities $\phi(K_F x)$ and $\phi(K_J x)$ satisfy Assumption 1. Therefore, the generic relation (5) can be particularized for both $\phi(K_F x)$ and $\phi(K_J x)$ as follows

$$\phi(K_F x)^\top S_F (\phi(K_F x) - \Omega_F K_F x) \leq 0 \quad \forall x \in \mathbb{R}^n \quad (8)$$

$$\phi(K_J x)^\top S_J (\phi(K_J x) - \Omega_J K_J x) \leq 0 \quad \forall x \in \mathbb{R}^n, \quad (9)$$

which holds for any diagonal positive matrices $S_F \in \mathbb{R}^{m \times m}$, $S_J \in \mathbb{R}^{m \times m}$ and where $\Omega_F \in \mathbb{R}^{m \times m}$ and $\Omega_J \in \mathbb{R}^{m \times m}$ are suitable positive definite diagonal matrices. Matrices Ω_F and Ω_J are supposed to be known.

In this note we address 1) the stability analysis of system (1)-(6), or equivalently system (7), when K_F are K_J are given, and 2) the design problem where K_F are K_J must be designed.

These two complementary problems can be summarized as follows.

Problem 1. Given the gains K_F are K_J , devise conditions to guarantee that the compact set \mathcal{A} defined as

$$\mathcal{A} = \{0\} \times [0, 2\rho] \subset \mathbb{R}^n \times [0, 2\rho] \quad (10)$$

is globally asymptotically stable for the closed loop (1)-(6) (equivalently, system (7)).

Problem 2. Design the gains K_F are K_J in (4) such that the compact set \mathcal{A} defined as in (10) is globally asymptotically stable for the closed loop (1)-(6) (equivalently system (7)).

3. STABILITY ANALYSIS RESULTS

In this section, theoretical results addressing Problem 1 are proposed by exploiting some ingredients provided in Fichera et al. (2016a), Prieur et al. (2018).

Recall that due to the dwell-time, the solutions (x, τ) to system (7) may flow outside the flow set as emphasized for example in Zaccarian et al. (2005), Prieur et al. (2018). Hence, to deal with the effects of dwell-time on trajectories and in order to allow for more design flexibility consider the following definitions,

$$\tilde{\mathcal{F}} = \{x \in \mathbb{R}^n : x^\top \tilde{M} x \leq 0\} \quad (11)$$

$$\tilde{\mathcal{F}}_\epsilon = \{x \in \mathbb{R}^n : x^\top \tilde{M} x - \epsilon x^\top x \leq 0\} \quad (12)$$

with $\tilde{M} = \tilde{M}^\top \in \mathbb{R}^{n \times n}$ and $\epsilon > 0$ to be designed.

Note that (12) is the ϵ -inflated version of (11), therefore the inclusion $\tilde{\mathcal{F}} \subset \tilde{\mathcal{F}}_\epsilon$ is always satisfied.

Stability conditions to solve Problem 1 are first proposed by focusing on a generic Lyapunov function, and then they are specialized to a quadratic Lyapunov function, thus leading to a convenient formulation involving convex linear matrix inequalities (LMI).

Theorem 1. Consider system (7) and the sets defined in (11) and (12). Assume that there exist a continuously differentiable function $V : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$, positive real scalars $\alpha_1, \alpha_2, \alpha_3$ and a nonnegative scalar $\underline{\rho}$ satisfying

$$\alpha_1 |x|^2 \leq V(x) \leq \alpha_2 |x|^2, \quad \forall x \in \mathbb{R}^n, \quad (13)$$

$$\begin{aligned} \langle \nabla V(x), A_F x + B_F \phi_F \rangle + \alpha_3 V(x) \\ - 2\phi_F^\top S_F (\phi_F - \Omega_F K_F x) < 0, \end{aligned} \quad \forall x \in \tilde{\mathcal{F}}_\epsilon \setminus \{0\} \quad (14)$$

$$\begin{aligned} V(A_J x + B_J \phi_J) - \exp(\alpha_3 \underline{\rho}) V(x) \\ - 2\phi_J^\top S_J (\phi_J - \Omega_J K_J x) \leq 0, \end{aligned} \quad \forall x \in \mathcal{J} \quad (15)$$

$$x^+ \in \tilde{\mathcal{F}}, \quad \forall x \in \mathcal{J} \quad (16)$$

$$\mathcal{F} \subset \tilde{\mathcal{F}}_\epsilon \quad (17)$$

with ϕ_F and ϕ_J being shorthands for $\phi(K_F x)$ and $\phi(K_J x)$, respectively. Then there exists $\bar{\rho} > \underline{\rho}$ such that, for any $\rho \in (\underline{\rho}, \bar{\rho})$, the set $\mathcal{A} := \{0\} \times [0, 2\rho] \subset \mathbb{R}^n \times [0, 2\rho]$ is globally asymptotically stable for the hybrid closed-loop system (7).

Proof. The proof is omitted in this extended abstract. However, the key ingredients of the proof emerge from a careful expansion of the proof of Theorem 5.1 in Prieur et al. (2018) to the case of system (7) subject to the nonlinearities $\phi(K_F x)$ and $\phi(K_J x)$ satisfying the cone-bounded conditions (8) and (9), respectively. \square

An interesting way to particularize Theorem 1 is to select a quadratic Lyapunov function $V(x) = x^\top P x$, in order to derive LMI-based conditions. Then, the following result can be stated in the context of Problem 1.

Proposition 1. Given System (7) and positive scalars $\alpha_3 > 0$, $\underline{\rho} > 0$, assume that there exist matrices $P = P^\top > 0$, $\tilde{M} = \tilde{M}^\top$, S_F, S_J and \tilde{S}_J diagonal positive definite matrices, non-negative scalars $\tau_S, \tau_R, \tilde{\tau}_C, \tilde{\tau}_F$ and a positive scalar ϵ such that the following inequalities hold,

$$\begin{pmatrix} \text{He}(P A_F) + \alpha_3 P - \tau_S (\tilde{M} - \epsilon I) & P B_F + K_F^\top \Omega_F S_F \\ B_F^\top P + S_F \Omega_F K_F & -2S_F \end{pmatrix} < 0 \quad (18)$$

$$\begin{pmatrix} A_J^\top P A_J - \exp(\alpha_3 \rho) P + \tau_R M & A_J^\top P B_J + K_J^\top \Omega_J S_J \\ B_J^\top P A_J + S_J \Omega_J K_J & B_J^\top P B_J - 2S_J \end{pmatrix} \leq 0 \quad (19)$$

$$\begin{pmatrix} A_J^\top \tilde{M} A_J + \tilde{\tau}_C M & A_J^\top \tilde{M} B_J + K_J^\top \Omega_J \tilde{S}_J \\ B_J^\top \tilde{M} A_J + \tilde{S}_J \Omega_J K_J & B_J^\top \tilde{M} B_J - 2\tilde{S}_J \end{pmatrix} \leq 0 \quad (20)$$

$$\tilde{M} - \tilde{\tau}_F M \leq \epsilon I. \quad (21)$$

Then there exists $\bar{\rho} > \rho$ such that for any $\rho \in (\rho, \bar{\rho})$ the set \mathcal{A} is globally asymptotically stable for the hybrid closed-loop system (7).

Proof. We only present a sketch of the proof, which consists in rewriting the conditions of Theorem 1 for the case where $V(x) = x^\top P x$. Indeed, let us consider the case of relation (14), which reads

$$2x^\top P(A_F x + B_F \phi_F) + \alpha_3 x^\top P x - 2\phi_F^\top S_F(\phi_F - \Omega_F K_F x) < 0, \forall x \in \tilde{\mathcal{F}}_\epsilon \setminus \{0\}$$

with ϕ_F the shorthands for $\phi(K_F x)$. We can handle the fact that the condition has to be satisfied for any $x \in \tilde{\mathcal{F}}_\epsilon \setminus \{0\}$ by using the S-procedure (see Boyd et al. (1994)) and the definition (12), which leads to

$$2x^\top P(A_F x + B_F \phi_F) + \alpha_3 x^\top P x - 2\phi_F^\top S_F(\phi_F - \Omega_F K_F x) - \tau_S x^\top (\tilde{M} - \epsilon I) x < 0$$

with $\tau_S \geq 0$. Hence, if relation (18) is verified then the above inequality and therefore condition (14) are satisfied.

The same reasoning holds for relation (15). In the case of condition (16), since ϕ_J is involved, one has to consider that this relation holds for ϕ_J satisfying the cone-bounded condition (9). This leads to relation (20). \square

Remark 2. The inequalities presented in Proposition 1 are not LMIs in the matrix decision variables P , \tilde{M} , S_F , S_J , \tilde{S}_J and the scalar decision variables τ_S , τ_R , $\tilde{\tau}_C$, $\tilde{\tau}_F$, ϵ due to the product between certain scalar and matrix decision variables. These products can be eliminated, thereby transforming the conditions into authentic LMIs, when multiplying by τ_S inequalities (20) and (21), and performing the following change of variables:

$$\bar{M} = \tau_S \tilde{M}, \bar{\epsilon} = \tau_S \epsilon, \bar{\tau}_C = \tau_S \tilde{\tau}_C, \bar{\tau}_F = \tau_S \tilde{\tau}_F, \tau_S \tilde{S}_J = \bar{S}_J.$$

so that the emerging conditions become linear in the transformed decision variables P , \bar{M} , S_F , S_J , \bar{S}_J , τ_R , $\bar{\tau}_C$, $\bar{\tau}_F$ and $\bar{\epsilon}$.

It still is required to fix a priori the two scalars α_3 and ρ but we emphasize that these quantities should be selected small enough for the construction to be effective and this helps in an iterative selection.

4. DESIGN RESULTS

In this section we take inspiration from Proposition 1 and Remark 2, to propose sufficient conditions to solve Problem 2. Recall that the matrices Ω_F and Ω_J involved in relations (8) and (9) are supposed to be known.

Proposition 2. Given System (7) and positive scalars $\alpha_3 > 0$ and $\rho > 0$, if there exist matrices $P = P^\top > 0$, $\bar{M} = \bar{M}^\top$, \bar{K}_F , \bar{K}_J , diagonal matrices $S_F > 0$ and $S_J > 0$, non-negative scalars $\bar{\tau}_F, \bar{\tau}_C, \tau_R \in \mathbb{R}_{\geq 0}$ and positive scalar $\bar{\epsilon}$ such that the following inequalities hold,

$$\begin{pmatrix} A_F^\top P + P A_F + \alpha_3 P - \bar{M} + \bar{\epsilon} I & P B_F + \bar{K}_F^\top \\ B_F^\top P + \bar{K}_F & -2S_F \end{pmatrix} < 0 \quad (22)$$

$$\begin{pmatrix} A_J^\top P A_J - \exp(\alpha_3 \rho) P + \tau_R M & A_J^\top P B_J + \bar{K}_J^\top \\ B_J^\top P A_J + \bar{K}_J & B_J^\top P B_J - 2S_J \end{pmatrix} \leq 0 \quad (23)$$

$$\begin{pmatrix} A_J^\top \bar{M} A_J + \bar{\tau}_C M & A_J^\top \bar{M} B_J + \bar{K}_J^\top \\ B_J^\top \bar{M} A_J + \bar{K}_J & B_J^\top \bar{M} B_J - 2S_J \end{pmatrix} \leq 0 \quad (24)$$

$$\bar{M} - \bar{\tau}_F M \leq \bar{\epsilon} I, \quad (25)$$

Then there exists $\bar{\rho} > 0$ such that for any $\rho \in (\rho, \bar{\rho})$ the set \mathcal{A} is globally exponentially stable for the hybrid closed-loop system (7) with the gains $K_F = S_F^{-1} \Omega_F^{-1} \bar{K}_F$ and $K_J = S_J^{-1} \Omega_J^{-1} \bar{K}_J$.

Proof. The proof follows the same lines as in the proof of Proposition 1 by modifying the conditions thanks to the change of variables suggested in Remark 2. Furthermore, one can also use the change of variables $\bar{K}_F = S_F \Omega_F K_F$ and $\bar{K}_J = S_J \Omega_J K_J$, allowing to recover the gains K_F and K_J because matrices S_F , S_J , Ω_F and Ω_J are all diagonal positive definite and therefore nonsingular. \square

5. CONCLUSION

This note introduced a new class of hybrid Lur'e dynamical systems where a sector nonlinearity may affect both the continuous-time evolution and the reset map acting on suitable closed-loop states, under a time-regularization mechanism ensuring dwell time of solutions. For this class of systems, both the stability analysis and the control design problems have been addressed by exploiting Lyapunov-based stability conditions and homogeneity of the closed loop. By selecting a quadratic Lyapunov certificate these conditions can be cast as linear matrix inequalities. In the control design problem, both the feedback gains acting on the continuous evolution and the reset action can be designed. For this control design problem the synthesis can also be performed by solving a set of linear matrix inequalities.

The studies proposed are preliminary but pave the way for future directions of research including addressing regional stability properties and taking into account nonlinearities affecting also the shape of the flow and jump sets. Another interesting problem could be to study how the conditions change if the flow and jump maps are subject to different input-nonlinearity, with independent sector bounds.

ACKNOWLEDGEMENTS

This work has been supported in part by the ANR Labex CIMI (grant ANR-11-LABX-0040) within the French State Program ‘‘Investissement d’Avenir’’, by ANR via project HANDY, number ANR-18-CE40-0010 and by IRT Tremplin HYQPI.

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