



HAL
open science

General methodology for observer-based control of power electronic converters with measured perturbations

Manon Dore, Yassine Ariba, Germain Garcia

► To cite this version:

Manon Dore, Yassine Ariba, Germain Garcia. General methodology for observer-based control of power electronic converters with measured perturbations. Conference on Control Technology and Applications CCTA 2023, Aug 2023, Bridgetown, Barbados. hal-04219916

HAL Id: hal-04219916

<https://laas.hal.science/hal-04219916>

Submitted on 27 Sep 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

General methodology for observer-based control of power electronic converters with measured perturbations

Manon Doré*, Yassine Ariba* and Germain Garcia*

*LAAS-CNRS, Université de Toulouse, CNRS, INSA, Toulouse, France

Email: mdore@laas.fr, yariba@laas.fr, garcia@laas.fr

Abstract—This paper deals with the control and observation of DC output power electronic converters. In order to propose a general approach, a unified modeling methodology is first introduced. A switched state feedback control law using perturbed measurements and a full-order observer are then designed separately based on LMI conditions. The observer is used to compute the control law while the input and output voltages are measured, as well as perturbations. The resulting conditions ensure the exponential convergence of the reconstruction error towards zero. Then, showing the input-to-state stability (ISS) property of the system, we prove that the estimated state can be used instead to asymptotically stabilize the system. Therefore, the control scheme satisfies the separation principle property. Results are illustrated by simulations in PLECS with a non-inverting Buck-Boost converter (DC-DC) and a Half-Bridge Boost PFC converter (AC-DC).

I. INTRODUCTION

With the development of electrical applications, the use of switched power converters has increased over the last years. For many applications, the required level of efficiency implies the need for appropriate control laws. This context has led to revisit control problems of power converters with many efforts from the automatic control community [1].

Power converters are often modeled as hybrid systems [2], and more specifically switched affine systems (SAS). They are characterized by a set of continuous dynamic subsystems, and a switched control law that selects the active subsystem (or mode) [3]. This nature comes from the fact that the dynamic of a converter depends on the state of switches in its circuit. The analysis and design of control laws for these systems are difficult to handle, but fortunately, many results have been proposed in recent years (see [1], [3], [4] and references therein).

In the literature, most widely studied control solutions for power converters are based on averaged linearized models. Linearization allows to apply well-known and mastered linear control methods, to handle model uncertainties for example, and to avoid difficulties stemming from discontinuous nature of the system [4], [5]. Simplified models were also considered, with nonlinear control techniques [6]–[8].

More recently, an effort has been done to propose control methods that take into account the discontinuous behavior of power converters. Several results have been proposed under bilinear systems formalism [5], [9], hybrid framework [2], [10] or more specifically SAS [3], [11].

Regarding the observer based control problems, much less papers have investigated this topic for power converters

[12]–[14]. This topic may be relevant when the state is not measured or available to compute the control input, in consistence with real implementation issues [15]. Observers are also useful to limit the number of sensors and therefore the production cost, or to estimate relevant information in the circuit for maintenance. Finally, they can be used in an adaptive control scheme, to reconstruct unmeasured perturbations or model mismatches that may affect a converter in practical problems [16]–[18].

In this paper, we first reformulate a unified modeling methodology for power converters [19] so as to address the observer based control problem with a generic approach where disturbances are measured. The well-known argmin state feedback control law [3], [18] is recalled and adapted for the aforementioned model. It belongs to the so-called class of direct control. The design of the observer is based on a set of linear matrix inequalities (LMIs) built from a Lyapunov analysis. The main contribution of this paper is to prove that the reconstructed state can be used to compute the control law. The proof exploits the Input-to-State Stability (ISS) property of the system and we show that the separation principle applies. The methodology is illustrated with two converters: a non-inverting Buck-Boost and a Half Bridge Boost PFC converter. The simulations, carried out with the PLECS software, show the effectiveness of the proposed results.

II. MODELING AND STATE FEEDBACK CONTROL

A. Modeling

The modeling approach in this paragraph intends to formalize the process to get a switched affine system representation, widely used in the control community, from physical equations, for a class of converters. Power converters considered in this paper satisfy the following assumption:

Assumption 1: The framework chosen in this paper considers that the input supply voltage v_{in} , the system output y as well as the disturbances ω are measured. Moreover, the circuit operates only in a continuous conduction mode.

Considering Assumption 1 and applying Kirchhoff's laws, a physical modeling leads to the following general bilinear model:

$$\begin{cases} \dot{x} = (A_0 + \sum_{i=1}^m u_i A_{u_i})x + (B_0 + \sum_{i=1}^m u_i B_{u_i})v_{in} \\ \quad + (B_{\omega 0} + \sum_{i=1}^m u_i B_{\omega u_i})\omega \\ y = (C_0 + \sum_{i=1}^m u_i C_{u_i})x + (D_{\omega 0} + \sum_{i=1}^m u_i D_{\omega u_i})\omega \end{cases} \quad (1)$$

where:

- $x \in \mathbb{R}^n$ is the state vector, usually composed of inductor currents and capacitor voltages
- $y \in \mathbb{R}^p$ is the output vector
- $\omega \in \mathbb{R}^d$ is the perturbation signal
- v_{in} is the supply voltage
- matrices $A_0, A_{u_i}, B_0, B_{u_i}, B_{\omega_0}, B_{\omega u_i}, C_0, C_{u_i}, D_{\omega_0}$ and $D_{\omega u_i}$ are constant matrices of appropriate dimensions composed of electrical component values.

Since there are m binary variables u_i , there exists $M = 2^m$ possible combinations, and therefore M possible configurations of the circuit. Then, M new variables can be introduced, denoted by λ_i . By ordering u_i 's combinations in binary order, let us define a mode number to each configuration. In this approach, λ_i equals one if and only if the combination of u_i correspond to mode i . Basically, variables λ_i 's select the current circuit configuration. Table 1 summarizes the mode definitions, change of variables and expressions of λ_i and associated matrices A_{λ_i} (other matrices expressions follow the same logic). With this change of variable, the model can be rewritten in the following polytopic form:

$$\begin{cases} \dot{x} = (A_0 + \sum_{i=1}^M \lambda_i A_{\lambda_i})x + (B_0 + \sum_{i=1}^M \lambda_i B_{\lambda_i})v_{in} \\ \quad + (B_{\omega_0} + \sum_{i=1}^M \lambda_i B_{\omega \lambda_i})\omega \\ y = (C_0 + \sum_{i=1}^M \lambda_i C_{\lambda_i})x + (D_{\omega_0} + \sum_{i=1}^M \lambda_i D_{\omega \lambda_i})\omega \end{cases} \quad (2)$$

Let us now gather the λ_i variables in a vector λ . The latter belongs to the set:

$$\Lambda_S = \{\lambda \in \{0, 1\}^M : \sum_{i=1}^M \lambda_i = 1\} \quad (3)$$

Finally, to obtain a compact matrix form, let us define the following matrices:

$$\begin{aligned} B_0(x) &= [A_{\lambda_1}x + (B_0 + B_{\lambda_1})v_{in} \ \dots \\ &\quad \dots \ A_{\lambda_M}x + (B_0 + B_{\lambda_M})v_{in}] \\ B_\omega(\omega) &= [(B_{\omega_0} + B_{\omega \lambda_1})\omega \ \dots \ (B_{\omega_0} + B_{\omega \lambda_M})\omega] \\ D_0(x) &= [C_{\lambda_1}x \ \dots \ C_{\lambda_M}x] \\ D_\omega(\omega) &= [(D_{\omega_0} + D_{\omega \lambda_1})\omega \ \dots \ (D_{\omega_0} + D_{\omega \lambda_M})\omega] \end{aligned}$$

and the model is then expressed as:

$$\begin{cases} \dot{x} = A_0x + (B_0(x) + B_\omega(\omega))\lambda \\ y = C_0x + (D_0(x) + D_\omega(\omega))\lambda \end{cases} \quad (4)$$

This model belongs to the class of SAS since λ selects a configuration of the circuit and the expression is affine in x .

B. State feedback control using measured perturbations

Considering converters with DC output voltage, the aim of the control is to stabilize the output at a given reference, denoted y_e . First, an operating trajectory associated to the reference needs to be defined. The notion of operating trajectory has no meaning for a switched system, but it does for a relaxed version of the model, where λ belongs to:

$$\Lambda_R = \{\lambda \in [0, 1]^M : \sum_{i=1}^M \lambda_i = 1\} \quad (5)$$

It represents the so-called averaged model. An interesting property to be emphasized is that trajectories of the switched model are dense into trajectories of the averaged model [20]. As a consequence, trajectories of the averaged model can be arbitrarily approximated by those of the switched system (4). Therefore, the set of operating trajectories of the averaged model is of great interest for our problem. Consequently, solutions of system (4) will be next considered in the sense of Filippov [20]. In order to maintain the output constant, the effect of perturbations has to be taken into account with an appropriate set of operating trajectories:

$$X_{e\omega} = \{x_e : \mathbb{R} \rightarrow \mathbb{R}^n, \lambda_e : \mathbb{R} \rightarrow \Lambda_R : A_0x_e + (B_0(x_e) + B_\omega(\omega))\lambda_e = \dot{x}_e\} \quad (6)$$

In relation with this set, the set of reachable outputs can also be defined as:

$$Y_{e\omega} = \{y_e \in \mathbb{R}^p, x_e \in X_{e\omega}, \lambda_e \in \Lambda_R : y_e = C_0x_e + (D_0(x_e) + D_\omega(\omega))\lambda_e\}$$

Due to the converter gain between the input voltage and the output voltage being limited, disturbances should be limited to ensure that output reference is always reachable:

Assumption 2: The perturbation signal ω belongs to the following set:

$$\mathbb{D} = \{\omega \in \mathbb{R}^d \mid y_e \in Y_{e\omega}\}$$

The control problem to address in this section is to design a switching state feedback control law $\lambda : \mathbb{R} \rightarrow \Lambda_S$ to asymptotically stabilize system (4) at an operating trajectory $x_e \in X_{e\omega}$ related to the desired output $y_e \in Y_{e\omega}$. Assuming the state x is temporarily available, Theorem 1 is proposed to solve this problem.

Theorem 1: Consider system (4) and an operating trajectory $x_e : \mathbb{R} \rightarrow X_{e\omega}$ as defined in (6). Let $e := x - x_e$ be the error signal. Under Assumptions 1 and 2 and assuming x is measured, for a given symmetric positive definite matrix $Q_C \in \mathbb{R}^{n \times n}$, if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ such that the LMI

$$P(A_0 + A_{\lambda_i}) + (A_0 + A_{\lambda_i})^T P + 2Q_C < 0 \quad (7)$$

holds $\forall i = 1, \dots, M$, then the control law

$$\lambda^* = \arg \min_{d \in \Lambda_S} \left(e^T P (A_0x + (B_0(x) + B_\omega(\omega))d) \right) \quad (8)$$

makes the origin of e globally asymptotically stable (GAS). Thus the state x converges to the operating trajectory x_e .

Proof: Consider the Lyapunov function $V(e) = \frac{1}{2}e^T P e$ where $P = P^T > 0$, $V(0) = 0$ and $V(e) > 0, \forall e \neq 0$. Let us calculate the time derivative:

$$\dot{V}(e) = e^T P (A_0x + (B_0(x) + B_\omega(\omega))\lambda - \dot{x}_e)$$

For the proposed switching control λ^* (8), by construction:

$$e^T P ((B_0(x) + B_\omega(\omega))\lambda^*) \leq e^T P ((B_0(x) + B_\omega(\omega))\lambda_e)$$

Therefore:

$$\begin{aligned} \dot{V}(e) &\leq e^T P (A_0x + (B_0(x) + B_\omega(\omega))\lambda_e - \dot{x}_e) \\ &= e^T P (A_0 + \sum_{i=1}^M \lambda_{ei} A_{\lambda_i})e \leq -e^T Q_C e < 0 \end{aligned}$$

TABLE I: Modes and change of variables from the physical model (1) to the polytopic model (2).

Mode(i)	u_1	u_2	...	u_{m-1}	u_m	λ_i	A_{λ_i}
1	0	0	...	0	0	$(1-u_1)\dots(1-u_m)$	0
2	0	0	...	0	1	$(1-u_1)\dots(1-u_{m-1})u_m$	A_{u_m}
3	0	0	...	1	0	$(1-u_1)\dots u_{m-1}(1-u_m)$	$A_{u_{m-1}}$
4	0	0	...	1	1	$(1-u_1)\dots u_{m-1}u_m$	$A_{u_{m-1}} + A_{u_m}$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
$2^m - 1$	1	1	...	1	0	$u_1 u_2 \dots u_{m-1} (1 - u_m)$	$A_{u_1} + \dots + A_{u_{m-1}}$
2^m	1	1	...	1	1	$u_1 u_2 \dots u_{m-1} u_m$	$A_{u_1} + \dots + A_{u_m}$

The last inequality is obtained from LMI condition (7). Since $V(e) < 0, \forall e \neq 0$ and the dynamic of e does not jump when the control input changes, we can conclude that the origin of e is globally asymptotically stable (GAS). ■

III. OBSERVER DESIGN

In this section, we want to design an observer to asymptotically reconstruct the state of system (4). To do so, Theorem 2 is proposed:

Theorem 2: Under Assumption 1, for a given symmetric positive definite matrix $Q_O \in \mathbb{R}^{n \times n}$, if there exists a symmetric positive definite matrix $S \in \mathbb{R}^{n \times n}$ and matrices $W_i \in \mathbb{R}^{n \times p}$ such that the LMI

$$\begin{aligned} & (A_0 + A_{\lambda_i})^T S + S(A_0 + A_{\lambda_i}) \\ & - (C_0 + C_{\lambda_i})^T W_i^T - W_i(C_0 + C_{\lambda_i}) + 2Q_O < 0 \end{aligned} \quad (9)$$

holds $\forall i = 1, \dots, M$, then the observer

$$\begin{cases} \dot{\hat{x}} = A_0 \hat{x} + (B_0(\hat{x}) + B_\omega(\omega))\lambda + \sum_{i=1}^M \lambda_i L_{\lambda_i} (y - \hat{y}) \\ \hat{y} = C_0 \hat{x} + (D_0(\hat{x}) + D_\omega(\omega))\lambda \end{cases} \quad (10)$$

where $L_{\lambda_i} = S^{-1}W_i$ ensures the convergence of \hat{x} to x .

Proof: Let $\varepsilon := x - \hat{x}$ be the reconstruction error signal. Let's first evaluate its dynamic:

$$\begin{aligned} \dot{\varepsilon} &= A_0 x + (B_0(x) + B_\omega(\omega))\lambda - A_0 \hat{x} \\ &\quad - (B_0(\hat{x}) + B_\omega(\omega))\lambda - \sum_{i=1}^M \lambda_i L_{\lambda_i} (C_0 + C_{\lambda_i})\varepsilon \\ &= \sum_{i=1}^M \lambda_i (A_0 + A_{\lambda_i} - L_{\lambda_i} (C_0 + C_{\lambda_i}))\varepsilon \end{aligned}$$

Consider the Lyapunov function $V(\varepsilon) = \varepsilon^T S \varepsilon$ where $S = S^T > 0$. $V(0) = 0$ and $V(\varepsilon) > 0, \forall \varepsilon \neq 0$. Then,

$$\begin{aligned} \dot{V}(\varepsilon) &= \left(\sum_{i=1}^M \lambda_i (A_0 + A_{\lambda_i} - L_{\lambda_i} (C_0 + C_{\lambda_i})) \right)^T S \varepsilon \\ &\quad + \varepsilon^T S \left(\sum_{i=1}^M \lambda_i (A_0 + A_{\lambda_i} - L_{\lambda_i} (C_0 + C_{\lambda_i})) \right) \varepsilon \\ &= \varepsilon^T \left(\sum_{i=1}^M \lambda_i (t) \left((A_0 + A_{\lambda_i})^T S + S(A_0 + A_{\lambda_i}) \right. \right. \\ &\quad \left. \left. - S L_{\lambda_i} (C_0 + C_{\lambda_i}) - (C_0 + C_{\lambda_i})^T L_{\lambda_i}^T S \right) \right) \varepsilon \\ &< -2\varepsilon^T Q_O \varepsilon < 0 \end{aligned}$$

The last inequality is obtained from condition (9) by defining $W_i = S L_{\lambda_i}$. Since $\dot{V}(\varepsilon) < 0, \forall \varepsilon \neq 0$ and the dynamics of ε does not jump when the control input changes, we can conclude that the origin of ε is GAS. Moreover, it decreases exponentially with decay $\frac{\lambda_{\min}(Q_O)}{\lambda_{\max}(S)}$ ([21] Chapter 4.5) ■

IV. OBSERVER BASED CONTROL

Let us now exploit the estimated state \hat{x} to compute an output feedback control law. Indeed, we want to design a switching output feedback control law $\lambda : \mathbb{R} \rightarrow \Lambda_S$ to asymptotically stabilize the system (4) at an operating trajectory $x_e \in X_{e\omega}$ related to the desired output $y_e \in Y_{e\omega}$. The following theorem shows that for system (4), the separation principle applies, and thus the observer (10) can be designed independently from the control law.

Theorem 3: Consider system (4) and an operating trajectory $x_e \in X_{e\omega}$ as defined in (6). Let $\tilde{e} := \hat{x} - x_e$ be the estimated error signal, where \hat{x} is the state estimated by the observer (10). Under Assumption 1 and 2, for a given symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ solution of (7), the control law

$$\lambda^* = \arg \min_{d \in \Lambda_S} \left(\tilde{e}^T P (A_0 \hat{x} + (B_0(\hat{x}) + B_\omega(\omega))d) \right) \quad (11)$$

makes the origin of e GAS.

Proof: Consider the Lyapunov function $V(\tilde{e}) = \frac{1}{2} \tilde{e}^T P \tilde{e}$ where $P = P^T > 0$ is solution of (7), $V(0) = 0$ and $V(\tilde{e}) > 0, \forall \tilde{e} \neq 0$. Then,

$$\begin{aligned} \dot{V}(\tilde{e}) &= \tilde{e}^T P \left(A_0 \hat{x} + (B_0(\hat{x}) + B_\omega(\omega))\lambda - \dot{x}_e \right. \\ &\quad \left. + \sum_{i=1}^M \lambda_i L_{\lambda_i} (C_0 + C_{\lambda_i})\varepsilon \right) \\ &= \tilde{e}^T P \left(A_0 \tilde{e} + B_0(\hat{x})\lambda - B_0(x_e)\lambda_e \right. \\ &\quad \left. + B_\omega(\omega)(\lambda - \lambda_e) + \sum_{i=1}^M \lambda_i L_{\lambda_i} (C_0 + C_{\lambda_i})\varepsilon \right) \end{aligned}$$

For the proposed switching control λ^* (11), by construction:

$$\tilde{e}^T P \left((B_0(\hat{x}) + B_\omega(\omega))\lambda^* \right) \leq \tilde{e}^T P \left((B_0(\hat{x}) + B_\omega(\omega))\lambda_e \right)$$

Therefore:

$$\begin{aligned} \dot{V}(\tilde{e}) &\leq \tilde{e}^T P \left(A_0 \tilde{e} + (B_0(\hat{x}) - B_0(x_e))\lambda_e \right) \\ &\quad + \tilde{e}^T P \sum_{i=1}^M \lambda_i L_{\lambda_i} (C_0 + C_{\lambda_i})\varepsilon \\ &= \tilde{e}^T P \left(A_0 + \sum_{i=1}^M \lambda_{e_i} A_{\lambda_i} \right) \tilde{e} \\ &\quad + \tilde{e}^T P \sum_{i=1}^M \lambda_i L_{\lambda_i} (C_0 + C_{\lambda_i})\varepsilon \end{aligned}$$

Inequality (7) implies

$$\dot{V}(\tilde{e}) \leq -\tilde{e}^T Q_C \tilde{e} + \tilde{e}^T P \sum_{i=0}^M \lambda_i L_{\lambda_i} (C_0 + C_{\lambda_i})\varepsilon$$

Consequently,

$$\dot{V}(\tilde{e}) \leq -(1 - \theta) \tilde{e}^T Q_C \tilde{e} < 0$$

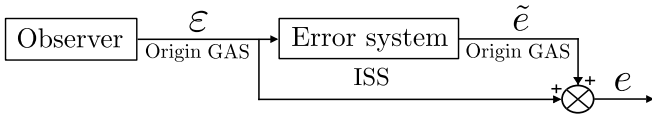


Fig. 1: Scheme of the proof.

if

$$-\theta \tilde{e}^T Q_C \tilde{e} + \tilde{e}^T P \sum_{i=1}^M \lambda_i L_{\lambda_i} (C_0 + C_{\lambda_i}) \varepsilon \leq 0$$

for some scalar $0 < \theta < 1$. This condition holds for

$$\theta \tilde{e}^T Q_C \tilde{e} \geq \tilde{e}^T P \sum_{i=1}^M \lambda_i L_{\lambda_i} (C_0 + C_{\lambda_i}) \varepsilon$$

This latter inequality is implied by the norm condition

$$\theta \|\tilde{e}\|^2 \lambda_{\min}(Q_C) \geq \|\tilde{e}\| \lambda_{\max}(P) \max_{i=1, \dots, M} (\|L_{\lambda_i}\| \|C_0 + C_{\lambda_i}\|) \|\varepsilon\|$$

Hence, a sufficient condition is

$$\|\tilde{e}\| \geq \frac{\lambda_{\max}(P) \max_{i=1, \dots, M} (\|L_{\lambda_i}\| \|C_0 + C_{\lambda_i}\|)}{\theta \lambda_{\min}(Q_C)} \|\varepsilon\|$$

It has been proven that the error system with state \tilde{e} has the ISS (Input-to-State Stability) property with respect to the input ε (see [21] Chapter 4.9). Since the origin of ε is GAS and the error system \tilde{e} is ISS, it implies that the origin of \tilde{e} is GAS. Finally, noticing that $e = \tilde{e} + \varepsilon$, we can conclude that the origin of e is GAS too (see Figure 1 for a scheme of the proof). ■

Remark 1: Papers in the literature are either specific to a particular converter or very general in the context of SAS or bilinear systems, with illustration on simple DC-DC converters [14], [18]. In this paper, we aim at proposing a methodology that can be directly apply to address modeling and control design of different type of converters. For instance, in the next section, a DC-DC converter with two switches and a AC-DC converter are considered. In our approach, perturbations are measured. This makes it possible to manage non-piecewise-constant perturbations, unlike conventional integral approaches.

V. SIMULATION EXAMPLES

To illustrate the proposed results, two examples are considered: a non-inverting Buck-Boost Converter and a Half-Bridge Boost PFC Converter. Simulations are performed using Simulink and PLECS software.

A. Non-inverting Buck-Boost Converter

As a first example, a Buck-Boost Non-inverting Converter shown in Figure 2 is considered. There are 4 switches for two control variables u_1 and u_2 . The input voltage V_{in} is affected by some perturbations ω that are measured. Let us first establish the electrical equations via Kirchhoff's laws:

$$\begin{cases} L \dot{i}_L = -r_L i_L - (1 - u_2) V_{out} + u_1 (V_{in} + \omega) \\ C \dot{v}_C = (1 - u_2) i_L - \frac{V_{out}}{R} \\ V_{out} = v_C + r_C C \dot{v}_C \end{cases}$$

TABLE II: Parameters of simulations.

Buck-Boost		Half-Bridge Boost PFC	
Parameters	Values	Parameters	Values
L	220 μ H	L	5 mH
C	22 μ F	C	2 mF
R	100 Ω	R	200 Ω
r_L	0.3 Ω	r_L	0.4 Ω
r_C	0.02 Ω	r_C	20 k Ω
$V_{in} + \omega$	[5;11.4] V	V	120 V
		ω_0	$2\pi \times 50$ rad/s

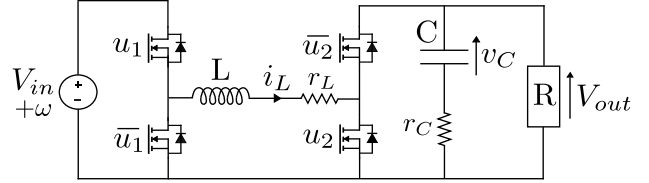


Fig. 2: Scheme of the Buck-Boost noninverting Converter.

Taking $x = [i_L \ v_C]^T$ as a state vector, $\alpha = \frac{R}{R+r_C}$ and V_{out} as the output, a state space representation of the form (1) is obtained with matrices:

$$\begin{aligned} A_0 &= \begin{bmatrix} \frac{-r_L - \alpha r_C}{L} & -\frac{\alpha}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} & A_{u_1} &= 0 & A_{u_2} &= \begin{bmatrix} \frac{\alpha r_C}{L} & \frac{\alpha}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} \\ B_0 &= B_{\omega_0} = B_{u_2} = B_{\omega_{u_2}} = 0 & B_{u_1} &= B_{\omega_{u_1}} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \\ C_0 &= [\alpha r_C \ \alpha] & C_{u_1} &= 0 & C_{u_2} &= [-\alpha r_C \ 0] \end{aligned}$$

Following the methodology presented in Section II to obtain a model of the form (2) and then (4), the change of variable according to Table I yields:

$$\begin{aligned} A_{\lambda_1} &= 0 & B_{\lambda_1} &= B_{\omega \lambda_1} = 0 & C_{\lambda_1} &= 0 & \lambda_1 &= \bar{u}_1 \bar{u}_2 \\ A_{\lambda_2} &= A_{u_2} & B_{\lambda_2} &= B_{\omega \lambda_2} = 0 & C_{\lambda_2} &= C_{u_2} & \lambda_2 &= \bar{u}_1 u_2 \\ A_{\lambda_3} &= 0 & B_{\lambda_3} &= B_{\omega \lambda_3} = B_{u_1} & C_{\lambda_3} &= 0 & \lambda_3 &= u_1 \bar{u}_2 \\ A_{\lambda_4} &= A_{u_2} & B_{\lambda_4} &= B_{\omega \lambda_4} = B_{u_1} & C_{\lambda_4} &= 0 & \lambda_4 &= u_1 u_2 \end{aligned}$$

where $\bar{u}_i = 1 - u_i$. The set of operating trajectories is:

$$X_{e\omega} = \left\{ x_e = \begin{bmatrix} \frac{(V_{in} + \omega)(\lambda_{e3} + \lambda_{e4})}{\gamma} \\ \frac{(V_{in} + \omega)R(\lambda_{e1} + \lambda_{e3})(\lambda_{e3} + \lambda_{e4})}{\gamma} \end{bmatrix}, \lambda_e \in \Lambda_R \right\}$$

where $\gamma = r_L + r_C \alpha (\lambda_{e1} + \lambda_{e3}) + \alpha R (\lambda_{e1} + \lambda_{e3})^2$. Considering y_e , it can also be written as:

$$X_{e\omega} = \left\{ x_e = \begin{bmatrix} \frac{2\alpha y_e^2}{\beta + \sqrt{\beta^2 - 4\alpha R y_e^2 r_L}} \\ y_e \end{bmatrix}, \lambda_e \in \Lambda_R \right\}$$

where $\beta = (V_{in} + \omega)R(\lambda_{e3} + \lambda_{e4}) - \alpha r_C y_e$. Component parameters are summarized in Table II. The output voltage reference is $y_e = 24V$ and the initial conditions of the system are $x_0 = [0.1 \ 5]^T$. We set arbitrarily:

$$Q_C = \begin{bmatrix} 10 & 0 \\ 0 & 30 \end{bmatrix} \quad Q_O = \begin{bmatrix} 2 & 0 \\ 0 & 0.1 \end{bmatrix}$$

Using MATLAB and a SDP solver to minimize the trace of P and S while satisfying (7), (9) and $S > 10^{-4} \mathbb{I}$, we obtain:

$$P = \begin{bmatrix} 6 \times 10^{-1} & 9.4 \times 10^{-3} \\ 9.4 \times 10^{-3} & 6.63 \times 10^{-2} \end{bmatrix}$$

$$L_{\lambda_1 | \lambda_2 | \lambda_3 | \lambda_4} = \begin{bmatrix} -1.46 & 0 & -1.44 & 0 \\ 5.76 & 5.79 & 5.81 & 5.79 \end{bmatrix} \times 10^3$$

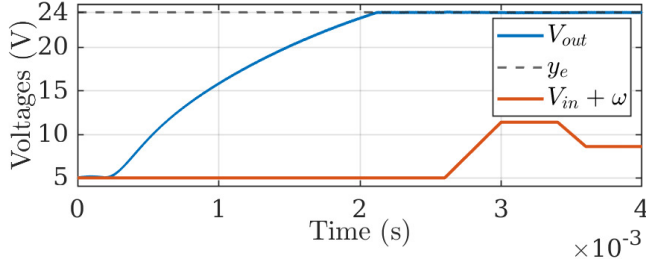


Fig. 3: V_{out} , its reference $y_e = 24$ V and perturbed V_{in} .

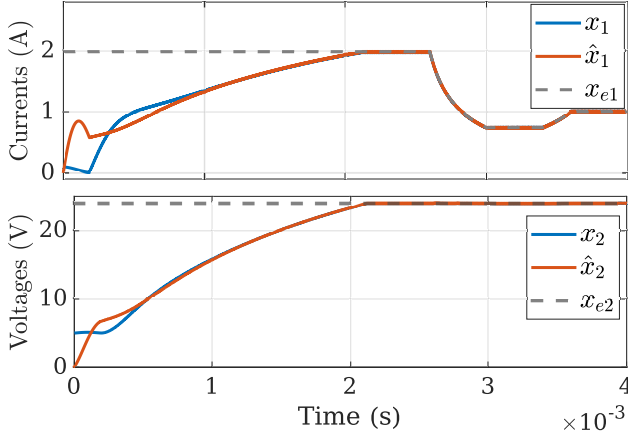


Fig. 4: System and observer states with references.

The output voltage and its reference are presented in Figure 3. As expected, it can be observed the output converges to the reference even with the perturbation on V_{in} , shown in Figure 3 too.

The evolution of the states of the system and the observers with their references is plotted in Figure 4. We can observe that for both state variables, the observer state \hat{x} reaches the system state x as predicted. As desired, the first state follows its operating trajectory updated with the perturbation variations, while the second state stabilizes at its constant reference from the prescribed reference y_e .

B. Half-Bridge Boost PFC Converter

As a second example, the Half Bridge Boost PFC Converter shown in Figure 5 is considered. There are two perfect switches associated to the binary control variable u_1 . The electrical equations of the circuit are:

$$\begin{cases} L\dot{i}_L = V_{in} - r_L i_L + (1 - u_1)v_{C1} - u_1 v_{C2} \\ C\dot{v}_{C1} = -(1 - u_1)i_L - \frac{V_{out}}{R} - \frac{v_{C1}}{r_C} \\ C\dot{v}_{C2} = u_1 i_L - \frac{V_{out}}{R} - \frac{v_{C2}}{r_C} \\ V_{out} = v_{C1} + v_{C2} \end{cases} \quad (12)$$

For the sake of control, the chosen state vector is $x = [i_L \ v_+ \ v_-]^T$ where $v_+ = v_{C1} + v_{C2}$ and $v_- = v_{C1} - v_{C2}$. With V_{out} as the output and $R_e = \frac{Rr_C}{R+2r_C}$, a state space representation of the form (1) is obtained with matrices:

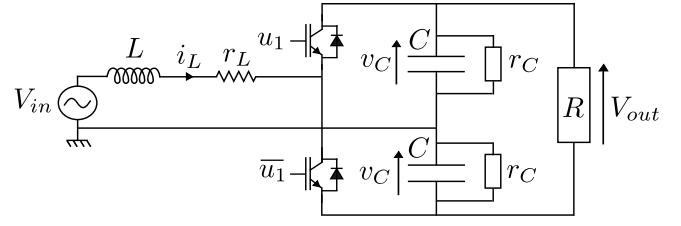


Fig. 5: Scheme of the Half Bridge Boost PFC Converter.

$$A_0 = \begin{bmatrix} \frac{-r_L}{L} & \frac{1}{2L} & \frac{1}{2L} \\ -\frac{1}{C} & -\frac{1}{R_e C} & 0 \\ -\frac{1}{C} & 0 & -\frac{1}{Cr_C} \end{bmatrix} \quad A_{u_1} = \begin{bmatrix} 0 & -\frac{1}{L} & 0 \\ \frac{2}{C} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} \quad B_{u_1} = 0 \quad C_0 = [0 \ 1 \ 0] \quad C_{u_1} = 0$$

As previously, Table I enables to derive straightforwardly the switch affine systems (2) and (4) with matrices:

$$\begin{aligned} A_{\lambda_1} &= 0 & B_{\lambda_1} &= 0 & C_{\lambda_1} &= 0 & \lambda_1 &= (1 - u_1) \\ A_{\lambda_2} &= A_{u_1} & B_{\lambda_2} &= 0 & C_{\lambda_2} &= 0 & \lambda_2 &= u_1 \end{aligned}$$

The computation of the operating trajectory for a AC-DC converter is more involved. A way to deal with it, is to use (12) to study the power balance of the system:

$$Li_L \dot{i}_L + \frac{C}{2}(\dot{v}_- v_- + \dot{v}_+ v_+) = i_L(V_{in} - r_L i_L) - \frac{v_+^2}{2R_e} - \frac{v_-^2}{2r_C}$$

The input voltage is $V_{in}(t) = V \sin(\omega_0 t)$, so i_L must be in phase with it to ensure a unit power factor. Therefore, its reference is $i_{L_e} = I \sin(\omega_0 t)$ where I has to be determined. Other objectives are $v_+ = y_e$ and $v_- = 0$. With those conditions, the power balance at the reference is:

$$LI^2 \omega \sin(\omega t) \cos(\omega t) = I(V - r_L I) \sin^2(\omega t) - \frac{y_e^2}{2R_e}$$

Integrating between 0 and $T = \frac{2\pi}{\omega_0}$, we obtain:

$$0 = \frac{I}{2}(V - r_L I) - \frac{y_e^2}{2R_e} \Leftrightarrow I = \frac{V}{2r_L} \left(1 \pm \sqrt{1 - \frac{4r_L y_e^2}{R_e V^2}} \right)$$

The smallest solution is selected to minimize power. Component parameters are summed up in Table II. The output voltage reference y_e is 300V and the initial conditions of the system are $x_0 = [3 \ 20 \ 2]^T$. The approximated operating trajectory is $x_e = [7.7 \sin(\omega_0 t) \ 300 \ 0]^T$. We set arbitrarily:

$$Q_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 100 \end{bmatrix} \quad Q_O = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Using MATLAB and a SDP solver to minimize the trace of P and S while satisfying (7), (9) and $S > \mathbb{I}$, we obtain:

$$P = \begin{bmatrix} 85.12 & 0 & -1.96 \\ 0 & 20.01 & 0 \\ -1.96 & 0 & 20.27 \end{bmatrix} \quad L_{\lambda_1|\lambda_2} = \begin{bmatrix} -39.78 & 39.77 \\ 44.26 & 44.34 \\ -36.17 & 36.16 \end{bmatrix}$$

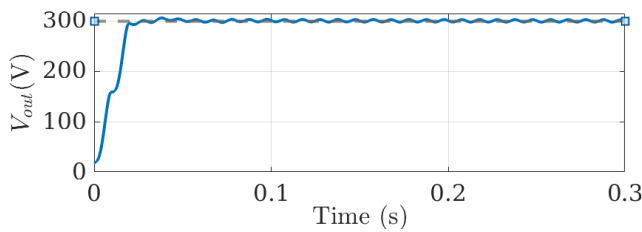


Fig. 6: V_{out} and its reference $y_e = 300V$.

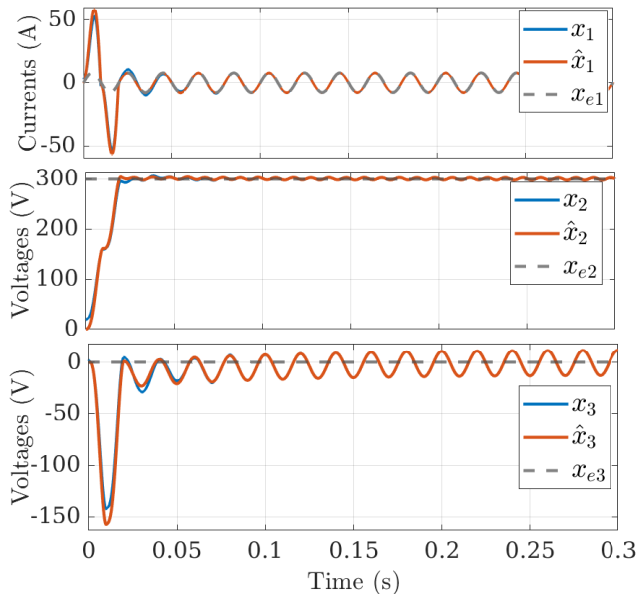


Fig. 7: System and observer states with references.

The output voltage and its reference are presented in Figure 6. As expected, it can be observed that the output converges to reference with a remaining AC part.

The evolution of observer and system states are reported in Figure 7. We can see that the first state follows its reference, which means that the input current is in phase with the input voltage as desired. The second and third states oscillate around their references, which is normal since those references are averaged approximations.

VI. CONCLUSION

In this paper, a unified approach for observer based control of DC output power converters has been presented. The proposed methodology is based on two separate LMI conditions for the control and the observer parts, efficiently solved with numerical tools. The stability of the system with the proposed control is assessed with a Lyapunov analysis and using the ISS property. The applicability of the approach as been illustrated with two examples using PLECS. The next step is to validate those results on an experimental setup. In this paper, perturbations are measured, we aim at considering unknown perturbations in a future work.

REFERENCES

[1] A. H. R. Rosa, L. M. F. Morais, G. O. Fortes, and S. I. Seleme Júnior, "Practical considerations of nonlinear control techniques applied to

static power converters: A survey and comparative study," *International Journal of Electrical Power & Energy Systems*, vol. 127, p. 106545, May 2021.

[2] C. Albea, G. Garcia, and L. Zaccarian, "Hybrid dynamic modeling and control of switched affine systems: Application to DC-DC converters," in *2015 54th IEEE Conference on Decision and Control (CDC)*, Dec. 2015, pp. 2264–2269.

[3] G. Deaecto, J. Geromel, F. Saldanha Garcia, and J. Pomilio, "Switched affine systems control design with application to DC-DC converters," *Control Theory & Applications, IET*, vol. 4, pp. 1201–1210, Aug. 2010.

[4] H. J. Sira-Ramirez and R. Silva-Ortigoza, *Control Design Techniques in Power Electronics Devices*. Springer Science & Business Media, Sept. 2006.

[5] C. Olalla, I. Queinnec, R. Leyva, and A. El Aroudi, "Optimal State-Feedback Control of Bilinear DC-DC Converters With Guaranteed Regions of Stability," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 10, pp. 3868–3880, Oct. 2012.

[6] A. Ndoye, R. Delpoux, L. Hetel, A. Kruszewski, J.-F. Tréguët, and X. Lin-Shi, "Robust relay control for buck converters : experimental application," in *2019 IEEE 58th Conference on Decision and Control (CDC)*, Dec. 2019, pp. 8124–8129.

[7] D. Cortes, J. Alvarez, and J. Alvarez, "Robust sliding mode control for the boost converter," in *VIII IEEE International Power Electronics Congress, 2002. Technical Proceedings. CIEP 2002.*, Oct. 2002, pp. 208–212.

[8] O. Lopez-Santos, L. Martinez-Salamero, G. Garcia, H. Valderrama-Blavi, and T. Sierra-Polanco, "Robust Sliding-Mode Control Design for a Voltage Regulated Quadratic Boost Converter," *IEEE Transactions on Power Electronics*, vol. 30, no. 4, pp. 2313–2327, Apr. 2015.

[9] V. Spinu, N. Athanasopoulos, M. Lazar, and G. Bitsoris, "Stabilization of Bilinear Power Converters by Affine State Feedback Under Input and State Constraints," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 59, no. 8, pp. 520–524, Aug. 2012.

[10] T. A. F. Theunisse, J. Chai, R. G. Sanfelice, and W. P. M. H. Heemels, "Robust Global Stabilization of the DC-DC Boost Converter via Hybrid Control," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 62, no. 4, pp. 1052–1061, Apr. 2015.

[11] M. Serieye, C. Albea-Sanchez, and A. Seuret, "Free-matrices min-projection control for high frequency DC-DC converters," in *2019 IEEE 58th Conference on Decision and Control (CDC)*, Dec. 2019.

[12] N. Gonzalez-Fonseca, J. de Leon-Morales, and J. Leyva-Ramos, "Observer-Based Controller for Switch-Mode DC-DC Converters," in *Proceedings of the 44th IEEE Conference on Decision and Control*, Dec. 2005, pp. 4773–4778.

[13] J. Wu and Z. Sun, "Observer-driven switching stabilization of switched linear systems," *Automatica*, vol. 49, no. 8, pp. 2556–2560, Aug. 2013.

[14] G. S. Deaecto, J. C. Geromel, and J. L. N. Brito, "Asymptotic stability of continuous-time switched affine systems with unknown equilibrium points*," in *2022 IEEE 61st Conference on Decision and Control (CDC)*, Dec. 2022, pp. 679–684.

[15] A. Jaafar, A. Alawieh, R. Ortega, E. Godoy, and P. Lefranc, "PI Stabilization of Power Converters With Partial State Measurements," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 2, pp. 560–568, Mar. 2013.

[16] S. Hadjeras, C. A. Sanchez, and G. Garcia, "Hybrid adaptive control of the boost converter," in *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, Dec. 2017, pp. 5720–5725.

[17] S. Baldi, A. Papachristodoulou, and E. B. Kosmatopoulos, "Adaptive pulse width modulation design for power converters based on affine switched systems," *Nonlinear Analysis: Hybrid Systems*, vol. 30, pp. 306–322, Nov. 2018.

[18] G. Buneux, P. Riedinger, J. Daafouz, and L. Grimaud, "Adaptive stabilization of switched affine systems with unknown equilibrium points: Application to power converters," *Automatica*, vol. 99, pp. 82–91, Jan. 2019.

[19] G. Garcia and O. Lopez Santos, "A Unified Approach for the Control of Power Electronics Converters. Part I—Stabilization and Regulation," *Applied Sciences*, vol. 11, no. 2, p. 631, Jan. 2021.

[20] B. Ingalls, E. D. Sontag, and Y. Wang, "An Infinite-Time Relaxation Theorem for Differential Inclusions," *Proceedings of the American Mathematical Society*, vol. 131, no. 2, pp. 487–499, 2003.

[21] H. K. Khalil, *Nonlinear systems*, 3rd ed. Upper Saddle River: Prentice Hall, 2002.