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► To cite this version:

Yassir Idmessaoud, Didier Dubois, Jérémie Guiochet. Confidence assessment in safety argument structure - Quantitative vs. qualitative approaches. International Journal of Approximate Reasoning, 2024, 165, pp.109100. 10.1016/j.ijar.2023.109100 . hal-04342922

HAL Id: hal-04342922
https://laas.hal.science/hal-04342922

Submitted on 13 Dec 2023

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Confidence assessment in safety argument structure - Quantitative vs. qualitative approaches

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Abstract

Some safety standards (e.g., ISO 26262 in automotive industry) propose the use of argument structures to justify that the high-level safety properties of a system have been ensured. The goal structuring notation (GSN) is a graphical tool used to represent these argument structures. However, this approach does not address the uncertainties that may affect the validity of the arguments. Thus, some authors proposed to complement GSN patterns with a quantitative confidence assessment procedure. In this paper, we first present a refined procedure that expresses the relation between premises (pieces of evidence) and the conclusion (top-goal to be demonstrated) using logical expressions. Then using Dempster-Shafer theory, we quantify uncertainty on each expression to build an explicit mathematical formula for propagating uncertainty to the conclusion. Inputs for the propagation model are collected from experts and transformed into numerical values using an improved elicitation model. Afterwards, we introduce a purely qualitative alternative to the quantitative procedure based on the theory of qualitative capacities. Finally, we adapt the propagation and elicitation models to this framework.

Keywords: Assurance cases, Safety cases, Dempster-Shafer Theory (DST), Confidence propagation, Expert elicitation, Goal Structuring Notation (GSN), Qualitative Capacity Theory (QCT).

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1. Introduction

Safety critical systems developed in industries such as aerospace, railway or automotive are more and more complex. Hence more reliable safety assessment procedures are needed. For instance, the development of technologies based on artificial intelligence (e.g. machine learning) rises the issue of safety, since these techniques are not yet fully understood nor covered by safety standards. Argument structures are often used to argue in favor of safety. The so-called Goal Structuring Notation (GSN) is used to replace textual argument structures that are not always easy to operate. This graphical tool is well structured, clear and concise (compared to the text format). It allows a better visualization and understanding of the arguments. It explains how a top goal is reached by providing a body of evidence supporting it. Nevertheless, it has been noticed [1] that this representation poorly describes how the pieces of evidence interact to support a goal nor does it assess how much evidence-based confidence can be granted to them. This point raises the issue of uncertainty management in argument structures. Two main challenges emerge: (1) how to propagate uncertainties in argument structures, and (2) how to feed these structures with expert opinions ?

Several works already proposed mathematical models to quantify and propagate confidence/uncertainty in graphical representations of argument structures like GSN. Graydon and Holloway [1], as well as Duan et al.[2], stated and discussed proposals that deal with the issue of uncertainty. An important number of approaches used probability theory to model uncertainty and propagate it with Bayesian networks [3, 4]. More specifically, some authors [5] transform a GSN into a Bayesian network (BBN) and propagate probabilities accordingly. Due to the limited expressiveness of the probabilistic framework, such approaches can properly deal with uncertainties due to aleatory phenomena, but they poorly represent epistemic uncertainties due to incomplete information. In addition, these methods are also very greedy in terms of data, which requires a lot of time in order to collect and process. Other approaches, like in

Yuan et al. [6], use subjective logic to also define some basic argument types and rules of confidence propagation. In contrast, we use propositional logic to define basic argument types considering the possible relationships between premises in their support of the conclusion, and Dempster-Shafer Theory (DST) to quantify and propagate confidence in GSN-based arguments.

Building such a confidence propagation model relies on input values, usually provided by experts in a qualitative form, then transformed into quantitative values. Hence, there is a need of developing an "expert opinion elicitation" approach. This method is more often used with probabilistic models. For instance, in [7], authors used an expert elicitation procedure in a risk assessment approach to fault trees. However, such a procedure can also be used in evidence theory. Ben Yaghlane et al. [8] generate belief functions from a preference relation between events provided by experts. In relation to our framework, few authors augmented their confidence assessment method by such a data elicitation procedure in order to provide quantitative values for their models. Only some authors such as [9, 10, 11] used an elicitation method that transforms expert opinions, about pieces of evidence, given in the form of qualitative values, into quantitative ones. In this paper, we also develop an elicitation model based on DST which improves those proposed in the literature, notably the works of [9, 12].

However, the naive translation of qualitative information into quantitative information is somewhat arbitrary. Furthermore, the reverse transformation back to the qualitative, using the elicitation model, can also introduce additional uncertainty to the argument. For these reasons and more listed in this paper, investigating an approach with a qualitative (as in [13]) counterpart of belief functions seems promising, both for uncertainty propagation and expert elicitation.

This paper relies on and completes preliminary works presented at conferences [14, 15, 16]. It is structured as follows. Section 2 introduces the theoretical and practical background needed to develop our approaches. Then in sections 3 and 4, we respectively present our quantitative and qualitative propagation and elicitation models. Finally, in section 5, we present our confidence assessment procedure which relies on the two previous models and conduct a preliminary comparison between the quantitative procedure and its qualitative counterpart.

2. Background

This section defines safety argument structures and introduces a graphical formalism used to model them known as *Goal Structuring Notation* (GSN). We also present some basic concepts and tools of *Dempster-Shafer Theory* (DST) and *Qualitative Capacity Theory* (QCT).

2.1. Safety argument structures

A safety argument structure, or safety case is a document that gathers a body of solid and reliable evidence demonstrating that a system is acceptably safe to accomplish a given function (or task) under given circumstances. These documents can be used, for instance, as certification tools in safety critical fields, such as automotive, railway and aerospace industries. Goal structuring notation (GSN), defined in [17], is a non-formal graphical tool, inspired from Toulmin argumentation studies [18]. As presented in Figure 1, it breaks down a top claim, called "*goal*", into elements, called "*sub-goals*" according to a "*strategy*" (that justifies this decomposition choice) and following a specific "*context*" and "*assumptions*" (restricting the argumentation to their contents). It also provides each sub-goal with a reference to pieces of evidence supporting it, called "*solutions*". The "*justification*" component gives the rationale behind the adoption of a strategy or the presentation of a goal. These seven elements can either be connected with an "*in context of*" link relating an item to the context component or by a "*supported by*" link which relates the remaining items.

Figure 1, represents a typical hazard avoidance GSN pattern. To be considered as "*acceptably safe*" (G_1) all hazards (from G_2 to G_n) of the system (X), listed in the context box (C_1), should be provably handled (Sn_1, Sn_2, ...) following the strategy (S_1). The diamonds connected to the sub-goals (i.e, G_n) means that they have not yet been achieved.

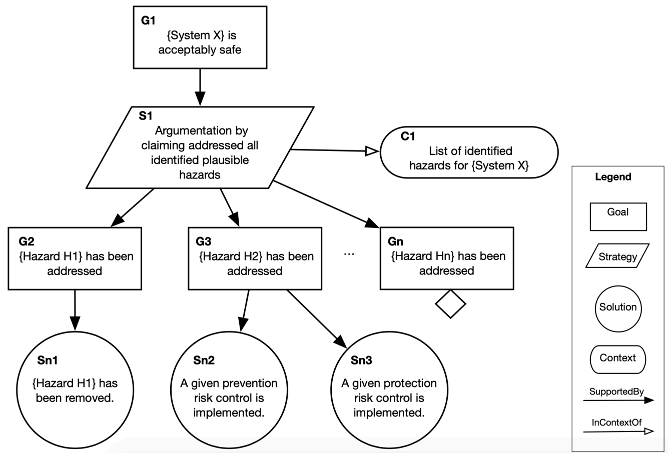

Figure 1: GSN example adapted from Hazard Avoidance Pattern [19]

"Assurance case" is a general term that includes argument structures justifying properties beyond safety (e.g, security, reliability, etc).

2.2. Dempster-Shafer Theory

Dempster-Shafer theory (DST) (aka Theory of Evidence) [20] was developed to address the issue of imprecise evidence [21]. It represents a form of generalized probability theory where probability masses are assigned to sets of possible values, instead of singletons. The idea is that there is not enough information to share a probability mass assigned to a subset among its elements. DST offers tools to model and propagate both aleatory (due to random events) and epistemic uncertainty (due to ignorance).

A mass function, or basic belief assignment (BBA), is a probability distribution over the power set of the universe of possibilities (Ω), known as the "*frame of discernment*". Formally, a mass function $m^\Omega : 2^\Omega \rightarrow [0, 1]$ is such that $\sum_{E \subseteq \Omega} m(E) = 1$, and $m(\emptyset) = 0$. Any subset E of Ω such as $m(E) > 0$ is called a focal set of m. $m(E)$ quantifies the probability that we only know that the truth lies in E; in particular $m(\Omega)$ quantifies the amount of ignorance.

A mass assignment induces a so-called belief function ($Bel : 2^\Omega \rightarrow [0, 1]$).

It represents the sum of all the masses supporting a statement of the form $x \in A \subseteq \Omega$ (x is the ill-known entity of interest); the belief function is defined by:

$$Bel(A) = \sum_{E \subseteq A, E \neq \emptyset} m(E). \tag{1}$$

Belief in the negation $\neg A$ of the statement A is called disbelief and is represented by: $Disb(A) = Bel(\neg A)$; the value $Uncer(A) = 1 - Bel(A) - Disb(A)$ quantifies the lack of information about A. It is maximal where there is neither belief nor disbelief in A. In this paper, we shall use *non-dogmatic* belief functions such that $m(\Omega) > 0$, *categoric* ones, such that $m(E) = 1$ for a single non-empty set, and *simple support* belief functions that are non-dogmatic ones such that $0 < m(E) < 1$ for a single non-empty set $E \subset \Omega$, and 0 for other subsets of Ω. In this work, a conjunctive rule of combination is used for uncertainty propagation. It combines multiple pieces of evidence (represented by mass functions m_i, with $i = 1, 2, ..., n$) coming from independent sources of information. When $n = 2$, we define $m_\cap = m_1 \otimes m_2$ such that:

$$m_\cap(A) = \sum_{E_1 \cap E_2 = A} m_1(E_1) \cdot m_2(E_2) \tag{2}$$

This combination rule is commutative and associative. The value $m_\cap(\emptyset)$ represents the degree of conflict between m_1 and m_2. Note that for the calculation of the belief function (1) when $m_\cap(\emptyset) > 0$, the condition $E \neq \emptyset$ makes full sense since otherwise $m_\cap(\emptyset)$ would appear in the expressions of belief and disbelief, then potentially violating the consistency condition $Bel(A) + Disb(A) \leq 1$.

Dempster rule of combination requires an additional normalization step when $m_\cap(\emptyset) > 0$: $m(A) = m_\cap(A)/(1 - m_\cap(\emptyset))$. However, this normalisation is questionable when the value of $m_\cap(\emptyset)$ is close to 1. Indeed, it enforces the division by a very small number, which makes the result numerically unstable. A high level of conflict indicates we must reconsider the sources of information.

2.3. Qualitative Capacity theory

In contrast, we outline the qualitative approach in [22, 23, 24]. Let L be a finite totally ordered set representing certainty levels. In Ω, the universe of possibilities (frame of discernment), a qualitative capacity (q-capacity, for short) is a function $\gamma : 2^\Omega \to L$ such that:

$$\gamma(\emptyset) = 0; \quad \gamma(\Omega) = 1; \quad A \subseteq B \Rightarrow \gamma(A) \leq \gamma(B)$$

Any q-capacity can be put in the form:

$$\gamma(A) = \max_{\emptyset \neq E \subseteq A} \rho(E), \forall A \subseteq \Omega \tag{3}$$

where ρ is formally a basic possibility assignment (BΠA) [22], namely, a possibility distribution $\rho : 2^\Omega \to L$ on the power set of Ω, such that $\max_{E \subseteq \Omega} \rho(E) = 1$ and $\rho(\emptyset) = 0$. The value $\rho(E)$ is the strength of piece of evidence E. Note the similarity between (3) and the definition of the belief function (replacing the sum by the maximum). However, several BΠA's can generate the same γ, the least of which is the qualitative Moebius transform (QMT) of γ such that:

$$\gamma_\#(A) = \begin{cases} \gamma(A) \text{ if } \gamma(A) > \gamma(A \setminus \{w\}), \forall w \in A; \\ 0 \text{ otherwise.} \end{cases} \tag{4}$$

In the paper, we shall use *non-dogmatic* q-capacities such that $\rho(\Omega) = 1$, and *simple support* q-capacities that are non-dogmatic ones such that $0 < \rho(E) \leq 1$ for a single non-empty set $E \subset \Omega$, and 0 for other subsets of Ω. The value $\gamma(A)$ (resp. $\gamma(\neg A)$) qualifies the support in favor of (resp. against) A, i.e. belief (resp. disbelief) in A using an element in the qualitative scale L. The pair $(\gamma(A), \gamma(\neg A))$ thus describes our epistemic stance with respect to A in terms of belief and disbelief, ranging from no information (i.e., (0, 0)), to full conflicting information (i.e., (1, 1)), from full belief (i.e., (1, 0)) to full disbelief (i.e., (0, 1)). This is more general than possibility theory where the case $(1, 1)$ is not allowed.

Figure 2 presents the credibility and information orderings on pairs (belief, disbelief) including extreme cases [24]. A proposition A is at least as credible as B if A is at least as supported as B, and $\neg B$ is at least as supported as $\neg A$, i.e., $\gamma(A) \geq \gamma(B)$ and $\gamma(\neg A) \leq \gamma(\neg B)$ (solid arrows from B to A); it thus ranges from certainty of falsity $(0, 1)$ up to certainty of truth $(1, 0)$. A proposition

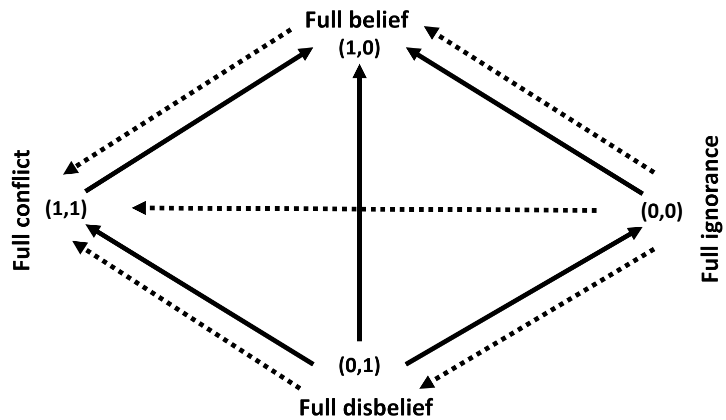

Figure 2: Evolution of certainty and information in pairs (belief, disbelief)

A is at least as informed as B if A is at least as supported as B, and $\neg A$ is at least as supported as $\neg B$, i.e., $\gamma(A) \geq \gamma(B)$ and $\gamma(\neg A) \geq \gamma(\neg B)$ (dotted arrows from B to A); it thus ranges from ignorance ($(0,0)$, no information) up to conflict ($(1,1)$, full contradictory information). When the amount of evidence supporting the conclusion is equal to the one rejecting it, we are in the situation of indecision. Equipped with these two orderings, the set $L \times L$ then possesses a bilattice structure, well-known in inconsistency-tolerant logics (more details in [24]).Note that the credibility and information orderings make sense in the quantitative case as well, comparing pairs $(Bel(A), Disb(A))$. However, the full conflict situation corresponds to $(Bel(A), Disb(A)) = (0.5, 0.5)$.

In order to qualitatively combine pieces of evidence represented by possibilistic mass functions, i.e., BΠA's ρ_i, coming from several sources of information, the qualitative counterpart of the conjunctive rule of combination for belief functions is: $\rho_\cap = \rho_1 \odot \rho_2$ such that [24]:

$$\rho_\cap(A) = \max_{E_1 \cap E_2 = A} \{\min[\rho_1(E_1), \rho_2(E_2)]\} \tag{5}$$

This is similar to (2), replacing sum by maximum and product by minimum. However, due to the use of the (idempotent) minimum operation, the combined pieces of evidence are not supposed to be independent. The result is not always a BΠA, strictly speaking. First we may have that $\rho_\cap(A) < 1$ for all A. So we must add the condition $\rho_\cap(\Omega) = 1$, which makes the combination non-associative [24]. This will not occur if we restrict to non-dogmatic BΠA's such that $\rho_i(\Omega) = 1$, which we assume in this paper. Under this restriction, the qualitative combination rule is associative. Besides, we may have that $\rho_\cap(\emptyset) > 0$, indicating conflict between the pieces of evidence.

3. Quantitative confidence elicitation and propagation models

In order to assess confidence in GSN, we first need to collect assessments about its components and transform them into usable format in the setting of DST (i.e. belief and disbelief measures). Then, we need to propagate them to the top-goal while taking into account the relation between GSN elements, notably those between a goal and its sub-goals that we will respectively call *conclusion* and *premises* in the following.

3.1. A logical approach to GSN

Apart from the relationship "*supported by*" (between a goal and its sub-goals) or "*in context of*" (between a goal and its context), a GSN does not explicitly specify the nature of the relation between a top-goal and its sub-goals. These relations are essential to justify the choice of the uncertainty propagation schemes to be used. Wang et al. [11, 12] proposed the use of equivalence ($\equiv$) to express the relation between a conclusion and its premise(s). This choice assumes that the expert has information to both validate or refute the conclusion, based on the truth or the falsity of the premise. To remain more flexible about the state of knowledge where the expert could have only information to validate, or the one to refute the conclusion or both, we decided to use material implication (denoted by $\Rightarrow$). In the following, we use Boolean variables [1] to represent premises (p) and the conclusion (C). When the premise

[1]Each variable is denoted by x when it takes the value "*True*" and $\neg x$ when it takes the value "*False*".

is sufficient to validate the conclusion, we use $p \Rightarrow C = \neg p \vee C$; when the falsity of the premise is sufficient to invalidate the conclusion, we use $\neg p \Rightarrow \neg C = p \vee \neg C$; when both statements are true, using two implications is the same as using equivalence, i.e. $([p \Rightarrow C] \wedge [\neg p \Rightarrow \neg C]) \Leftrightarrow (p \equiv C)$.

On the other hand, the GSN formalism does not either specify the relation between sub-goals (premises) in their support to the goal (conclusion). From a purely logical standpoint, when a conclusion is supported by more than one premise, we identify two types of links between them: logical *conjunction* (e.g. $p_1 \wedge p_2$) and logical *disjunction* (e.g. $p_1 \vee p_2$).

3.2. Quantitative confidence elicitation model

Experts can be uncertain about the truth or the falsity of pieces of information they specify in a GSN. In this section, we present how to qualitatively express an expert judgment on a statement and how to turn qualitative uncertainty into quantitative belief values in DST.

3.2.1. Collecting expert data

To assess uncertainty of a proposition, say x, the expert is supposed to provide two pieces of qualitative information:

- A level of *decision* Dec on a qualitative scale, which describes a trend from acceptance (maximal belief) to rejection (maximal disbelief) of a proposition.

- A level of *confidence* $Conf$, on a qualitative scale, which reflects the amount of information an expert possesses that can justify his/her decision.

Each expert judgment, expressed in the form of a decision and the degree of confidence associated to it, is collected using an evaluation matrix presented in Figure 3. Each dot in this matrix corresponds to a pair $(Dec, Conf)$.

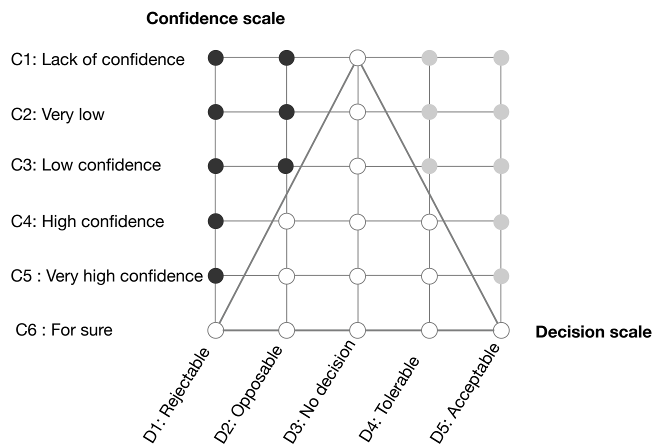

Figure 3: Evaluation matrix

3.2.2. From symbolic to numerical data

The symbolic scale of confidence (from $C1$ to $C6$ in Figure 3) is associated to a numerical scale between 0 and 1. We choose a linear scale for transforming qualitative pairs $(Dec, Conf)$ into numerical values like in header columns and rows of Table 1. Choosing a scale for $(Dec, Conf)$ and translating such pairs into numerical degrees is not trivial, we thus make the equidistance assumption for simplicity and to be comparable to previous works [9, 25]. This choice will be discussed in section 4.

When $Conf(x) = 1$, it means that the expert has full information supporting his choice of $Dec(x)$. While, when $Conf(x) = 0$, it means that he has no information to accept or deny x.

3.2.3. Deriving belief and disbelief degrees

Formally, confidence is defined as summation of belief (evidence in favor of the proposition) and disbelief (evidence against the proposition) degrees.

$$Conf(x) = Bel(x) + Disb(x) \tag{6}$$

In the same way each item of the decision scale (from $D1$ to $D5$ in Figure 3) is associated with a numerical value, which can be understood as the probability of acceptance. For instance, when $Dec(x) = 1$ it indicates a full certainty on the truth of x. On the other hand, when $Dec(x) = 0$ it indicates a full certainty of its falsity. And, when for some reason the expert cannot take side, $Dec(x) = 1/2$.

Formally, we define the decision as the result of the *Pignistic* transform [26] that turns a mass function m on a set Ω_x (the frame of discernment) into a probability, changing the focal sets into uniform distributions. When $\Omega_x = \{x, \neg x\}$ has two possible states, as it is the case here, $Dec(x)$ is the midpoint of uncertainty interval between belief and plausibility of x.

$$Dec(x) = \frac{1 + Bel(x) - Disb(x)}{2} \tag{7}$$

Note that when $Bel(x) = Disb(x)$ ($= 0$, in particular), we get $Dec(x) = 1/2$. From equations (7) and (6), we get:

$$Bel(x) = \frac{Conf(x) - 1}{2} + Dec(x),\ \ Disb(x) = \frac{Conf(x) + 1}{2} - Dec(x) \tag{8}$$

The degree of uncertainty can easily be deduced ($Uncer(x) = 1 - Bel(x) - Disb(x)$). The expert is allowed to choose any pair (decision, confidence) - a dot in Fig 3. However, some of these assessments cannot be interpreted by pairs $(Bel(x), Disb(x))$, in particular clear-cut decisions (acceptance or rejection, i.e., $Dec(x) = 1$ or 0) with low confidence degree. A constraint known as "*Josang constraint*" [27] needs to be respected between $Conf(x)$ and $Dec(x)$. Otherwise, the transformation $(Dec, Conf)$ to $(Bel, Disb)$ will lead to values outside the unit interval $[0, 1]$.

Enforcing $Bel(x), Disb(x) \in [0, 1]$, expressions in (8) yield $1 - Conf(x) \leq \min(2Dec(x), 2(1 - Dec(x))$. Hence, we can express the feasibility range of $Dec(x)$ for a given confidence level as:

$$\frac{1 - Conf(x)}{2} \leq Dec(x) \leq \frac{1 + Conf(x)}{2} \tag{9}$$

which defines a triangle in the evaluation matrix (Figure 3). Graphically, the assessments outside the triangle are those that do not respect the constraint and thus lead to negative belief (black dots) or negative disbelief (grey dots) values, which make no sense.

As a consequence, when the expert makes a clear-cut decision (acceptance $Dec(x) = 1$ or rejection $Dec(x) = 0$) the confidence must be maximal ($Conf(x) = 1$, fully informed expert), otherwise his/her decision assessment will have no grounds. The closer you get to the midpoint value ($Dec(x) = 1/2$, no decision), the larger the allowed confidence interval will be. The pair ($Dec(x) = 1/2$, $Conf(x) = 0$) means that the expert cannot take side because he/she has no information (*total ignorance*), while ($Dec(x) = 1/2$, $Conf(x) = 1$) means that he/she cannot take side because he/she has as much evidence in favor of the premise as against (*total conflict*).

When the pair $(Dec(x), Conf(x))$ provided by an expert is situated outside the triangle, we make a correction. As confidence reflects the amount of information, we keep it and modify the Dec value. When $Dec(x) < \frac{1-Conf(x)}{2}$ (rejection: black dots on Figure 3), we set $Dec(x) = \frac{1-Conf(x)}{2}$. On the other hand, when $Dec(x) > \frac{1+Conf(x)}{2}$ (acceptance: grey dots on Figure 3), we set $Dec(x) = \frac{1+Conf(x)}{2}$.

Example 1. Suppose we have the following assessments on two premises (p_1) and (p_2):

- p_1: Opposable with high confidence ($Dec(p_1) = 0.25, Conf(p_1) = 0.6$).
- p_2: Acceptable with very high confidence ($Dec(p_2) = 1, Conf(p_2) = 0.8$).

We calculate $Bel(p_i)$ and $Disb(p_i)$ using (8). We notice that the assessment for p_1 is inside the triangle in the matrix (Figure 3). We can therefore claim that there is no need to adjust the values according to Josang constraint (Eq. 9): $Bel(p_1) = \frac{0.6-1}{2} + 0.25 = 0.05$, $Disb(p_1) = \frac{0.6+1}{2} - 0.25 = 0.55$ and $Uncer(p_1) = 1 - Bel(p_1) - Disb(p_1) = 0.4$.

On the other hand, the assessment for p_2 is situated outside the triangle. In this case, we can be sure that the decision degree must be adjusted in accordance with the confidence value to get correct inputs. Before adjustment, we find: $Bel(p_2) = \frac{0.8-1}{2} + 1 = 0.9$ and $Disb(p_2) = \frac{0.8+1}{2} - 1 = -0.1$, a negative

Table 1: Values from (*Dec*, *Conf*) to (*Bel*,*Disb*) pairs on premises (see Figure 3 for symbol meaning)

Conf \ *Dec*	D1 (0)	D2 (0.25)	D3 (0.5)	D4 (0.75)	D5 (1)
C1 (0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
C2 (0.2)	(0,0.20)	(0,0.20)	(0.10,0.10)	(0.20,0)	(0.20,0)
C3 (0.4)	(0,0.40)	(0,0.40)	(0.20,0.20)	(0.40,0)	(0.40,0)
C4 (0.6)	(0,0.60)	(0.05,0.55)	(0.30,0.30)	(0.55,0.05)	(0.60,0)
C5 (0.8)	(0,0.80)	(0.15,0.65)	(0.40,0.40)	(0.65,0.15)	(0.80,0)
C6 (1)	(0,1)	(0.25,0.75)	(0.50,0.50)	(0.75,0.25)	(1,0)

value of disbelief, which does not make sense. Following the discussion above, we set $Dec(p_2) = \frac{1+Conf(p_2)}{2} = \frac{1+0.8}{2} = 0.9$. Then we find that $Bel(p_2) = 0.8$, $Disb(p_2) = 0$ and $Uncer(p_2) = 1 - Bel(p_2) - Disb(p_2) = 0.2$. □

In Table 1, we grouped all possible (*Dec*, *Conf*) pairs on premises with their appropriate (*Bel*, *Disb*) counterparts. We notice that when no information is available (C1: Lack of confidence), no matter what choice is made the degree of uncertainty is maximal ($Uncer(p) = 1$). On the other hand, in the case of a fully informed expert (C6: For sure) the decision value varies from rejection to acceptance, which follows the restrictions imposed by "Josang constraint".

3.2.4. Comparison with other approaches

There are other formulas to transform a mass function into a probability distribution that can be employed instead of the pignistic transform. *Shenoy transform* [28] is one of them. It consists in renormalizing the plausibilities of singletons, dividing each by their sum. Applying it to a two-state frame of discernment $\Omega_x = \{x, \neg x\}$, we get the following formula:

$$Dec(x) = \frac{1 - Disb(x)}{2 - [Bel(x) + Disb(x)]} \tag{10}$$

Notice that like the pignistic formula (equation 7) when $Bel(x) = Disb(x)$,

$Dec(x) = 1/2$ which expresses the situation of indecision. The decision value calculated from Shenoy transform needs also to respect Josang constraint in order to give correct belief and disbelief degrees (included in the unit interval $[0, 1]$). Hence, we also frame each decision between to values:

$$\frac{1}{2 - Conf(x)} \leq Dec(x) \leq \frac{1 - Conf(x)}{2 - Conf(x)} \tag{11}$$

However, the pignistic transform is a better choice to use in our situation for the following reasons:

1. The decision interval obtained by applying Josang constraint to the pignistic transform (Eq. 9) is larger than the one obtained by the Shenoy transform (Eq. 11). Figure 4 presents the feasibility areas for each transformation (triangle for Pignistic Eq. 9 and curvy inner triangle for Shenoy Eq. 11). Notice that, indeed, the usable values (not requiring adjustment) represented by the area inside the two triangles is larger in the case where we use the Pignistic transform (triangle with plain edges) than in the Shenoy case (curvy triangle with dashed edges).
2. The decision value obtained by the pignistic transform represents the midpoint of the uncertainty interval. This is not the case with Shenoy transform, which may sound counterintuitive. Hence, with the formula of confidence (Eq. 6), which gives the length of the uncertainty interval, we can recover the uncertainty interval from its midpoint using the pignistic transform.

On the other hand, Cyra and Gorski [9], and Wang et al. [11, 12] formally defined decision as:

$$\begin{cases} \text{If } Conf(x) > 0: \ Dec(x) & = \frac{Bel(x)}{Bel(x)+Disb(x)} \\ \text{If } Conf(x) = 0: \ Dec(x) & = 1 \text{ (Cyra \& Gorski), or } 0 \text{ (Wang et al.)} \end{cases} \tag{12}$$

Notice that in situation of total ignorance ($Conf = Bel + Disb = 0$) Cyra and Gorski adopted an optimistic point of view by accepting the conclusion ($Dec = 1$), while Wang et al. choose a more conservative approach by rejecting

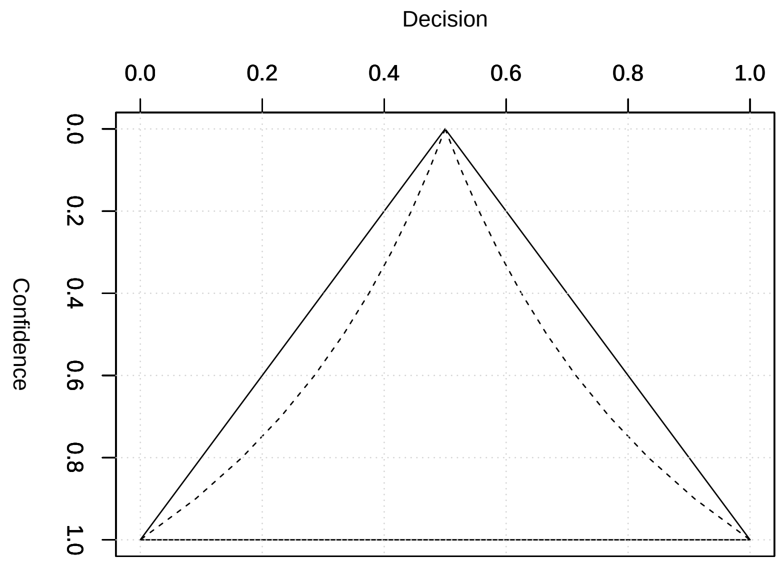

Figure 4: Pignistic (plain line) and Shenoy (dashed line) transforms constraint

the conclusion ($Dec = 0$). These choices seem arbitrary since they are based on no evidence. It would be better to take a neutral position because no evidence is provided to justify taking side (for or against). Therefore we replace this model by the *Pignistic* transform.

3.3. Quantitative confidence propagation model

Once we have defined the process of transforming experts judgments into degrees of belief, disbelief and uncertainty, we need to define how to propagate these values to the conclusion.

3.3.1. Sources of uncertainty in a GSN

In order to define propagation formulas to compute the confidence in the conclusion, we need to identify the sources of uncertainty in a GSN. Thus, we consider two sources:

- Uncertainty in a premise, known as *trustworthiness* [9, 11], in which the truth and falsity of pieces of evidence supporting the conclusion are

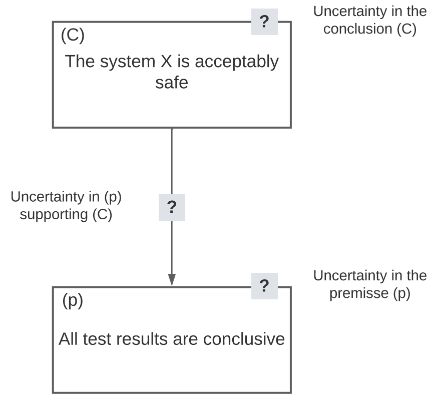

Figure 5: Sources of uncertainty in GSN - A simple argument type (S-Arg) example (Note that the arrow direction is not intuitive regarding the inference p supports C)

questioned. For instance, in Figure 5 one can doubt the test results because the experimental protocol was not respected. Formally, to assess confidence/uncertainty of a premise p, we define a mass function m_p on $\Omega_p = \{p, \neg p\}$, assigning masses to the premise (p), its negation ($\neg p$) and the tautology ($\top$ standing for Ω_p) summing to 1 (i.e. $m_p(p) + m_p(\neg p) + m_p(\top) = 1$, where $m_p(\top)$ represents the amount of ignorance). Note that $Bel(p) = m_p(p)$, and $Disb(p) = m_p(\neg p)$

- Uncertainty in the support relation between a premise and its conclusion, known as *appropriateness* [11], in which one can doubt the truth of this support. For instance, in Figure 5 an expert may argue whether the truth of the premise is sufficient to conclude the safety of the system. This support relation is expressed in propositional logic by material implications that we call *rules*. They are used to justify the propagation schemes for different argument types. Formally, each rule r is assigned a simple support mass function m_r [20], such that: $m_r(r) + m_r(\top) = 1$. Explanation of why, unlike a premise, the negation of a rule ($\neg r$) is not taken into consideration will be given later.

Note that assessing the support relationship in GSN between a goal (a conclusion

C) and its sub-goals (premises p) means assessing the chosen *strategy*, taking into account all the restrictions associated with it which define its scope (i.e. *context*, *assumption* and *justification* components).

3.3.2. Argument types

Looking through the literature, four common argument types were identified. In the following, we present and explain how confidence is propagated for a situation of one premise (p) supporting a conclusion (C), then for two premises (p_1) and (p_2) . Finally, we deduce formulas for "n" premises.

Simple argument type (S-Arg):

This argument describes the case of a conclusion (C) supported by a single premise (p), like in Figure 5. If the premise is true, then so is the conclusion. Material implication ($p \Rightarrow C$) is used to express the inference (support) between the premise and its conclusion. It is called *direct rule*: it propagates the truth (belief) in p. However, implication can only infer the acceptance of the conclusion when p holds (*Modus ponens* pattern). When p is not true, whether the conclusion holds is unknown. An additional rule, called *reverse rule* in opposition to the first one, which can infer the rejection of the conclusion, will be added: $\neg p \Rightarrow \neg C$. Direct and reverse rules are designed to respectively propagate belief and disbelief degrees of the premises. Figure 5 presents an example of a simple argument. In this case, the top-goal (or conclusion) "The system is acceptably safe" is achieved, only if the premise "The test results are conclusive" is true (direct rule). Otherwise, the conclusion is false (reverse rule). For each rule (direct and reverse), we assign resp. a simple support mass function ($m_{\Rightarrow}$ and $m_{\Leftarrow}$), which puts a mass on the rule and the rest on the tautology ($\top$)[2] summing to 1. We also assign another function to the premise (m_p), which puts a mass on the premise (p), its negation ($\neg p$) and the tautology ($\top$) summing to 1. Using the conjunctive rule of combination, we merge masses on rules

[2] Also denotedy b $m(\Omega)$ according to the set theory syntax, $[\top] = \Omega$.

($m_r = m_{\Rightarrow} \otimes m_{\Leftarrow}$) in Table 2; then its result is merged with m_p in Table 3: $m_C = m_r \otimes m_p$. This approach is a special case of the general reasoning method with uncertain propositional logic formulas proposed in [29].

Table 2: Combination of direct ($m_{\Rightarrow}$) and reverse ($m_{\Leftarrow}$) rules for S-Arg

$m_r = m_{\Rightarrow} \otimes m_{\Leftarrow}$	$m_{\Rightarrow}(p \Rightarrow C)$	$m_{\Rightarrow}(\top)$
$m_{\Leftarrow}(\neg p \Rightarrow \neg C)$	$p \equiv C$	$\neg p \Rightarrow \neg C$
$m_{\Leftarrow}(\top)$	$p \Rightarrow C$	$\top$

Table 3: Combination of the mass on premise (m_p) with its rule (m_r) for S-Arg

$m = m_p \otimes m_r$	$m_r(p \equiv C)$	$m_r(p \Rightarrow C)$	$m_r(\neg p \Rightarrow \neg C)$	$m_r(\top)$
$m_p(p)$	$p \wedge C$	$p \wedge C$	p	p
$m_p(\neg p)$	$\neg p \wedge \neg C$	$\neg p$	$\neg p \wedge \neg C$	$\neg p$
$m_p(\top)$	$p \equiv C$	$p \Rightarrow C$	$\neg p \Rightarrow \neg C$	$\top$

To calculate the belief degree of the conclusion, we sum the masses of all formulas that infer the conclusion (C) in Table 3 (dark grey areas).

$$
\begin{aligned}
Bel_C(C) &= \sum_{\phi:\phi \ implies \ C} m(\phi) = m(p \wedge C) \\
&= m_p(p) \times m_r(p \equiv C) + m_p(p) \times m_r(p \Rightarrow C) \\
&= m_p(p) \cdot [m_r(p \equiv C) + m_r(p \Rightarrow C)] \\
&= m_p(p) \cdot [m_{\Rightarrow}(p \Rightarrow C) \times m_{\Leftarrow}(\neg p \Rightarrow \neg C) + m_{\Rightarrow}(p \Rightarrow C) \times (1 - m_{\Leftarrow}(\neg p \Rightarrow \neg C))] \\
&= m_p(p) \cdot m_{\Rightarrow}(p \Rightarrow C)
\end{aligned}
$$

To calculate the disbelief degree of the conclusion, we sum the masses of all formulas that infer the negation of the conclusion ($\neg C$) in Table 3 (bright grey areas). The calculation is similar to the previous one, and we get

$$
\begin{aligned}
Disb_C(C) &= \sum_{\phi:\phi \ implies \ \neg C} m(\phi) = m(\neg p \wedge \neg C) \\
&= m_p(\neg p) \cdot m_{\Leftarrow}(\neg p \Rightarrow \neg C)
\end{aligned}
$$

Hence, we obtain the propagation formulas for an S-Arg, in 13:

$$\text{S-Arg}: \begin{cases} Bel_C(C) & = & Bel_p(p) \cdot Bel_{\Rightarrow}(p \Rightarrow C) \\ Disb_C(C) & = & Disb_p(p) \cdot Bel_{\Leftarrow}(\neg p \Rightarrow \neg C) \end{cases} \tag{13}$$

Remarks:

- As said earlier, since, we work on a two states frame of discernment (True, False) for both premises $\Omega_p = \{p_i, \neg p_i\}$ and the conclusion $\Omega_C = \{C, \neg C\}$, masses and degrees of (dis-)belief in premises, rules and conclusion, are equal. For instance, $m_p(p) = Bel_p(p)$ and $m_p(\neg p) = Bel_p(\neg p) = Disb_p(p)$.

- Unlike mass functions of premises, we do not assign a mass on negations of rules $p \wedge \neg C = \neg(p \Rightarrow C)$ (resp. $\neg p \wedge C = \neg(\neg p \Rightarrow \neg C)$). First, the negation of a rule is not a rule. Moreover, we do not assume that the truth of a premise can lead to the negation of the conclusion (at least in the GSN context). Components like *rebuttals*, which bring evidence against the conclusion or *defeaters*, which specify some exceptions to the rule, do not exist in GSN. However, it is allowed to question the truth of the premise for valid reasons (e.g., expert cannot trust the experiment because test conditions are not acceptable), which is more likely to occur. Therefore, we limit ourselves only to rules that can propagate belief and disbelief in the premises. Taking disbelief in rules into consideration will only make the calculations more complex and create conflict between mass functions.

- The belief in the conclusion $Bel_C(C)$ (resp. disbelief) only depends on the belief of the direct rule $Bel_{\Rightarrow}(p \Rightarrow C)$ (resp. reverse rule) and the belief (resp. disbelief) of the premise $Bel_p(p)$. Indeed, the direct rule $p \Rightarrow C$ (resp. reverse rule $\neg p \Rightarrow \neg C$) and the falsity (resp. truth) of the premise p cannot infer the rejection (resp. acceptance) of the conclusion C.

Conjunctive argument type (C-Arg):

This argument type describes the case when all premises are needed to support the conclusion. The direct rule is obtained by translating this definition into a logical expression: $(\wedge_{i=1}^{n} p_i) \Rightarrow C$. On the other hand, the reverse one is obtained by reversing the direct one: $\neg(\wedge_{i=1}^{n} p_i) \Rightarrow \neg C$, which is equivalent to $\wedge_i^n(\neg p_i \Rightarrow \neg C)$, a conjunction of elementary reverse rules. For instance, in

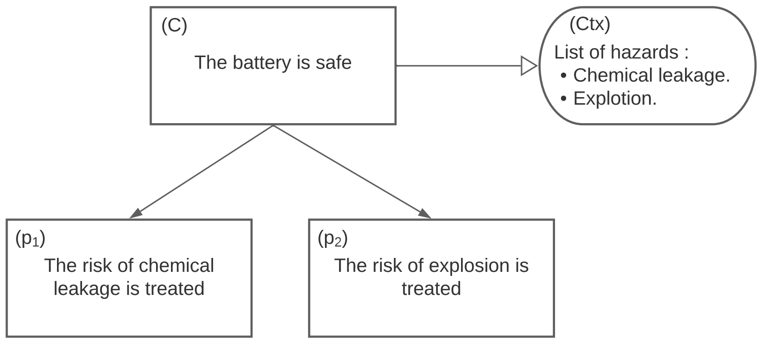

Figure 6: A conjunctive argument type (C-Arg) example

the example of Figure 6 both risks (listed in the context box) should be treated to guarantee the safety of the battery. However, if at least one premise is not treated, the system is no longer safe. To get propagation formulas (Eqs. 14) of a C-Arg for n premises, we combine the mass on rules ($m_r = m_{\Rightarrow} \otimes [\otimes_{i=1}^{n} m_{\Leftarrow}^{i}]$) with those on premises ($m_p = \otimes_{i=1}^{n} m_p^i$) using the conjunctive rule of combination ($m_C = m_p \otimes m_r$):

$$\text{C-Arg}: \begin{cases} Bel_C(C) & = & Bel_{\Rightarrow}([\wedge_{i=1}^{n} p_i] \Rightarrow C) \prod_{i=1}^{n} Bel_p(p_i) \\ Disb_C(C) & = & 1 - \prod_{i=1}^{n} [1 - Disb_p^i(p_i) Bel_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)] \end{cases} \tag{14}$$

We can notice from the equations in (14), that the belief formula takes the form of a general conjunction (the product of belief degrees in promises weighted by the mass in the conjunctive rule). On the other hand, the disbelief formula takes the form of a general disjunction (when $n = 2$, the probabilistic sum $1-(1-Disb_C^1(C)) \cdot (1 - Disb_C^2(C)) = Disb_C^1(C) + Disb_C^2(C) - Disb_C^1(C) \cdot Disb_C^2(C)$, where: $Disb_C^i(C) = Disb_p^i(p_i) \cdot Bel_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)$ the disbelief degree in the conclusion induced by the failure of one premise). The propagation scheme of this argument type favors the premise with the least strength (minimal belief obtained by the product of the belief on premises and maximal disbelief obtained by the probabilistic sum of disbeliefs of premises).

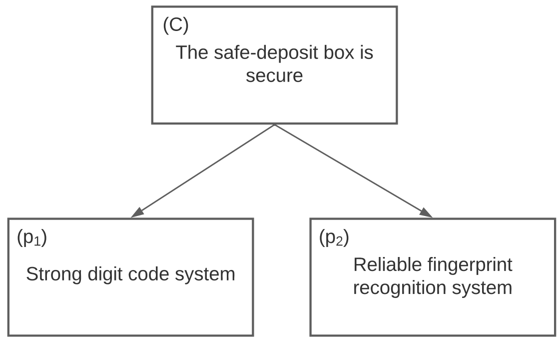

Figure 7: A disjunctive argument type (D-Arg) example

Disjunctive argument type (D-Arg):

This argument type describes the case when one premise is enough to support the whole conclusion. The corresponding rules are: $\wedge_{i=1}^{n}(p_i \Rightarrow C)$ (direct), and $(\wedge_{i=1}^{n}\neg p_i) \Rightarrow \neg C$ (reverse). In the example of Figure 7, each premise (digit code or fingerprint recognition systems) can guarantee the security of the safe-deposit box. Their conjunction does not, in any case, improve the degree of support in the conclusion (C). To get propagation formulas (Eqs. 15) of a D-Arg for n premises, we combine the mass on rules ($m_r = m_{\Leftarrow} \otimes [\otimes_{i=1}^{n} m_{\Rightarrow}^{i}]$) with those on premises ($m_p = \otimes_{i=1}^{n} m_p^i$) using the conjunctive rule of combination ($m_C = m_p \otimes m_r$).

$$\text{D-Arg} : \begin{cases} Bel_C(C) & = & 1 - \prod_{i=1}^{n}[1 - Bel_p^i(p_i) Bel_{\Rightarrow}^i(p_i \Rightarrow C)] \\ Disb_C(C) & = & Bel_{\Leftarrow}([\wedge_{i=1}^{n}\neg p_i] \Rightarrow \neg C) \prod_{i=1}^{n} Disb_p^i(p_i) \end{cases} \tag{15}$$

In opposition to the C-Arg, the belief (resp. disbelief) formula in equation (15) expresses a general disjunction (resp. conjunction). The propagation scheme of this argument (D-Arg) favors the premise with the greatest strength.

Hybrid argument type (H-Arg):

This argument type describes the case where each premise supports the conclusion to some extent, but their conjunction does it to a larger extent. This argument type could be considered as a general type which includes the two

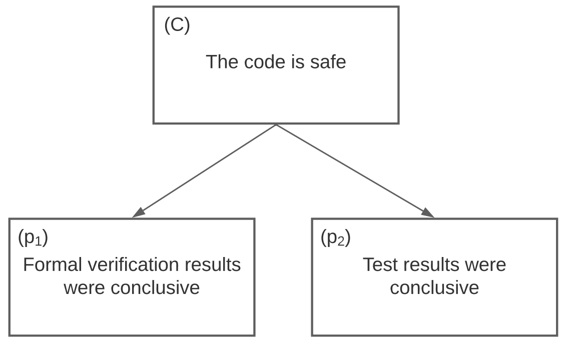

Figure 8: A hybrid argument type (H-Arg) example

previous ones. In fact, conjunctive and disjunctive types correspond to limit cases of the hybrid one. In the example of Figure 8, the premise "*Test results were conclusive*" supports the conclusion to some point. Since evidence based on formal verification was also provided, which allows to identify some unsafe states that the system will never reach, experts usually do not conduct lot of tests (which are limited by issues such as cost, feasibility, etc). On the other hand, tests can cover issues that formal verification might not capture. Unlike the D-Arg, the conjunction of these two premises improves the support degree in the conclusion. To get propagation formulas (Eqs. 16) of a H-Arg for n premises, we combine the mass on rules ($m_r = m_{\Rightarrow} \otimes [\otimes_{i=1}^{n} m_{\Leftarrow}^{i}] \otimes m_{\Leftarrow} \otimes [\otimes_{i=1}^{n} m_{\Rightarrow}^{i}]$) with those on premises ($m_p = \otimes_{i=1}^{n} m_p^i$) using the conjunctive rule of combination. See Appendix A.1 for the detailed calculation.

$$\text{H-Arg}: \begin{cases} Bel_C(C) & = Bel_{\Rightarrow}([\wedge_{i=1}^{n} p_i] \Rightarrow C) \times \prod_{i=1}^{n} Bel_p^i(p_i) \cdot [1 - Bel_{\Rightarrow}^i(p_i \Rightarrow C)] \\ & +\{1 - \prod_{i=1}^{n}[1 - Bel_p^i(p_i) \cdot Bel_{\Rightarrow}^i(p_i \Rightarrow C)]\} - m_C^{(n)}(\perp) \\ Disb_C(C) & = Bel_{\Leftarrow}([\wedge_{i=1}^{n} \neg p_i] \Rightarrow \neg C) \times \prod_{i=1}^{n} Disb_p^i(p_i) \cdot [1 - Bel_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)] \\ & +\{1 - \prod_{i=1}^{n}[1 - Disb_p^i(p_i) \cdot Bel_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)]\} - m_C^{(n)}(\perp) \end{cases} \tag{16}$$

where the mass $m_C^{(n)}(\bot)^3$ represents the conflict degree for (n) premises. We propose a recursive formula to calculate conflict mass for $n \geq 2$ premises:

$$m_C^{(n)}(\bot) = Bel_C^{(n-1)}(C)\times m_n(\neg p_n \wedge \neg C) + Disb_C^{(n-1)}(C)\times m_n(p_n \wedge C) + m_C^{(n-1)}(\bot) \tag{17}$$

where:

- $m_C^{(1)}(\bot) = 0$
- $Bel_C^{(n-1)}(C) = \{1 - \prod_{i=1}^{n-1}[1 - Bel_p^i(p_i) \cdot Bel_{\Rightarrow}^i(p_i \Rightarrow C)]\} - m_C^{(n-1)}(\bot)$
- $Disb_C^{(n-1)}(C) = \{1 - \prod_{i=1}^{n-1}[1 - Disb_p^i(p_i) \cdot Bel_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)]\} - m_C^{(n-1)}(\bot)$
- $m_i(p_i \wedge C) = Bel_p^i(p_i) \cdot Bel_{\Rightarrow}^i(p_i \Rightarrow C)$
- $m_i(\neg p_i \wedge \neg C) = Disb_p^i(p_i) \cdot Bel_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)$.

See Appendix A.2 for the detailed calculation of the conflict mass.

Note that in the expression (16) of $Bel(C)$ and $Disb(C)$, we first directly calculate expressions of the form $b(C) = \sum_{\phi \models C} m^{(n)}(\phi)$ and $d(C) = b(\neg C) = \sum_{\phi \models \neg C} m^{(n)}(\phi)$ that include the term $m^{(n)}(\bot)$, but are easier to compute than $Bel(C)$ and $Disb(C)$. Then we subtract the term $m^{(n)}(\bot)$ calculated recursively in a separate way (see Appendix A.2). Remember that we may have $b(C) + d(C) > 1$ since $m^{(n)}(\bot)$ is then counted twice. Additional explanation is given through an example of computation of belief and disbelief propagation formulas for the case of two premises supporting one conclusion in Appendix A.1.

We can notice that belief and disbelief propagation formulas for an H-Arg (Eqs. 16) have the same form. They sum two parts. A first one which expresses the conjunctive component of this type, weighted by the uncertainty on the direct elementary rules ($1 - Bel_{\Rightarrow}^i(p_i \Rightarrow C)$, resp. the reverse ones). The second part represent its disjunctive component. To deduce the formulas of a C-Arg from H-Arg, all you need is to set the elementary rules ($Bel_{\Rightarrow}^i(p_i \Rightarrow C)$) and the reverse conjunctive one ($Bel_{\Leftarrow}([\wedge_{i=1}^n \neg p_i] \Rightarrow \neg C)$) to zero. Conversely, if we set to zero the masses of the direct conjunctive rule and those of the reverse elementary rules, we get the formulas of a D-Arg. Finally, in the case of a

[3] Also denoted by $m(\emptyset)$ according to the set theory syntax: $[\bot] = \emptyset$.

conclusion supported by one premise, both masses of the direct and reverse conjunctive rules are equal to zero. Thus, we also deduce the formula of an S-Arg. Hence, the propagation formulas of the H-Arg can also be used to calculate confidence of the other argument types (S-Arg, C-Arg and D-Arg).

A conflict situation represents the case when one or more premises lead to opposite assessments of the conclusion (e.g., a premise p_i supports a conclusion C, while the negation of another premise p_j supports its negation). Formally, it always takes the form of a combination of four items: p_i, $p_i \Rightarrow C$, $\neg p_j$ and $\neg p_j \Rightarrow \neg C$, which produce empty intersections (noticeable if we combine the masses of elementary rules and those on the premises). In the case of the C-Arg and D-Arg this combination never occurs. Indeed, since the definition of a C-Arg (resp. D-Arg) does not use direct (resp. reverse) elementary rules that combine with reverse (resp. direct) elementary rules, we can see that the value of $m_C^{(n)}(\perp)$ is always zero for these types.

Facing a conflict situation, when we have opposite assessment on premises supporting the same goal, the conjunctive type adopts a cautious behavior in favor of the propagation of the premises that does not support the conclusion. On the contrary, the disjunctive type takes a more optimistic view, which favors the propagation of the premises that support the conclusion. The hybrid type (H-Arg) stands between these two limit cases, and the mass $m_C^{(n)}(\perp)$ may have a positive value.

Remarks

- In the case of a minor conflict ($m_C^{(n)}(\perp)$ small), instead of subtracting $m_C^{(n)}(\perp)$, we could normalise the result, dividing by the consistency degree $(1 - m_C^{(n)}(\perp))$ as proposed in the usual Dempster rule of combination. It would eliminate the conflict and proportionally increase the contradiction-free degrees of the belief $Bel_C(C)$ and disbelief $Disb_C(C)$ in a misleading way. In the case of full conflict, this mass is equal to 1 making this normalisation meaningless (division by zero).[4] In fact the term $m_C^{(n)}(\perp)$, if high enough, provides additional information in the sense that it detects a contradiction in the expert data, which may lead to questioning the argument structure.

- Our confidence propagation scheme can be addressed by standard existing belief function software based on results in [30] (e.g., the belief function machine implemented in MatLab), but the GSNs we study have a particular tree-like structure that enables an explicit symbolic calculation of the belief function on the conclusion space. The explicit formulas make the calculation more efficient, are easy to interpret and liable to sensitivity analysis, thus better explaining the obtained results, and validating the approach.

- Unlike our proposal, which uses simple support functions to quantify confidence in rules separately, Wang et al. [12] represent the relation between the premises and the conclusion with a single mass function for all rules with weights summing to 1. This choice makes the confidence in one premise dependent on confidence in the other ones. The more confidence in the rule corresponding to a particular premise, the less confidence in the other rules. Our approach has more degrees of freedom and is closer to logic.

3.4. Application

A real world case study is provided in [16], but we propose here a smaller generic example to illustrate our approach. We consider a situation with two premises p_1 and p_2 supporting a conclusion C. Then, we set the belief on rules to the values in Table 4 according, respectively, to the C-Arg, D-Arg and H-Arg and propagate confidence from premises to the conclusion. We consider three different settings of premise assessments. A first one, with positive evaluations

[4]This can be the case with masses on formulas equal to 1.

Table 4: Example of belief values for the three argument types with two premises (p_1 and p_2) supporting a conclusion (C)

Rules \ Types	H-Arg	C-Arg	D-Arg
$Bel_{\Rightarrow}([p_1 \wedge p_2] \Rightarrow C)$	1	1	0
$Bel_{\Leftarrow}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C)$	1	0	1
$Bel^1_{\Rightarrow}(p_1 \Rightarrow C)$	0.75	0	1
$Bel^1_{\Leftarrow}(\neg p_1 \Rightarrow \neg C)$	0.75	1	0
$Bel^2_{\Rightarrow}(p_2 \Rightarrow C)$	0.75	0	1
$Bel^2_{\Leftarrow}(p_2 \Rightarrow \neg C)$	0.75	1	0

Table 5: Three examples of (Belief, Disbelief) propagation for a conclusion (C) supported by two premises (p_1 and p_2) and their (Decision, Confidence) counterpart

Configuration	G_1	G_2	G (C-Arg)	G (D-Arg)	G (H-Arg)
1^{st}	$(0.75, 0.25)$	$(0.75, 0.25)$	$(0.56, 0.44)$	$(0.94, 0.06)$	$(0.63, 0.13)$
	(Tol,c_6)	(Tol,c_6)	(ND,c_6)	(Acc,c_6)	(Tol,c_5)
2^{nd}	$(0.25, 0.75)$	$(0.25, 0.75)$	$(0.06, 0.94)$	$(0.44, 0.56)$	$(0.13, 0.63)$
	(Opp,c_6)	(Opp,c_6)	(Rej,c_6)	(ND,c_6)	(Opp,c_5)
3^{rd}	$(0.75, 0.25)$	$(0.25, 0.75)$	$(0.19, 0.81)$	$(0.81, 0.19)$	$(0.30, 0.30)$
	(Tol,c_6)	(Opp,c_6)	(Opp,c_6)	(Tol,c_6)	(ND,c_4)

(both tolerable, for sure), negative evaluations (both opposable, for sure) and opposite evaluations. Then, we propagate confidence from premises (p_1 and p_2) to the conclusion (C) according to each argument type. Finally we transform and approximate the calculated belief and disbelief values into symbolic decision and confidence pairs. The results are gathered in Table 5.

Notice that C-Arg, as expected, favors the propagation of the premise with least strength. The effect of attenuation is due, on the one hand, to the product between belief degrees on premises, which decreases the belief of the conclusion, and, on the other hand, to the probabilistic sum of disbelief degrees of premises, which increases the disbelief of the conclusion. In Table 5, we can notice that the decision degree of the conclusion respectively moves from *tolerable* and *opposable* to *no decision* and *rejectable* due to the attenuation effect in the first and second case.

In opposition to C-Arg, we can notice that D-Arg favors the propagation of the premise with the strongest assessment. The effect of amplification is due to the use of probabilistic sum for belief degrees and the product for the disbelief degrees. In Table 5, we can notice that the decision of the conclusion respectively moves from *tolerable* and *opposable* to *acceptable* and *no decision* due to the amplification effect in the first and second case.

Finally, we can notice that the H-Arg achieves a balance between the assessments by maintaining respectively *tolerable* and *opposable* decisions in the first and second cases and giving *no decision* in the third case when we have opposite assessments. But in return, it degrades the level of confidence.

3.5. Argument types in the literature - Comparison

As discussed above, the logical definition of the H-Arg includes many argument types. In the following, we compare some argument types proposed in the literature with those we propose.

Using the informal definition of the argument types given in [9, 31] and the formal one [25], we placed those types with respect to ours in Figure 9. We can notice that in each work there is a pure conjunctive type and a pure disjunctive one. The rest of the types can be considered as a special case of the hybrid (H-arg) one. Some are close to the C-Arg, while others are more close to the D-Arg. The "complementary and alternative combination type", proposed by Cyra and Gorski [9], by definition is at equal distance between C-Arg and D-Arg. However, these authors consider this type either as an alternative type or a complementary type according to each situation, instead of proposing a unified formula that cover these two types.

These works also use DST to model and propagate confidence in GSN. As we did, they proposed different propagation formulas for different identified types of arguments [31, 9, 12]. However, they poorly describe how premises interact to support the conclusion (imprecise argument types). For instance,

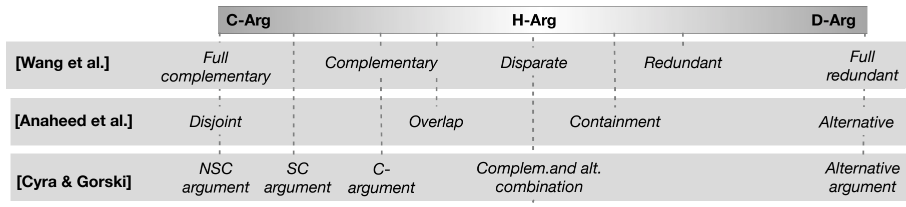

Figure 9: Proposed argument types compared to Wang et al. [12], Anaheed et al. [31] and Cyra & Gorski [9]

Anaheed et al. [31] try to explain these interactions using Venn diagrams where the logical (conjunction, disjunction, etc) and confidence (mass assignment) aspects of these argument types are mixed up. Therefore, we cannot guess the argument type from its propagation formulas. They all take the form of a weighted average. Similar remarks can be made on Cyra and Gorski's work [9], where one cannot identify, from the formulas, the logic that leads to them. The work of Wang et al. [12] is closer to ours. A separation between the logic (argument type definition) and confidence (mass assignment definition) was made, which facilitates the understanding of their formulas. They use a mix between conjunction and disjunction to model their types and logical equivalence to link premises to the conclusion. Using equivalence assumes that information about both the acceptance and denial of the conclusion is available and they have the same weight [14]. These models do not consider the case where a single type of information is available, hence the interest of breaking it down into two implications, as we do.

4. Qualitative confidence propagation and elicitation models

In this section, we present reasons to question the quantitative approach to uncertainty elicitation. It motivates a purely qualitative counterpart of the quantitative elicitation and propagation models presented in previous section. Then, we introduce the new approach based on qualitative capacity theory.

4.1. Rationale for the qualitative approach

There are three reasons to develop a purely qualitative confidence elicitation and propagation model in argument structures.

Source independence assumption. Using Dempster rule of combination (with or without normalization) supposes that the belief functions defined to quantify confidence (or uncertainty) in a conclusion (C) are coming from independent sources. Hence, the confidence provided by multiple pieces of evidence supporting the same conclusion (acceptance of x or its rejection) is higher than the one provided by each one individually: i.e, for two pieces of evidence we get a degree of confidence $\alpha_1 + \alpha_2 - \alpha_1 \cdot \alpha_2 \geq \max(\alpha_1, \alpha_2)$, where α_i is the confidence provided by the ith piece of evidence.

In this framework, we often rely on the judgment of a single expert to assess the confidence in each node of a GSN (a conclusion supported by one or more premises). Hence, we cannot always assume independence of sources. It is not systematically verified in practice. Therefore, it is important to investigate combination rules that do not suppose independence of sources. Destercke et al. [32] and Denoeux [33] propose some *idempotent* rules of combination to merge belief functions induced by dependent sources. In the qualitative setting the counterpart of the conjunctive rule of combination is idempotent.

Attenuation and amplification effects. As presented in the application of section 3.4, in the case of a C-Arg (the most encountered argument in our state of the art study), we can have premises with high credibility assessment and yet end up with a conclusion with little credibility [16]. A similar problem (this time amplification) occurs with D-Arg. To overcome this problem, we can change the numerical scale of decision and confidence. The choice of the linear scale adopted in section 3.2 is adopted only to compare with previous studies. For instance, using a logarithmic scale, commonly encountered in railway safety applications, may partially solve this issue, but the results of the calculation will always strongly rely on the choice of the scale. As seen later this phenomenon will not occur in the qualitative setting.

Transformation of qualitative assessments into quantitative ones. Encoding qualitative uncertainty assessments with real numbers contains a part of arbitrariness, whatever the chosen target scale. As suggested previously, changing the linear scale into a logarithmic one can partially mitigate the attenuation and amplification effects respectively in C-Arg and D-Arg. But this transformation can introduce instability in the propagation model: the results of the propagation steps may depend upon the choice of the transformation from qualitative to quantitative values, including when approximating back the calculated values of belief, disbelief, and uncertainty of the conclusion into qualitative decision and confidence pairs.

For all these reasons, we find it valuable to investigate a purely qualitative approach of confidence elicitation and propagation, which avoids the qualitative-numerical translation step.

4.2. Qualitative confidence elicitation model

In order to elicit qualitative capacities, we use a modified version of the quantitative method proposed in sub-section 3.2, already outlined in [24]. Thus, the same types of information will be collected to assess the GSN pattern:

- The decision index $Dec(x)$, takes values in a bipolar scale $D = \{0_D = d_{-n}, d_{n-1}, \ldots, d_0 = e, d_1, \ldots, d_n = 1_D\}$ with $2n+1$ values, the bottom of which (0_D) expresses rejection, the top (1_D) acceptance, and the midpoint (e_D) a neutral position. In the application, we assume $n = 2$.

- The confidence index $Conf(x)$ lies in a positive uni-polar scale with $n+1$ values $C = \{0_C = c_0, c_1, \ldots, c_n = 1_C\}$ (the top 1_C expresses full confidence, the bottom 0_C is neutral- no information). For $n = 2$: the levels c_0 to c_2 may respectively express lack of confidence, moderate confidence and full confidence.

We now turn pairs $(Dec, Conf)$ into qualitative capacity assessments on the scale L. To make decision and confidence scales compatible with the transformation formulas presented hereafter, we make several assumptions:

(i) The bipolar scale D is equipped with an order-reversing map ν_D such that $\nu_D(d_{-i}) = d_i$. Especially we have that $\nu_D(Dec(x)) = Dec(\neg x)$.

(ii) The unipolar scale C is isomorphic to the positive part of D, and is equipped with an order-reversing map ν such that: $\nu(c_i) = c_{n-i}$.

In order to switch from a pair $(Dec(x), Conf(x))$ to a pair of capacity values $(\gamma(x), \gamma(\neg x))$, we define a function f that maps $D \times C$ to the belief-disbelief scale $L \times L$ containing pairs $(\gamma(x), \gamma(\neg x))$. The scale L must have the same number of elements as C (i.e., 3 here $L = \{0, \lambda, 1\}$). The mapping $f : D \times C \to L \times L$: must satisfy some conditions [24]:

- If the expert declares lack of confidence, the result is $f(Dec(x), 0_C) = (0, 0)$, whatever the trend expressed on the decision scale.
- If the expert is fully confident, then $f(1_D, 1_C) = (\gamma(x), \gamma(\neg x)) = (1, 0)$, and likewise, $f(0_D, 1_C) = (0, 1)$, $f(e_D, 1_C) = (1, 1)$. Indeed, for the latter, there is a total conflict: the expert is maximally informed, and cannot decide between x and its negation.
- $\max(\gamma(x), \gamma(\neg x)) = Conf(x)$: the belief in x or its negation cannot be stronger than the confidence.
- if $Dec(x)$ is the midpoint of D, then $\gamma(x) = \gamma(\neg x)(= Conf(x))$ (no reason to take side).
- if $Dec(x)$ is less than the midpoint of D, then the trend is to reject x, so $\gamma(x) < \gamma(\neg x) = Conf(x)$, and the smaller $D(x)$, the smaller $\gamma(x)$.
- if $Dec(x)$ is greater than the midpoint of D, then the trend is to accept x, so $\gamma(x) = Conf(x) > \gamma(\neg x)$, and the greater $D(x)$, the smaller $\gamma(\neg x)$.

Table 6: Values from (Dec, $Conf$) to (Bel,$Disb$) pairs on premises for $n = 2$

$Conf$ \ Dec	d_{-2} (Rej)	d_{-1} (Opp)	$d_0(ND)$	d_1 (Tol)	d_2 (Acc)
c_0 (Lack of conf.)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
c_1 (Moderate conf.)	(0,λ)	(λ,λ)	(λ,λ)	(λ,λ)	(λ,0)
c_2 (For sure)	(0,1)	(λ,1)	(1,1)	(1,λ)	(1,0)

Keep in mind $\gamma : 2^\Omega \to L = \{0, \lambda, 1\}$.

These conditions lead to propose the following translation formulas [24]:

if $Dec(x) < e_D,$ then $\gamma(x) = \min[\nu_C(Dec(\neg x)), Conf(x)]$ and $\gamma(\neg x) = Conf(x)$

if $Dec(x) > e_D,$ then $\gamma(x) = Conf(x)$ and $\gamma(\neg x) = \min[\nu_C(Dec(x)), Conf(x)]$

if $Dec(x) = Dec(\neg x) = e_D,$ then $\gamma(x) = \gamma(\neg x) = Conf(x)$

In Table 1, we grouped all possible (Dec, $Conf$) pairs on premises with their appropriate ($\gamma(x)$, $\gamma(\neg x)$) counterparts, using the formulas above. We can notice an anti-symmetry between belief and disbelief degrees with respect to the central column (d_0: No decision). We also notice that when no information is available (c_0: Lack of confidence), no matter what choice is made, the degrees of belief and disbelief take the minimal value. On the other hand, in the case of a fully informed expert (c_2: For sure) the decision value varies from rejection to acceptance and is reflected by the pair $(\gamma(x), \gamma(\neg x))$. We can see that the values in the table respect the conditions imposed above.

4.3. Qualitative confidence propagation model

To build the qualitative confidence propagation model, we are going to use the same rules (argument types) as those defined for the quantitative models in sub-section 3.2. To represent uncertainty in premises and rules, we use non-dogmatic q-capacities instead of belief functions, and the qualitative rule of combination to merge these capacities.

Table 7: Combination of the focal sets of the premise (ρ_p) with its rules (ρ_r) for S-Arg

$\rho = \rho_p \odot \rho_r$	$\rho_r(p \equiv C)$	$\rho_r(p \Rightarrow C)$	$\rho_r(\neg p \Rightarrow \neg C)$	$\rho_r(\top)$
$\rho_p(p)$	$p \wedge C$	$p \wedge C$	p	p
$\rho_p(\neg p)$	$\neg p \wedge \neg C$	$\neg p$	$\neg p \wedge \neg C$	$\neg p$
$\rho_p(\top)$	$p \equiv C$	$p \Rightarrow C$	$\neg p \Rightarrow \neg C$	$\top$

Simple argument type (S-Arg):

To model this argument type, we associate to each rule (direct and reverse one) a simple support $B\Pi A$ (resp., $\rho_\Rightarrow$ and $\rho_\Leftarrow$), and a $B\Pi A$ on the premise space ρ_p, assigning a mass to its truth $\rho_p(p)$, its falsity $\rho_p(\neg p)$ and the tautology $\rho_p(\top) = 1$. Then, using the combination rule in equation (5), we merge the BΠAs on rules ($\rho_r = \rho_\Rightarrow \odot \rho_\Leftarrow$) with the one on the premise (ρ_p): $\gamma_C = \gamma_r \odot \gamma_p$ (Table 7). Similarly to the quantitative formulas, $\gamma_C(C) = \rho_C(C)$, $\gamma_p(p) = \rho_p(p)$ and $\gamma_r(r^*) = \rho_r(r^*)$, $\forall r^*$ (conjunctive, disjunctive, direct or reverse), since we work on a two states frame of discernment $\Omega_x = \{x, \neg x\}$. We get:

$$\begin{aligned}\gamma_C(C) &= \max_{\phi:\phi\vdash C,\phi\neq\emptyset} \rho(\phi)\\ &= \max(\min(\rho_p(p), \rho_r(p \equiv C)), \min(\rho_p(p), \rho_r(p \Rightarrow C)))\\ &= \min[\rho_p(p), \rho_\Rightarrow(p \Rightarrow C)]\end{aligned}$$

as $\rho_r(p \equiv C) = \min(\rho_\Rightarrow(p \Rightarrow C), \rho_\Leftarrow(\neg p \Rightarrow \neg C)) \leq \rho_\Rightarrow(p \Rightarrow C) = \rho_r(p \Rightarrow C)$. A similar computation can be done for $\gamma_C(\neg C)$.

So, we conclude for the uncertainty propagation in simple argument:

$$\text{S-Arg}: \begin{cases} \gamma_C(C) &= \min[\gamma_p(p), \gamma_\Rightarrow(p \Rightarrow C)] \\ \gamma_C(\neg C) &= \min[\gamma_p(\neg p), \gamma_\Leftarrow(\neg p \Rightarrow \neg C)] \end{cases} \tag{18}$$

We can notice that, like in the quantitative setting, the belief $\gamma_C(C)$ only depends on the direct rule $\gamma_\Rightarrow(p \Rightarrow C)$ and the belief degree of the premise $\gamma_p(p)$, while the disbelief $\gamma_C(\neg C)$ only depends on the reverse rule $\gamma_\Leftarrow(\neg p \Rightarrow \neg C)$ and the disbelief of the premise $\gamma_p(\neg p)$.

Conjunctive argument type (C-Arg):

We formally defined direct and reverse rules for this type by (resp.): $(\wedge_{i=1}^n p_i) \Rightarrow C$ and $\wedge_{i=1}^n(\neg p_i \Rightarrow \neg C)$. Following the same reasoning as for the previous argument type, we put a simple $B\Pi A$ on each rule (conjunctive direct rule: $\rho_{\Rightarrow}$ and elementary reverse rules: $\rho_{\Leftarrow}^i$), and a function $B\Pi A$ on each premise: ρ_p^i, which assigns one mass on the truth of (p_i), its falsity $(\neg p_i)$ and the tautology $(\top)$ such that $\rho_p^i(\top) = 1$. Then, using the combination rule in equation (5), we deduce $\gamma_C(C)$ and $\gamma_C(\neg C)$ from the combination: $\rho_C = \rho_p \odot \rho_r$, where: $\rho_p = (\odot_{i=1}^n \rho_p^i)$ and $\rho_r = \rho_{\Rightarrow} \odot (\odot_{i=1}^n \rho_{\Leftarrow}^i)$. Hence, we get the following confidence formulas for a C-Arg:

$$\text{C-Arg} : \begin{cases} \gamma_C(C) & = & \min\{\min_{i=1}^n \gamma_p^i(p_i), \gamma_{\Rightarrow}([\wedge_i^n p_i] \Rightarrow C)\} \\ \gamma_C(\neg C) & = & \max_{i=1}^n\{\min[\gamma_p^i(\neg p_i), \gamma_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)]\} \end{cases} \quad (19)$$

In the formulas of the quantitative approach (Eqs. 14), we use probabilistic sum $(a+b-ab)$ and the product (ab) instead of $\max, \min$, highlighting the similarity between the results obtained in both models. In fact, we can better see with min-max operators that the C-Arg favors the propagation of the premise with the lowest strength (minimal belief, with a maximal disbelief degree).

Disjunctive argument type (D-Arg):

Formally, the direct and reverse rules are defined as follows: $\wedge_{i=1}^n(p_i \Rightarrow C)$ and $(\wedge_{i=1}^n \neg p_i) \Rightarrow \neg C$. The calculation of $\gamma_C(C)$ and $\gamma_C(\neg C)$ is identical to the one above, swapping the two expressions ($\rho_C = \rho_p \odot \rho_r$, where $\rho_r = (\odot_{i=1}^n \rho_{\Rightarrow}^i) \odot \rho_{\Leftarrow}$ and $\rho_p = \odot_{i=1}^n \rho_p^i$). We get the confidence propagation formulas for a D-Arg:

$$\text{D-Arg} : \begin{cases} \gamma_C(C) & = & \max_{i=1}^n\{\min[\gamma_p^i(p_i), \gamma_{\Rightarrow}^i(p_i \Rightarrow C)]\} \\ \gamma_C(\neg C) & = & \min\{\min_{i=1}^n \gamma_p^i(\neg p_i), \gamma_{\Leftarrow}([\wedge_i^n \neg p_i] \Rightarrow \neg C)\} \end{cases} \quad (20)$$

We can notice that this model, like its quantitative counterpart (Eqs. 15), favors the propagation of the premise with the greatest strength (maximal belief and minimal disbelief degree).

Hybrid argument type (H-Arg):

Merging the q-capacity functions of all rules with those on premises, such that: $\rho_C = \rho_p \odot \rho_r$, where $\rho_r = \rho_{\Rightarrow} \odot (\odot_{i=1}^n \rho_{\Leftarrow}^i) \odot \rho_{\Leftarrow} \odot (\odot_{i=1}^n \rho_{\Rightarrow}^i)$ and $(\rho_p = \odot_{i=1}^n \rho_p^i)$, we get the confidence propagation formulas for an H-Arg:

$$\text{H-Arg}: \begin{cases} \gamma_C(C) = & \max\{\min[\min_{i=1}^n \gamma_p^i(p_i), \gamma_{\Rightarrow}([\wedge_{i=1}^n p_i] \Rightarrow C)], \\ & \max_{i=1}^n(\min[\gamma_p^i(p_i), \gamma_{\Rightarrow}^i(p_i \Rightarrow C)]\} \\ \gamma_C(\neg C) = & \max\{\min[\min_{i=1}^n \gamma_p^i(\neg p_i), \gamma_{\Leftarrow}([\wedge_{i=1}^n \neg p_i] \Rightarrow \neg C)], \\ & \max_{i=1}^n \min[\gamma_p^i(\neg p_i), \gamma_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)]\} \end{cases} \tag{21}$$

In order to get a non-trivial H-argument, we need that the weight of rules of the form $[\wedge_{i=1}^n p_i] \Rightarrow C$ be greater than the weight of individual rules $p_i \Rightarrow C$, and likewise, the weight of rules of the form $[\wedge_{i=1}^n \neg p_i] \Rightarrow \neg C$ be greater than the weight of individual rules $\neg p_i \Rightarrow \neg C$.

We can notice, as expected (analogy to quantitative formulas), that formulas of H-Arg (Eqs. 21), are more general than C-Arg formulas (Eqs. 19), and D-Arg (Eqs. 20). Assuming maximal belief ($= 1$) (resp. disbelief) on premises, it is enough that the simple direct rules take a null value (resp. the reversed conjunctive one) to get the conjunctive argument type. And conversely, to get the disjunctive argument type, we have to put null values on direct conjunctive and simple reverse rules. The S-Arg, represents a special case when only one premise is available ($n = 1$). In the following, only the H-Arg will be used since it covers the four types.

Conflict degree impact on the H-Arg. Remember that C-Arg and D-Arg argument types are conflict-free ($m^{(n)}(\bot) = 0$ in the quantitative setting). They respectively propagate the premise with lowest and highest assessments. However, it is not the same for H-Arg. Indeed, merging $B\Pi A$'s ρ_p^i (on p_i, $\neg p_i$ and $\top$), $\rho_{\Rightarrow}^i$, $\rho_{\Leftarrow}^i$, $i = 1, \ldots n$, as in its quantitative counterpart, the BΠA pertaining to the conclusion C obtained from this fusion may assign a mass to the contradiction. Conflict always appears when four items are merged of the form: p_i and

$p_i \Rightarrow C$ with $\neg p_j$ and $\neg p_j \Rightarrow \neg C, j \neq i$, whose conjunction is a contradiction $\perp$ with mass:

$$\begin{aligned}\rho_C^{ij}(\perp) &= \min[\rho_C^i(p_i \wedge C), \rho_C^j(\neg p_j \wedge \neg C)] \\ &= \min[\rho_p^i(p_i), \rho_\Rightarrow^i(p_i \Rightarrow C), \rho_p^j(\neg p_j), \rho_\Rightarrow^j(\neg p_j \Rightarrow \neg C)]]\end{aligned}$$

The final weight on contradiction for n premises takes the form $\rho_C(\perp) = \max_{i \neq j} \rho_C^{ij}(\perp)$. Besides, this capacity on contradiction does not affect the final results of belief and disbelief since $\gamma_C(C)$ and $\gamma_C(\neg C)$ are not less than $\gamma_C(\perp)$. For instance, for $n = 2$: $\rho_C(\perp) = \max(\rho_C^{12}(\perp), \rho_C^{21}(\perp))$. Using equation 3, we get:

$$\begin{aligned}\gamma_C(C) \geq \max(\min(\rho_p^1(p_1), \rho_\Rightarrow^1(p_1 \Rightarrow C)), \min(\rho_p^2(p_2), \rho_\Rightarrow^2(p_2 \Rightarrow C))) \\ \geq \max(\rho_C^{12}(\perp), \rho_C^{21}(\perp)).\end{aligned}$$

and likewise for $\gamma_C(\neg C)$.

5. Confidence assessment procedure for a GSN

In this section, we present our confidence assessment procedure, which includes both our elicitation and propagation models. This procedure is the same for the quantitative (section 3) and purely qualitative (section 4) settings.

5.1. Determination of belief weights for rules

Before presenting the details of the belief estimation procedure for rules, we draw your attention on an observation made for the propagation model presented in sub-section 3.3. The whole procedure is based on this observation. Assuming clear-cut knowledge about some (or all) premises ($Bel_p^i(p_i) = 1 - Disb_p^i(p_i) \in \{0, 1\}$) and total ignorance about the others ($Uncer_p^i(p_i) = 1$, i.e., $Bel_p^i(p_i) = Disb_p^i(p_i) = 0$), $Bel_C(C)$ and $Disb_C(C)$ respectively take the belief values of direct and reverse rules. For example, in the case of a conclusion (C) supported by two premises (p_1) and (p_2), assuming total acceptance of these two premises with maximal confidence, i.e. $Bel_p^1(p_1) = Bel_p^2(p_2) = 1$ then: $Bel_C(C) = Bel_\Rightarrow([p_1 \wedge p_2] \Rightarrow C)$ using equation (14). While assuming total rejection with maximal confidence of (p_1), i.e $Disb_p^1(p_1) = 1$ and total ignorance about (p_2), i.e. $Uncer_p^2(p_2) = 1$ then: $Disb_C(C) = Bel_{\Leftarrow}(\neg p_1 \Rightarrow \neg C)$ using equation (14).

We propose a procedure for collecting belief degrees of rules based on the elicitation model (as for the premises) and the observation of the paragraph above where we assume that the GSN pattern to be assessed is a C-Arg (resp. D-Arg) to estimate the values of $Bel_{\Rightarrow}([\wedge_{i=1}^{n} p_i] \Rightarrow C)$ and $Bel_{\Leftarrow}^{i}(\neg p_i \Rightarrow \neg C)$ (resp. $Bel_{\Rightarrow}^{i}(p_i \Rightarrow C)$ and $Bel_{\Leftarrow}([\wedge_{i=1}^{n} \neg p_i] \Rightarrow \neg C))$. Thus, from the expert assessment of the conclusion for predefined extreme premise assessments $(Bel, Disb)$, we can measure the belief values of the rules. These values can be used afterwards to calculate the confidence of a conclusion based on belief values of the premises.

Moreover, as mentioned before, no positive disbelief is assigned to a rule. This constraint impacts the allowed pairs (Dec, Conf) for the expert. The latter is constrained to choose only a decision on the positive side (from "*no decision*" to "*acceptable*") for direct rules. On the contrary, (s)he can only choose a negative decision (from "*rejectable*" to "*no decision*") for the reverse rules. Formulas in (6) and (7) are used to derive the degrees of belief on rules.

Example 2. Suppose the case of a conclusion (C) supported by one premise (p). Figure 10 describes the procedure of rule elicitation. To get the belief on the direct rule $R_1 : p \Rightarrow C$ and the reverse one $R_2 : \neg p \Rightarrow \neg C$, we ask an expert to give his/her assessment about the conclusion respectively when $(Dec(p) = 1,\ Conf(p) = 1)$ for R_1, and then when $(Dec(p) = 0,\ Conf(p) = 1)$ for R_2.

Suppose the expert gives the following assessments:

- When $(Dec(p) = 1,\ Conf(p) = 1)$, the expert assigns "Tolerable, with high confidence" to the conclusion (C), hence: $Dec(C) = 0.75$ and $Conf(C) = 0.6$.

- When $(Dec(p) = 0,\ Conf(p) = 1)$, the expert assigns "Opposable, with very high confidence" to the conclusion (C), hence: $Dec(C) = 0.25$ and $Conf(C) = 0.8$.

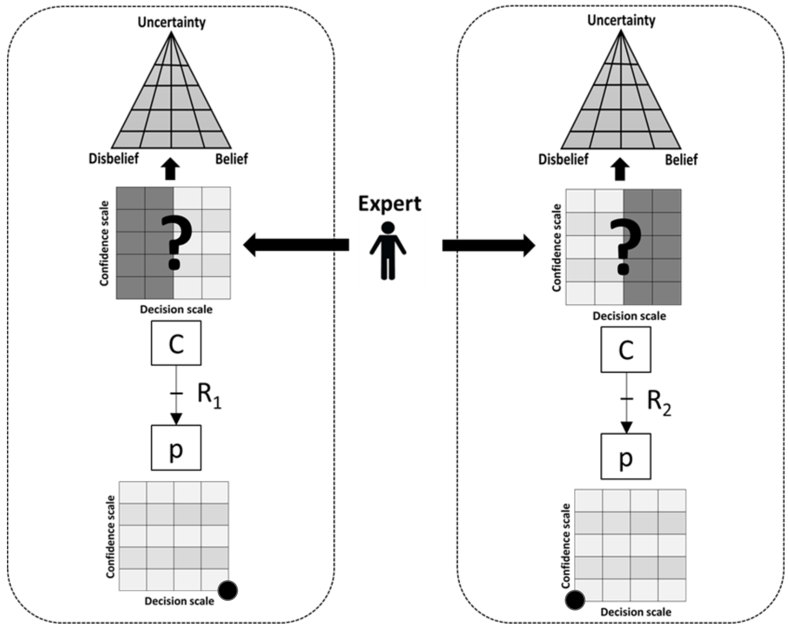

Figure 10: Belief elicitation of rules

We can notice in this example that both cases respect the Josang constraint (Eq. 9). Hence, there is no need to adjust the decision value. The first situation ($Dec(p) = 1$, $Conf(p) = 1$) for R_1 is equivalent to $Bel_p(p) = 1$. We thus are investigating the direct rule $p \Rightarrow C$. Using equations (6) and (7) for the direct rule R_1: $Bel_{\Rightarrow}(R_1) = Bel_C(C) = \frac{(0.6)-1}{2} + (0.75) = 0.55$ and we set $Bel_{\Rightarrow}(\neg R_1) = 0$. In the same way, for the reverse rule R_2: $Bel_{\Leftarrow}(R_2) = Disb_C(C) = \frac{(0.8)+1}{2} - (0.25) = 0.65$ and we set $Bel_{\Leftarrow}(\neg R_2) = 0$.

We note that the results obtained for these two rules are consistent with our expectations. Indeed, starting from high confidence values, we find that the belief value of these rules is indeed higher than the uncertainty value.

To collect the masses on rules, Wang et al. [12] choose to use identification techniques. They ask the expert to give his/her assessment about the conclusion (outputs) according to predefined assessment of premises (inputs). Then, using a non-linear least square method, they identify the values of the rules weight (denoted ω_i). However, we notice that this method could lead to values outside the unit interval $[0, 1]$, which makes no sense. Moreover, asking the expert to give his/here assessment of the conclusion according to predefined inputs (i.e, Supposing that we have "*tolerable, with high confidence*" and "*opposable, with low confidence*" assessment on premises, what this your assessment on the conclusion ?) can be disturbing and difficult, specially if you have several premises. In our case, we ask the expert to give his/her opinion only on extreme situations assuming that the argument is a C-Arg (resp. D-Arg) to define the direct (resp. reverse) conjunctive rule and the reverse (resp. direct) elementary rules.

5.2. Outline of the confidence assessment procedure

Figure 11 illustrates our assessment procedure structured in two phases. The first one is called *modeling phase*. It provides the beliefs on the rules. The second is called the *application phase*. It provides the beliefs on the premises and then propagates them with the beliefs on the rules, using the propagation model, up to the conclusion. This procedure is the same for both quantitative and purely qualitative methods.

5.2.1. Modeling phase

It will be conducted by asking $2n + 2$ questions to the assessor using the evaluation matrices, n being the number of premises. The first $2n$ questions concern masses on elementary rules (direct and reverse). For instance, to get (resp.) the values of $Bel^1_{\Leftarrow}(\neg p_1 \Rightarrow \neg C)$ and $Bel^1_{\Rightarrow}(p_1 \Rightarrow C)$ the expert will be asked the following questions (in the case $n = 2$):

1. Supposing no knowledge about the premise p_2: $(Dec = 0.5, Conf = 0)$ and total rejection (rejectable for sure) of p_1: $(Dec = 0, Conf = 1)$, what is your Decision/Confidence in the conclusion ?
2. Supposing no knowledge about the premise p_2: $(Dec = 0.5, Conf = 0)$ and total acceptance (acceptable for sure) of p_1 : $(Dec = 1, Conf = 1)$, what is your Decision/Confidence in the conclusion ?

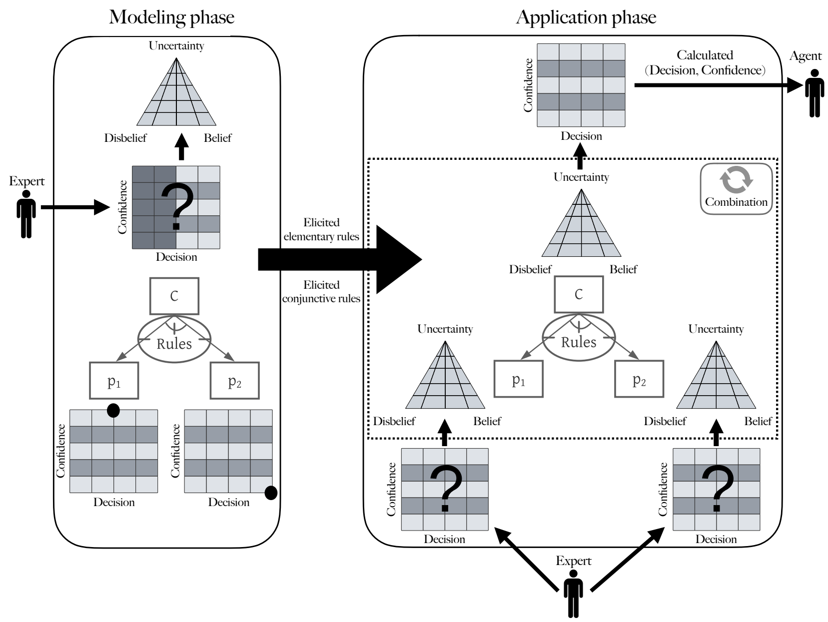

Figure 11: Outline of the assessment framework for safety argument

The additional two questions concern the conjunctive rules (resp. reverse and direct):

3. Supposing total reject of both premises p_1, p_2 : $(Dec = 0, Conf = 1)$, what is your Decision/Confidence in the conclusion ?
4. Supposing total acceptance of both premises p_1, p_2 : $(Dec = 1, Conf = 1)$, what is your Decision/Confidence in the conclusion ?

It is important to mention that the expert can only select pairs $(Dec, Conf)$ from the positive side of the evaluation matrix ("no decision" to "acceptable") while assessing direct rules. Conversely, he/she can only select negative assessment for the reverse rules ("rejectable" to "no decision"). For instance in Figure 11 (modelling phase), choosing a pair from this forbidden zone (shaded) will assign a mass to $Bel^2_{\Rightarrow}(\neg[p_2 \Rightarrow C])$ which is not a rule (see, sub-sections 3.1 and 3.3). Even if we set all the positive masses of this kind (disbelief on rules) to zero, we prefer not to allow access to this area to keep consistency with the rules definition. The implication used to define rules can only infer one side of the assessment at a time.

Once the masses on rules are obtained, one can deduce the argument type of the assessed GSN pattern (C-Arg, D-Arg or H-Arg). The case of C-Arg is simple to identify, since verification of one premise (i.e., true) cannot infer the conclusion C alone. Thus, $Bel^i_{\Rightarrow}(p_i \Rightarrow C) = 0$. Conversely, the denial of one premise (i.e., false) infers the conclusion denial $\neg C$. However, it is not the same for D-Arg. If a premise p_1 supports the conclusion C, then $p_1 \wedge p_2$ also supports it even if p_2 cannot. To keep consistency with the definition of D-Arg, we set the mass of the direct conjunction and the reverse elementary ones to zero ($Bel_{\Rightarrow}([\wedge^n_{i=1} p_i] \Rightarrow C) = Bel^i_{\Leftarrow}(\neg p_i \Rightarrow \neg C) = 0$), if at least the mass on one elementary rule is equal to the one on the conjunction. In this case, we can say that the conjunction of premises does not bring additional support to the conclusion.

Then, once the masses of the rules are acquired and the propagation formula is specified, we can proceed to the next step of this assessment procedure by following the instructions below, for the considered system.

Note: The number of pieces of information/questions to collect, is a critical point of this interaction. The smaller the amount of data required, the easier the approach will be for both user and evaluator. We did not specifically drive any study to assess the best number of information items. But what we have done so far is to compare the number of pieces of information needed in some similar approaches and in ours. For instance, when using Bayesian approaches [5, 34] to propagate confidence in the network, we need 2^n pieces of information to quantify the *inference between a parent node and its child nodes* (rules in our approach). Having only $2n + 2$ questions per node represents an asset in favor of our approach.

5.2.2. Application phase

In this phase, the expert will be asked again n additional questions (1 question per premise). Hence, we end-up with $3n + 2$ questions per node. These questions are grouped in a questionnaire [35].

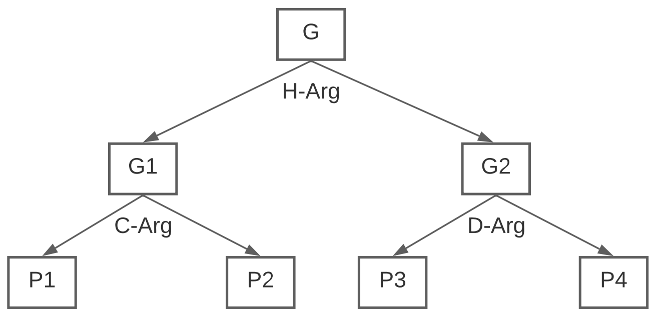

Figure 12: GSN artificial example

Then using the propagation formulas, we calculate the belief and disbelief in the conclusion. This procedure will be iterated for each node consisting of a conclusion (goal in GSN formalism) supported by its premises (sub-goals), starting from the bottom of the GSN, up to the top-goal.

Finally, we may transform the resulting pair (Belief, Disbelief), of the conclusion, into a pair (Decision, Confidence) (using formulas (6) and (7) in the numerical setting) and approximate them by choosing the qualitative values, of the closest pair $(Dec, Conf)$ to their corresponding numerical values.

5.3. Quantitative vs. qualitative assessment procedures

On an artificial example (Figure 12) that displays three argument types (C-Arg, D-Arg and H-Arg), we apply our approach in order to see how each type affects the propagation of uncertainty from premises to the overall goal (conclusion). We also apply the quantitative approach on this artificial case study. To compare results from both approaches, we will use the same decision and confidence scales presented in Figure 13.

The example in Figure 12, presents a top-goal (G) supported by two sub-goals (G1) and (G2) through a hybrid argument type (H-Arg). Each one of them is also supported, respectively, by two premises. Goal (G1) is supported by the premises (P1) and (P2) related by a conjunctive argument type (C-Arg). On the other hand, goal (G2) is supported by the premises (P3) and (P4) related by a disjunctive argument type (D-Arg).

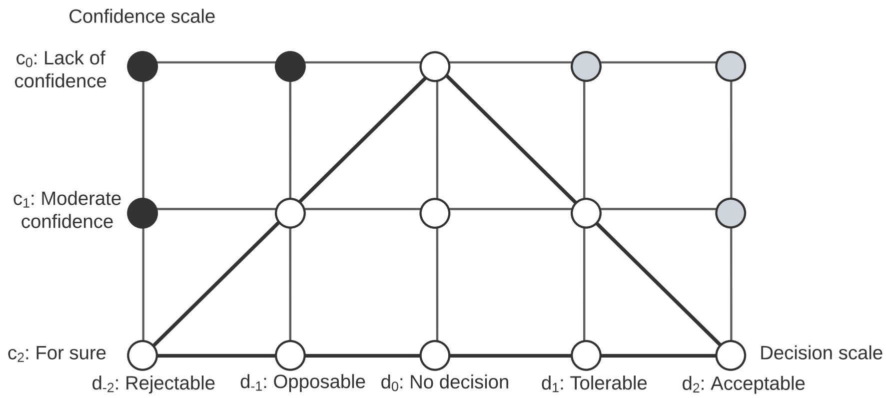

Figure 13: Evaluation matrix

For simplicity, we set all masses on rules of C-Arg, D-Arg, and the conjunctive ones of H-Arg to their maximal values ("acceptable for sure"). While we set the value of elementary rules of H-Arg to "tolerable, for sure". Then, we use four different cases of premises assessments and compute the confidence in the top goal for each. To get, respectively, belief degrees and q-capacities from the assessment of rules (see Table 8) and premises, we use the appropriate elicitation models in sub-sections 3.2 and 4.2.

In general, we can see from Table 9 that both approaches give close results, which fits well with our expectations. The only difference is in the confidence values. We can say that, in this case the qualitative approach gives results with higher levels of confidence than the quantitative one.

We notice from Table 9 in the 1^{st} and 2^{nd} lines that, as expected, the top goal keeps the same decision as premises respectively: "*tolerable*" and "*opposable*"., with a degradation of the degree of confidence in the quantitative case. Calculating the conflict mass in both cases, we can notice that their values are not null (i.e., $m_G(\perp) = 0.25$ for both cases). These values result from the difference between the calculated evaluations of the conjunctive argument in G_1 and disjunctive argument in G_2. In the 3^{rd} line, we get "*tolerable*" decision, because the D-Arg favors the propagation of the premise with the greatest weight (toler-

Table 8: Values of the belief degrees on rules

Type	Rule	Qualitative value (γ)	Quantitative value (Bel)
C-Arg	$(P1 \wedge P2) \Rightarrow G1$	1	1
	$\neg P1 \Rightarrow \neg G1$	1	1
	$\neg P2 \Rightarrow G1$	1	1
D-Arg	$(\neg P3 \wedge \neg P4) \Rightarrow \neg G2$	1	1
	$P3 \Rightarrow G2$	1	1
	$P4 \Rightarrow G2$	1	1
H-Arg	$(G1 \wedge G2) \Rightarrow G$	1	1
	$(\neg G1 \wedge \neg G2) \Rightarrow \neg G$	1	1
	$G1 \Rightarrow G$	λ	0.75
	$\neg G1 \Rightarrow \neg G$	λ	0.75
	$G2 \Rightarrow G$	λ	0.75
	$\neg G2 \Rightarrow \neg G$	λ	0.75

Remember: $Bel : 2^{\Omega} \rightarrow [0, 1]$ and $\gamma : 2^{\Omega} \rightarrow L = \{0, \lambda, 1\}$.

Table 9: Pairs (decision, confidence) using qualitative (Qual.) and quantitative (Quant.) methods

Case	P_1	P_2	P_3	P_4	G (Quant.)	G (Quali.)
1^{st}	(Tol,c_2)	(Tol,c_2)	(Tol,c_2)	(Tol,c_2)	(Tol,c_1)	(Tol,c_2)
2^{nd}	(Opp,c_2)	(Opp,c_2)	(Opp,c_2)	(Opp,c_2)	(Opp,c_1)	(Opp,c_2)
3^{rd}	(Tol,c_2)	(Tol,c_2)	(Tol,c_2)	(Opp,c_2)	(Tol,c_1)	(Tol,c_2)
4^{th}	(Opp,c_2)	(Tol,c_2)	(Tol,c_2)	(Tol,c_2)	(ND,c_1)	$(\lambda, \lambda) \equiv$ (ND,c_1)

Where: c_2: For sure, c_1: Moderately confident and c_0: Lack of confidence.

able) to G_2 (G_1: tolerable). The conflict mass in this situation is $m_G(\perp) = 0.26$. The opposite assessments in the disjunctive argument does not have a significant impact on the top goal G. On the contrary, in the 4^{th} line, we get a "*no decision*". This result is explained by the fact that we end up with two opposite judgments in the H-Arg (conflict situation) due to C-Arg that propagates the premise with least strength (opposable) to G_1 (G_2: tolerable). This also explains the high value of the conflict mass (i.e., $m_G(\perp) = 0.44$), in contrast to previous cases.. The degree of confidence also decreases in the quantitative case.

The difference in the degree of confidence between qualitative and quantitative approaches is due to the nature of the operations used. For example, the C-Arg favors the propagation of the weakest premise (weaker belief and stronger disbelief). In the quantitative setting, we use the product and the probabilistic sum. And in the qualitative case, we use min and max, which does not model attenuation or reinforcement effects in case of independent pieces of information.

Notice that in the 4^{th} case, we get the assessment for the qualitative setting: $\gamma_C(C) = \gamma_C(\neg C) = \lambda$. In this situation, we choose to translate these values to "*no decision*" (ND), with "*moderate confidence*" (c_1). However, if we go back to Table 6, we notice that this assessment could express three different decision values (opposable, no decision and tolerable). This is one limitation of the qualitative approach, a relative lack of discrimination.

6. Conclusion

In this article, we propose an approach to confidence assessment in assurance cases based on GSN. To do this, we formally define argument types using logical expressions, and model the inference of the goal to be proven from the pieces of evidence they support. Then, using belief functions, we quantify confidence (belief and disbelief) in these expressions, and merge them with a variant of Dempster rule of combination, thus defining a quantitative confidence propagation model. In contrast to similar approaches proposed in the literature, we propose a general model covering the different types of arguments. The propagation formula can be used to compute the confidence in the conclusion for the four types of arguments we have defined (H-Arg, C-Arg, D-Arg and S-Arg). According to the values taken by the degrees of belief of the rules (direct and inverse), the type is easily identifiable. This allows for easy implementation and fast execution of this model for possible future applications. The propagation model also takes into consideration conflict situations involving two or more opposite assessments regarding different pieces of evidence supporting the same goal. We also improved an existing expert information elicitation model, using the pignistic transform, and adding a neutral (indecisive) option to the possible uncertainty assessment choices. In addition, we propose a questionnaire to collect data about rules and premises.

To avoid the relative arbitrariness of the quantitative assessment approach, notably the issues related to the attenuation-amplification effects, choice of numerical scales, and the independence of sources assumption, we propose a new purely qualitative confidence assessment method which propagates qualitative belief and disbelief from the premises to the conclusion. This approach avoids the need for transforming expert assessments from natural language into numerical values which can be seen as a source of uncertainty.

The use of DST to quantify and propagate confidence in graphical models of argument structure such as GSN is not widespread and is relatively new compared to Bayesian approaches. However, the main problem to overcome lies in the elicitation procedure. The elicitation model for both the quantitative and qualitative approach need improvements. On one hand, it is important to develop specific scales calibrated to real assurance cases and verify that the results given by the elicitation model are intuitively correct. On the other hand, it is also necessary to develop the questionnaire by improving, for instance, the way in which the questions are asked. Indeed, it seems that the way in which the questions are asked so far encourages the experts to give extreme assessments, which leads to extreme argument types. In particular, most case studies we encountered only involved the C-Arg. However, for a robust validation of our approach, more experiments on general cases must be conducted for both quantitative and qualitative assessment methods. These experiments would be useful to further compare the quantitative assessment method and the qualitative one so as to determine which approach is best.

Finally, the approach we propose only takes into consideration components from the GSN formalism (goal, strategy, solutions, context, etc). Thus, it is not always easy to switch from one formalism to another. By integrating other components, our approach will become more generic. An interesting proposal is to take into account exceptions to rules (i.e. defeaters) or evidence against the conclusion (i.e. rebuttals) by considering disbelief respectively in direct and reverse rules.

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Appendix A. Propagation formulas - detailed calculation

In the following, we present a detailed calculation of propagation formulas for the hybrid argument type (H-Arg) in the case of a conclusion (C) supported by two premises (p_1) and (p_2) in the numerical setting. Then, we present how we calculate the mass of conflict for "n" premises.

To calculate propagation formulas in a pure qualitative setting, it is enough to replace, respectively, the mass functions (BPA: m_p, m_r and m_C) and the conjunctive combination rule (equation 2) by qualitative capacity functions (BΠA: ρ_p, ρ_r and ρ_C) and the qualitative combination rule (equation 5). Since we always combine the same rules for each type of argument, all the following combination tables are the same for the quantitative and qualitative approach.

Appendix A.1. Propagation formulas for the hybrid argument type

Considering the example in Figure 8, we define the rules of a H-Arg with $n = 2$ premises, which includes those of C-Arg and D-Arg defined above.

Using the conjunctive rule of combination (equation 2) we combine the mass on premises with those on rules: $m_C = (m_{\Leftarrow} \otimes m_{\Rightarrow}) \otimes ((m^1_{\Leftarrow} \otimes m^1_{\Rightarrow}) \otimes m^1_p) \otimes ((m^2_{\Leftarrow} \otimes m^2_{\Rightarrow}) \otimes m^2_p)$ (parentheses indicate the order of computations from Table A.10 to A.14) and deduce belief and disbelief propagation formulas.

Table A.10: Combination of direct and reverse conjunctive rules for H-Arg

$m_r = m_{\Rightarrow} \otimes m_{\Leftarrow}$	$m_{\Rightarrow}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C)$	$m_{\Rightarrow}(\top)$
$m_{\Rightarrow}([p_1 \wedge p_2] \Rightarrow C)$	$F_c = ([p_1 \wedge p_2] \Rightarrow C) \wedge ([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C)$	$[p_1 \wedge p_2] \Rightarrow C$
$m_{\Rightarrow}(\top)$	$[\neg p_1 \wedge \neg p_2] \Rightarrow \neg C$	$\top$

Table A.11: Combination of direct and reverse elementary rules for the H-Arg

$m^i_r = m^i_{\Rightarrow} \otimes m^i_{\Leftarrow}$	$m^i_{\Rightarrow}(p_i \Rightarrow C)$	$m^i_{\Rightarrow}(\top)$
$m^i_{\Leftarrow}(\neg p_i \Rightarrow \neg C)$	$p_i \equiv C$	$\neg p_i \Rightarrow \neg C$
$m^i_{\Leftarrow}(\top)$	$p_i \Rightarrow C$	$\top$

Table A.12: Combination of the i^{th} premise ($p_i,\ i = \{1,2\}$) with its elementary rules for H-Arg

$m_i = m_p^i \otimes m_r^i$	$m_r^i(p_i \equiv C)$	$m_r^i(p_i \Rightarrow C)$	$m_r^i(\neg p_i \Rightarrow \neg C)$	$m_r^i(\top)$
$m_p^i(p_i)$	$p_i \wedge C$	$p_i \wedge C$	p_i	p_i
$m_p^i(\neg p_i)$	$\neg p_i \wedge \neg C$	$\neg p_i$	$\neg p_i \wedge \neg C$	$\neg p_i$
$m_p^i(\top)$	$p_i \equiv C$	$p_i \Rightarrow C$	$\neg p_i \Rightarrow \neg C$	$\top$

Table A.13: Combination of the 1^{st} and 2^{nd} premises (p_i) with their elementary rules for H-Arg

$m_{12} = m_1 \otimes m_2$	$m_2(p_2 \wedge C)$	$m_2(\neg p_2 \wedge \neg C)$	$m_2(p_2)$	$m_2(\neg p_2)$	$m_2(p_2 \Rightarrow C)$	$m_2(\neg p_2 \Rightarrow \neg C)$	$m_2(p_2 \equiv C)$	$m_2(\top)$
$m_1(p_1 \wedge C)$	$p_1 \wedge p_2 \wedge C$	$\emptyset$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge \neg p_2 \wedge C$	$p_1 \wedge C$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge C$
$m_1(\neg p_1 \wedge \neg C)$	$\emptyset$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge p_2 \wedge \neg C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg C$
$m_1(p_1)$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge \neg p_2 \wedge \neg C$	$p_1 \wedge p_2$	$p_1 \wedge \neg p_2$	$p_1(p_2 \Rightarrow C)$	$p_1(\neg p_2 \Rightarrow \neg C)$	$p_1(p_2 \equiv C)$	p_1
$m_1(\neg p_1)$	$\neg p_1 \wedge p_2 \wedge C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge p_2$	$\neg p_1 \wedge \neg p_2$	$\neg p_1(p_2 \Rightarrow C)$	$\neg p_1(\neg p_2 \Rightarrow \neg C)$	$\neg p_1(p_2 \equiv C)$	$\neg p_1$
$m_1(p_1 \Rightarrow C)$	$p_2 \wedge C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$p_2(p_1 \Rightarrow C)$	$\neg p_2(p_1 \Rightarrow C)$	$(p_1 \Rightarrow C)(p_2 \Rightarrow C)$	$(p_1 \Rightarrow C)(\neg p_2 \Rightarrow \neg C)$	$(p_1 \Rightarrow C)(p_2 \equiv C)$	$(p_1 \Rightarrow C)$
$m_1(\neg p_1 \Rightarrow \neg C)$	$p_1 \wedge p_2 \wedge C$	$\neg p_2 \wedge \neg C$	$p_2(\neg p_1 \Rightarrow \neg C)$	$\neg p_2(\neg p_1 \Rightarrow \neg C)$	$(p_2 \Rightarrow C)(\neg p_1 \Rightarrow \neg C)$	$(\neg p_2 \Rightarrow \neg C)(\neg p_1 \Rightarrow \neg C)$	$(p_2 \equiv C)(\neg p_1 \Rightarrow \neg C)$	$\neg p_1 \Rightarrow \neg C$
$m_1(p_1 \equiv C)$	$p_1 \wedge p_2 \wedge C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$p_2(p_1 \equiv C)$	$\neg p_2(p_1 \equiv C)$	$(p_1 \equiv C)(p_2 \Rightarrow C)$	$(p_1 \equiv C)(\neg p_2 \Rightarrow \neg C)$	$(p_1 \equiv C)(p_2 \equiv C)$	$(p_1 \equiv C)$
$m_1(\top)$	$p_2 \wedge C$	$\neg p_2 \wedge \neg C$	p_2	$\neg p_2$	$p_2 \Rightarrow C$	$\neg p_2 \Rightarrow \neg C$	$p_2 \equiv C$	$\top$

Table A.14: Combination of premises and their elementary rules with the conjunctive rules (table A.10) for H-Arg

$m = m_{12} \otimes m_r$	$m_r(F_c)$	$m_r([p_1 \wedge p_2] \Rightarrow C)$	$m_r([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C)$	$m_r(\top)$
$m_{12}(p_1 \wedge p_2 \wedge C)$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2 \wedge C$
$m_{12}(\neg p_1 \wedge p_2 \wedge C)$	$\neg p_1 \wedge p_2 \wedge C$	$\neg p_1 \wedge p_2 \wedge C$	$\neg p_1 \wedge p_2 \wedge C$	$\neg p_1 \wedge p_2 \wedge C$
$m_{12}(p_1 \wedge \neg p_2 \wedge C)$	$p_1 \wedge \neg p_2 \wedge C$	$p_1 \wedge \neg p_2 \wedge C$	$p_1 \wedge \neg p_2 \wedge C$	$p_1 \wedge \neg p_2 \wedge C$
$m_{12}(p_1 \wedge C)$	$p_1 \wedge C$	$p_1 \wedge C$	$p_1 \wedge C$	$p_1 \wedge C$
$m_{12}(p_2 \wedge C)$	$p_2 \wedge C$	$p_2 \wedge C$	$p_2 \wedge C$	$p_2 \wedge C$
$m_{12}(\neg p_1 \wedge \neg p_2 \wedge \neg C)$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$
$m_{12}(\neg p_1 \wedge p_2 \wedge \neg C)$	$\neg p_1 \wedge p_2 \wedge \neg C$	$\neg p_1 \wedge p_2 \wedge \neg C$	$\neg p_1 \wedge p_2 \wedge \neg C$	$\neg p_1 \wedge p_2 \wedge \neg C$
$m_{12}(p_1 \wedge \neg p_2 \wedge \neg C)$	$p_1 \wedge \neg p_2 \wedge \neg C$	$p_1 \wedge \neg p_2 \wedge \neg C$	$p_1 \wedge \neg p_2 \wedge \neg C$	$p_1 \wedge \neg p_2 \wedge \neg C$
$m_{12}(\neg p_1 \wedge \neg C)$	$\neg p_1 \wedge \neg C$	$\neg p_1 \wedge \neg C$	$\neg p_1 \wedge \neg C$	$\neg p_1 \wedge \neg C$
$m_{12}(\neg p_2 \wedge \neg C)$	$\neg p_2 \wedge \neg C$	$\neg p_2 \wedge \neg C$	$\neg p_2 \wedge \neg C$	$\neg p_2 \wedge \neg C$
$m_{12}(p_1 \wedge p_2)$	$p_1 \wedge p_2 \wedge C$	$p_1 \wedge p_2 \wedge C$	-	-
$m_{12}(\neg p_1 \wedge \neg p_2)$	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	-	$\neg p_1 \wedge \neg p_2 \wedge \neg C$	-
...	...	...	...	...
$m_{12}(\emptyset)$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$m_{12}(\top)$	F_c	$[p_1 \wedge p_2] \Rightarrow C$	$[\neg p_1 \wedge \neg p_2] \Rightarrow \neg C$	$\top$

In Table A.14, we have chosen not to represent the focal elements resulting from Table A.13 which imply neither the conclusion nor its negation, so as to reduce the size of the table.

To calculate the belief degree of the conclusion, we sum the masses of all elements that imply the conclusion (C) in Table A.14 (dark grey areas).

$$\begin{aligned}
Bel_C(C) = {} & [m_{12}(p_1 \wedge p_2 \wedge C) + m_{12}(\neg p_1 \wedge p_2 \wedge C) + m_{12}(p_1 \wedge \neg p_2 \wedge C) + m_{12}(p_1 \wedge C) + m_{12}(p_2 \wedge C)] \cdot \sum_{E \subseteq \Omega_r} m_r(E) \\
& + m_{12}(p_1 \wedge p_2)[m_r(F_C) + m_r([p_1 \wedge p_2] \Rightarrow C)] \qquad \text{see Table A.14} \\
& = m_1(p_1 \wedge C) \cdot \sum_{E \subseteq \Omega_2} m_2(E) + m_2(p_2 \wedge C) \cdot \sum_{E \subseteq \Omega_1} m_1(E) \\
& - m_1(p_1 \wedge C) \cdot m_2(p_2 \wedge C) - [m_1(\neg p_1 \wedge \neg C) \cdot m_2(p_2 \wedge C) + m_1(p_1 \wedge C) \cdot m_2(\neg p_2 \wedge \neg C)]^* \\
& + m_1(p_1) \cdot m_2(p_2) \cdot m_{\Rightarrow}([p_1 \wedge p_2] \Rightarrow C) \qquad \text{see Tables A.10 and A.13} \\
& = [m_p^1(p_1) \cdot m_{\Rightarrow}^1(p_1 \Rightarrow C) + m_p^2(p_2) \cdot m_{\Rightarrow}^2(p_2 \Rightarrow C) - m_p^1(p_1) \cdot m_{\Rightarrow}^1(p_1 \Rightarrow C) \cdot m_p^2(p_2) \cdot m_{\Rightarrow}^2(p_2 \Rightarrow C)] - m_{12}(\emptyset) \\
& + m_{\Rightarrow}([p_1 \wedge p_2] \Rightarrow C) \cdot m_p^1(p_1) \cdot [1 - m_{\Rightarrow}(p_1 \Rightarrow C)] \cdot m_p^2(p_2) \cdot [1 - m_{\Rightarrow}(p_2 \Rightarrow C)] \qquad \text{see Tables A.11 and A.12} \\
& = \{1 - [1 - m_p^1(p_1) \cdot m_{\Rightarrow}^1(p_1 \Rightarrow C)] \cdot [1 - m_p^2(p_2) \cdot m_{\Rightarrow}^2(p_2 \Rightarrow C)]\} - m_{12}(\emptyset) \\
& + m_{\Rightarrow}([p_1 \wedge p_2] \Rightarrow C) \cdot m_p^1(p_1) \cdot [1 - m_{\Rightarrow}(p_1 \Rightarrow C)] \cdot m_p^2(p_2) \cdot [1 - m_{\Rightarrow}(p_2 \Rightarrow C)]
\end{aligned}$$

*To take advantage of $\sum_{E \subseteq \Omega_i} m_i(E) = 1,\ i = \{1, 2\}$ and simplify the calculation of the terms: $m_{12}(p_1 \wedge p_2 \wedge C) + m_{12}(\neg p_1 \wedge p_2 \wedge C) + m_{12}(p_1 \wedge \neg p_2 \wedge C) + m_{12}(p_1 \wedge C) + m_{12}(p_2 \wedge C)$ in Table A.13, we replace them by the sum of $m_1(p_1 \wedge C)$ and $m_2(p_2 \wedge C)$ from which we subtract the masses of the empty intersection and the redundant term $m_1(p_1 \wedge C) \times m_2(p_2 \wedge C)$.

Notice that $m_{12}(\emptyset)$ results from the combination in Table A.13 only, since the empty intersection does not appear when conjunctive rules are combined with themselves (i.e. direct and reverse rules in Table A.10) or with elementary rules (i.e. Table A.14), $m_C(\emptyset) = m_{12}(\emptyset)$ includes only the conflict mass resulting from the combination of elementary rules (i.e. white entries in Table A.13).

To calculate the conflict degree, we sum the masses of all elements that have an empty intersection ($\emptyset$) in Table A.14.

$$
\begin{aligned}
m_C(\emptyset) &= m_{12}(\emptyset) \cdot \sum_{E \subseteq \Omega_r} m_r(E) \\
&= [m_1(\neg p_1 \wedge \neg C) \cdot m_2(p_2 \wedge C) + m_2(\neg p_2 \wedge \neg C) \cdot m_1(p_1 \wedge C)] \times 1 \\
&= m_p^1(\neg p_1) \cdot m_{\Leftarrow}^1(\neg p_1 \Rightarrow \neg C) \cdot m_p^2(p_2) \cdot m_{\Rightarrow}^2(p_2 \Rightarrow C) + m_p^2(\neg p_2) \cdot m_{\Leftarrow}^2(\neg p_2 \Rightarrow \neg C) \cdot m_p^1(p_1) \cdot m_{\Rightarrow}^1(p_1 \Rightarrow C)
\end{aligned}
$$

Let's put: $b_C(C) = m_{\Rightarrow}([p_1 \wedge p_2] \Rightarrow C) \cdot m_p^1(p_1) \cdot [1 - m_{\Rightarrow}(p_1 \Rightarrow C)] \cdot m_p^2(p_2) \cdot [1 - m_{\Rightarrow}(p_2 \Rightarrow C)] + \{1 - [1 - m_p^1(p_1) \cdot m_{\Rightarrow}^1(p_1 \Rightarrow C)] \cdot [1 - m_p^2(p_2) \cdot m_{\Rightarrow}^2(p_2 \Rightarrow C)]\}$. For the conjunctive part $Bel(E) = b(E)$ because $m_r(\emptyset) = 0$. Keep also in mind that for frame of discernment with two elements (i.e. $|\Omega| = 2$): $Bel_p^i(p_i) = m_p^i(p_i)$, $Bel_{\Rightarrow}^i(p_i \Rightarrow C) = m_{\Rightarrow}^i(p_i \Rightarrow C)$ and $Bel_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C) = m_{\Leftarrow}^i(\neg p_i \Rightarrow \neg C)$

To calculate the disbelief degree of the conclusion, we sum the masses of all elements that imply the negation of the conclusion ($\neg C$) in Table A.14 (bright grey areas).

$$
\begin{aligned}
Disb_C(C) = {} & [m_{12}(\neg p_1 \wedge \neg p_2 \wedge \neg C) + m_{12}(\neg p_1 \wedge p_2 \wedge \neg C) + m_{12}(p_1 \wedge \neg p_2 \wedge \neg C) + m_{12}(\neg p_1 \wedge \neg C) \cdot m_{12}(\neg p_2 \wedge \neg C)] \cdot \sum_{E \subseteq \Omega_r} m_r(E) \\
& + m_{12}(\neg p_1 \wedge \neg p_2)[m_r(F_C) + m_r([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C)] \\
& = m_1(\neg p_1 \wedge \neg C) \cdot \sum_{E \subseteq \Omega_2} m_2(E) + m_2(\neg p_2 \wedge \neg C) \cdot \sum_{E \subseteq \Omega_1} m_1(E) \\
& - m_1(\neg p_1 \wedge \neg C) \cdot m_2(\neg p_2 \wedge \neg C)] - [m_1(\neg p_1 \wedge \neg C) \cdot m_2(p_2 \wedge C) + m_1(p_1 \wedge C) \cdot m_2(\neg p_2 \wedge \neg C)] \\
& + m_1(\neg p_1) \cdot m_2(\neg p_2) \cdot m_{\Leftarrow}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C) \\
& = [m_p^1(\neg p_1) \cdot m_{\Leftarrow}^1(\neg p_1 \Rightarrow \neg C) + m_p^2(\neg p_2) \cdot m_{\Leftarrow}^2(\neg p_2 \Rightarrow \neg C) - m_p^1(\neg p_1) \cdot m_{\Leftarrow}^1(\neg p_1 \Rightarrow \neg C) \cdot m_p^2(p_2) \cdot m_{\Leftarrow}^2(p_2 \Rightarrow C)] - m_{12}(\emptyset) \\
& + m_{\Leftarrow}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C) \cdot m_p^1(\neg p_1) \cdot [1 - m_{\Leftarrow}(\neg p_1 \Rightarrow \neg C)] \cdot m_p^2(\neg p_2) \cdot [1 - m_{\Leftarrow}(\neg p_2 \Rightarrow \neg C)] \\
& = \{1 - [1 - m_p^1(\neg p_1) \cdot m_{\Leftarrow}^1(\neg p_1 \Rightarrow \neg C)] \cdot [1 - m_p^2(\neg p_2) \cdot m_{\Leftarrow}^2(\neg p_2 \Rightarrow \neg C)]\} - m_{12}(\emptyset) \\
& + m_{\Leftarrow}([\neg p_1 \wedge \neg p_2] \Rightarrow \neg C) \cdot m_p^1(\neg p_1) \cdot [1 - m_{\Leftarrow}(\neg p_1 \Rightarrow \neg C)] \cdot m_p^2(\neg p_2) \cdot [1 - m_{\Leftarrow}(\neg p_2 \Rightarrow \neg C)]
\end{aligned}
$$

Let's put: $d_C(C) = b_C(\neg C) = m_{\Leftarrow}([\neg p_1 \wedge \neg p_2] \Rightarrow C) \cdot m_p^1(\neg p_1) \cdot [1 - m_{\Leftarrow}(\neg p_1 \Rightarrow \neg C)] \cdot m_p^2(\neg p_2) \cdot [1 - m_{\Leftarrow}(\neg p_2 \Rightarrow \neg C)] + \{1 - [1 - m_p^1(p_1) \cdot m_{\Leftarrow}^1(\neg p_1 \Rightarrow \neg C)] \cdot [1 - m_p^2(\neg p_2) \cdot m_{\Leftarrow}^2(\neg p_2 \Rightarrow \neg C)]\}$.

Hence, we show that for $n = 2$:

$$
\begin{cases}
Bel_C(C) & = & b_C(C) - m_{12}(\perp) \\
Disb_C(C) & = & d_C(C) - m_{12}(\perp)
\end{cases}
$$

Appendix A.2. Conflict mass calculation

In this section, we calculate the conflict mass formula for "n" premises. Let us enumerate all conflicting combinations of 4 formulas when merging mass functions on premises and rules.

The case $n = 2$ is already addressed in the previous section.

For $n = 3$, we can calculate $m_{123} = m_1 \otimes m_2 \otimes m_3$ directly but we have to calculate $8 \times 8 \times 8$ intersections and select the contradictory conjunctions. We can also calculate more easily:

$$\begin{aligned} m_{123}(\perp) &= \sum_{i \neq j; i,j=1,2,3} m_i(p_i \wedge C) \cdot m_j(\neg p_j \wedge \neg C) \\ &= m_{12}(\perp) + m_{13}(\perp) + m_{23}(\perp) \end{aligned}$$

However, this calculation counts contradictory terms several times. Thus, we may find that this mass ($m_{123}(\perp)$) is greater than 1. Denote the contradictory term as $m_1(p_1 \wedge C) \cdot m_2(\neg p_2 \wedge \neg C) \cdot m_3(\neg p_3 \wedge \neg C)$ with $1\bar{2}\bar{3}$, etc. We can write $m_1(p_1 \wedge C) \cdot m_2(\neg p_2 \wedge \neg C)$ as $m_1(p_1 \wedge C) \cdot m_2(\neg p_2 \wedge \neg C) \cdot \sum_{\phi \in \Omega_3} m_3(\phi)$, since $\sum_{\phi \in \Omega_3} m_3(\phi) = 1$. Hence, the term $m_1(p_1 \wedge C) \cdot m_2(\neg p_2 \wedge \neg C)$ of $m_{12}(\perp)$ includes $1\bar{2}\bar{3}$ and $1\bar{2}3$. Similarly, we get the remaining duplicate terms (in bold):

- $m_1(\neg p_1 \wedge \neg C) \cdot m_2(p_2 \wedge C)$ includes $\bar{1}2\bar{3}$ et $\bar{1}23$.
- $m_1(\neg p_1 \wedge \neg C) \cdot m_3(p_3 \wedge C)$ includes $\mathbf{\bar{1}23}$ et $\bar{1}\bar{2}3$.
- $m_1(p_1 \wedge C) \cdot m_3(\neg p_3 \wedge \neg C)$ includes $12\bar{3}$ et $\mathbf{1\bar{2}\bar{3}}$.
- $m_2(p_2 \wedge C) \cdot m_3(\neg p_3 \wedge \neg C)$ includes $\mathbf{12\bar{3}}$ et $\mathbf{\bar{1}2\bar{3}}$.
- $m_2(\neg p_2 \wedge \neg C) \cdot m_3(p_3 \wedge C)$ includes $\mathbf{1\bar{2}3}$ et $\mathbf{\bar{1}\bar{2}3}$.

Hence, the sum $m_{12}(\perp)+m_{13}(\perp)+m_{23}(\perp)$ counts twice the product of three terms. There are 12 such terms, so we have to delete 6 of them (the ones in bold). Thus we prove that:

$$\begin{aligned}
m_{123}(\perp) = m_{12}(\perp) &+ m_1(p_1 \wedge C)m_3(\neg p_3 \wedge \neg C)[1 - m_2(\neg p_2 \wedge \neg C)] \\
&+ m_1(\neg p_1 \wedge \neg C)m_3(p_3 \wedge C)[1 - m_2(p_2 \wedge C)] \\
&+ m_2(p_2 \wedge C)m_3(\neg p_3 \wedge \neg C)[1 - m_1(p_1 \wedge C) - m_1(\neg p_1 \wedge \neg C)] \\
&+ m_2(\neg p_2 \wedge \neg C)m_3(p_3 \wedge C)[1 - m_1(p_1 \wedge C) - m_1(\neg p_1 \wedge \neg C)] \\
&= m_{12}(\perp) + m_3(p_3 \wedge C) \cdot Disb_C^{(2)}(C) + m_3(\neg p_3 \wedge \neg C) \cdot Bel_C^{(2)}(C)
\end{aligned}$$

Where:

- $Bel_C^{(2)}(C) = 1 - [1 - Bel_p^1(p_1)Bel_{\Rightarrow}^1(p_1 \Rightarrow C)] \cdot [1 - Bel_p^2(p_2)Bel_{\Rightarrow}^2(p_2 \Rightarrow C)]$
- $Disb_C^{(2)}(C) = 1 - [1 - Disb_p^1(p_1)Bel_{\Leftarrow}^1(\neg p_1 \Rightarrow \neg C)] \cdot [1 - Disb_p^2(p_2)Bel_{\Leftarrow}^2(\neg p_2 \Rightarrow \neg C)]$

This calculation can be extended to $n > 1$ premises. Hence, we get as many as $\sum_{i=1}^{n-1} C_{n-1}^i(2^i - 1)$ focal sets inducing the conclusion C ($\{\Omega_{p_1} \times ... \times \Omega_{p_n}\} \wedge C$) and others of the same count inducing its negation $\neg C$ ($\{\Omega_{p_1} \times ... \times \Omega_{p_n}\} \wedge \neg C$). Combining these focal sets respectively with $(\neg p_n \wedge \neg C)$ and $(p_n \wedge C)$ generates an empty intersection. Summing the masses of these focal sets gives the general formula of conflict $m^{(n)}(\perp)$ (equation 17).

Notice that conjunctive rules $[\wedge_{i=1}^n p_i] \Rightarrow C$ and $[\wedge_{i=1}^n \neg p_i] \Rightarrow \neg C$ were not involved in this calculation because they do not generate an empty intersection (e.g., $([p_1 \wedge p_2] \Rightarrow C) \otimes \neg p_1 = \neg p_1$).