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Accurate ray tracing optical model for coded aperture spectral snapshot imagers

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The image formation in coded aperture spectral imagers is a key information to process the acquired compress data, and the optical system design and calibration of these instruments require great care. We propose an analytical model for CASSI systems that builds upon ray-tracing equations of each optical component. The model takes into account optical distortions, sampling effects and optical misalignments, and allows accurate modelling and fast calibration. Numerical comparisons with the simpler model usually exploited in the literature are provided, and an experimental validation is presented.

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1. INTRODUCTION

Hyperspectral imaging has been considerably renewed by the 2 advent of coded aperture imagers, known as CASSI systems 3 ("Coded-aperture Spectral Snapshot Imager" [1]). Unlike traditional spectral imagers where spectral bands are measured one 5 at a time, CASSI systems measure linear combinations of spec-6 tral bands. This combination results from spatio-spectral filter-7 ing of the scene by an optical system containing one or more 8 dispersive elements and a spatial light modulator that defines a coded aperture. 10

Various CASSI architectures have been proposed in the liter-11 ature, with variations on the number and position of dispersive 12 elements, the number of cameras, the nature and position of 13 the coded aperture [2]. Whatever the optical architecture and 14 the processing algorithms are, the extraction of relevant infor-15 mation from the measurements requires an accurate model of 16 the image formation, that takes into account both the charac-17 teristics of the optical components and the sampling effects at 18 the coded aperture and detector planes. This model plays a 19 key role in linking the coded aperture pattern to the resulting 20 21 spatio-spectral filtering occurring at the coded aperture plane, and hence on the information acquired in the encoded images. 22 Furthermore, a realistic model is needed to simulate acquisi-23 tions, e.g. to ease the definition of the processing algorithms, 24 or, following a co-design approach, to specify the characteris-25 26 tics of a system along with the processing algorithms.

Some authors consider simple models for CASSI systems
 [3, 4], with straightforward approximations of the spectral fil tering that degrade the quality of the extracted information. Ef-

forts have been made to improve the estimation of the spectral filtering: [5] focuses on the spatio-spectral discretization and model its impact, and [6] propose a precise spectral-dependant estimation of the point-spread function. Both works exploit a straightforward propagation model that does not consider optical distortions inherent in prism-based systems. Failing to explicitly consider these imperfections leads to artefacts in the reconstructed information, e.g. moiré effects in spectral planes, as we experienced in previous work [7]. The usual approach to account for optical distortions is to amend a simple model with experimental look-up tables (LUTs) obtained during calibration, once a prototype is built [8–10], a process that can be cumbersome and whose complexity increases with the spectral and spatial resolutions of the instrument. Calibration is deemed as one of the drawbacks of coded aperture hyperspectral imaging systems [11, 12].

We propose here an accurate *analytical model* of a CASSI system, that builds upon ray-tracing equations of each optical component. The model embraces optical distortions that are seldom considered in the literature (smile and stretching distortions) and also accounts for optical misalignments. It precisely represents the image formation process, and more specifically the spatial/spectral filtering in CASSI systems.

Besides, the advantages of an analytical model as compared to an experimental LUT are threefold. First, the model allows easy and fast calibration of CASSI prototypes, using a small set of calibration scenes to estimate the model physical parameters with an optimisation process. Second, by explicating the impact of the system physical parameters in its formulation, it

allows to dimension the system to reach given requirements, al-59 105 lowing fast parametric studies. And finally, it allows to easily 60 106 develop a coded-image generation simulator (we provide such 107 61 a simulator in [13]). Our work is complementary to [5] and [6] 108 62 as it includes a high-order discretization and an estimation of 109 63 64 the point-spread function, but also includes a modular, differ- 110 65 entiable and realistic optical propagation model. Its differentiability is key for efficient optimisation, including machine learn-66 ing approaches. The simulator thus generates realistic encoded 67 images and yields the possibility to jointly define and configure 68 the system and data processing algorithms along a co-design 69 paradigm, or to train inference models on synthetic data. 70

Paper outline: the next section recalls the principle of a 112 71 single-disperser CASSI system, and then describes two opti-113 72 cal models: the simplified model commonly used in the liter-114 73 74 ature, and the proposed analytical model. Section 3 compares 115 both models in relation to the very accurate results provided 116 75 by the optical design software Zemax, and show drastic im-117 76 provements provided by our model. In section 4, we estimate 118 77 the model parameters of a prototype CASSI system, depict the 119 78 spectral sampling defined by our model, and provide an ex-79 120 perimental validation. Finally, section 5 shows how the non-80 121 regular local spectral sampling induced by the distortions and 81 122 discretization effects can be harnessed to enhance the spectral 82 resolution of homogeneous regions in the scene. 83 123

2. MODEL OF A SINGLE DISPERSER CASSI 84



Fig. 1. Single-disperser CASSI architecture.

The studied architecture is a single-disperser prism-based 85 136 CASSI (SD-CASSI), originally proposed in [1] and depicted in 86 137 figure 1. Imaging optics are used to image the scene in the mask 87 138 plane of the instrument. A mask placed in this plane applies a 88 139 spatial modulation to the object according to a chosen coding 89 140 pattern. The masked object is then imaged in the detector plane 141 90 by an optical system containing relay optics and a prism, re-91 sulting in each wavelength forming on the detector a laterally 92 143 shifted image. As a result, the spatial information and the spec-93 144 tral information from the observed object are multiplexed on 94 145 the detector. 95

We focus throughout the paper on a prism-based SD-CASSI 146 96 system, but a similar approach can be trivially applied to a 147 97 grating-based SD-CASSI. Also, the proposed model can de- 148 98 scribe a dual-disperser CASSI sytem (DD-CASSI), by using it 149 99 twice: once for each of the two arms of the DD-CASSI. 100

A. Classical propagation model 101

The acquisition model widely used in the literature for an SD-102 CASSI describes the intensity collected on the detector at the 103 (x_d, y_d) position as: 104

where *H* is the filtering function, (x_d, y_d) the coordinates in the detector plane, λ the wavelength of interest and S_0 the spectrospatial power density of the scene.

For the sake of simplicity, we assume the optical elements and detector to be 100 % efficient: spectral filtering only occurs in the mask plane. In the models proposed by [4, 5, 14, 15], H is written as:

$$H(x_d, y_d, \lambda) = T(x_m, y_m)$$

with $M_c: \begin{cases} x_m = -x_d + \Omega(\lambda) \\ y_m = -y_d \end{cases}$ (2)

where the "classical" model M_c is the backward propagation model mapping positions (x_d, y_d) in the detector plane to positions (x_m, y_m) in the mask plane, $T(x_m, y_m)$ is the transmission efficiency of the mask at position (x_m, y_m) , independent of the wavelength λ , and the spectral spread function $\Omega(\lambda)$ models the spatial shift due to the spectral dispersion of the prism.

The underlying assumptions of the propagation model M_c are the following:

- ◊ assumption H1: The spectral dispersion induced by the prism is the same for all positions (x_d, y_d) in the detector plane.
- ◊ assumption H2: Whatever the wavelength, the magnification G_{χ} between the detector plane and the mask plane is constant in the dispersion direction *x*.
- ◊ assumption H3: Whatever the wavelength, the magnification G_{y} between the detector plane and the mask plane is constant in the direction perpendicular to the dispersion y.

The assumption **H1** is contained in the function $\Omega(\lambda)$ that only depends on the considered wavelength λ and not on the position (x_d, y_d) . The assumptions H2 and H3 are implicitly contained in the $\times(-1)$ factors in eq. Eq. (2) that correspond to $G_{\mathcal{X}} = G_{\mathcal{Y}} = -1.$

B. Accurate propagation model

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Our experimental observations [16] and simulations with the Zemax software detailed in section A indicate that the range of validity of the previous assumptions is limited. A more general propagation model is needed, that captures the specificities of a real prism-based CASSI system, such as geometric distortions induced by the prism and alignment errors. We define here the improved backward propagation model M_{p} , by integrating the parametrization of elementary transformations induced by the optical components (prism and lenses) and their relative orientations (rotation matrices).

B.1. Components models

From the detector back to the mask plane, light undergoes three transformations corresponding to the three optical components: a position-to-angle conversion $l_{p,a}$ through the first lens, an angle-to-angle conversion $p_{a,a}$ through the prism, and an angleto-position conversion $l_{a,p}$ through the second lens.

Lens model *l_{p,a}* (image plane to Fourier plane)

We assume the lenses are perfect thin lenses and the propagation distances before and after the lens are perfectly equal to their focal length F. When propagating from an image plane to a Fourier plane, such a lens performs a perfect position-to-angle conversion depending only on its focal length F, as depicted in figure 2.



Fig. 2. Position-to-angle conversion $l_{p,a}$ induced by a perfect thin lens between object and Fourier planes.

158 This assumption is well verified in practice when using achromatic lenses and working with a small aperture system. 159 This position-to-angle relation is: 160

$$l_{p,a}:\begin{cases} \alpha = -\arctan\left(x/F\right)\\ \beta = -\arctan\left(y/F\right) \end{cases}$$
(3)

161 where *F* is the focal length, (x, y) the horizontal and vertical 162 incoming positions in the object plane and (α, β) the horizontal and vertical outgoing angles after the lens. 163

In terms of normalized wave-vector $\vec{v} = \vec{k} \times \frac{\lambda}{2\pi}$, after the 164 lens, the model $l_{p,a}$ gives: 165

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \frac{1}{\sqrt{1 + \tan^2(\alpha) + \tan^2(\beta)}} \begin{bmatrix} \tan(\alpha) \\ \tan(\beta) \\ 1 \end{bmatrix}$$
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- Lens model $l_{a,p}$ (Fourier plane to image plane) 166
- Conversely, when propagating from a Fourier plane to an im-167
- age plane, a perfect thin lens performs an angle-to-position con-168 version as depicted in figure 3.



Fig. 3. Angle-to-position conversion $l_{a,p}$ induced by a perfect thin lens between Fourier and image planes.

This position-to-angle relation is: 170

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$$l_{a,p}:\begin{cases} x = F \tan(\alpha) \\ y = F \tan(\beta) \end{cases}$$
(5) 196
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where *F* is the focal length, (α, β) the horizontal and vertical 171 incoming angles and (x, y) the horizontal and vertical positions 172 in the image plane after the lens. 173

Note that using these definitions of $l_{p,a}$ and $l_{a,p}$ ensures that 174 a 4*F* telescope, described by $l_{p,a} \circ l_{a,p}$ has the expected magnifi-175 cation ratio of $G_x = G_y = -1$. 176

Prism model $p_{a,a}$ (spectral deviation)

Prisms perform an angle-to-angle conversion that depends on the wavelength. We describe the incident wavevector k_i as:

$$\vec{k}_i = \frac{2\pi}{\lambda} \times \vec{v}_i$$

where the normalized vector \vec{v}_i can be written as: 177

$$\begin{bmatrix} v_{i,y} \\ v_{i,z} \end{bmatrix} = \frac{1}{\sqrt{1 + \tan^2(\alpha_i) + \tan^2(\beta_i)}} \begin{bmatrix} \tan(\beta_i) \\ 1 \end{bmatrix}$$
(6)

with α and β corresponding to incident angles on the input in-178 terface of the prism P expressed in the corresponding frame 179 $\{R_{P_i}\}$, as illustrated in figure 4. Note that α_i and α_o belong to 180 the *xOz* plane and β_i and β_o belong to the *yOz* plane of the re-181 spective frames.



Fig. 4. Propagation of a wavevector through a prism. k_i is an incident wavevector on the input interface of the prism in the $\{R_{P_i}\}$ frame; \vec{k}_o is the corresponding output vector in frame $\{R_{P_o}\}.$

The refraction law ensures that the component of the wavevector lying in the air/glass interface (i.e. component within the z = 0 plane in $\{R_{P_i}\}$) is conserved. The same applies to its normalized vector \vec{v}_i . Combining this conservation of inplane components at both input and output interfaces with the fact that the normalized wavevector norm is 1 out of the prism and $n(\lambda)$ inside the prism, one can express the components of the output vector \vec{v}_o in the $\{R_{P_o}\}$ frame as:

$$\begin{bmatrix} v_{ox} \\ v_{oy} \\ v_{oz} \end{bmatrix} = \begin{bmatrix} \cos(A)v_{ix} - \sin(A)\sqrt{[n(\lambda)]^2 - v_{ix}^2 - v_{iy}^2} \\ v_{iy} \\ \sqrt{\|\vec{v}_o\|^2 - v_{ox}^2 - v_{oy}^2} \end{bmatrix}$$
(7)

where A is the apex angle of the prism, $\|\vec{v}_o\| = 1$, the norm of the \vec{v}_o vector, λ the considered wavelength and $n(\lambda)$ the corresponding refractive index.

Output angles in the frame of the output face $\{R_{P_a}\}$ can be expressed with vector \vec{v}_o components as:

$$p_{a,a}: \begin{cases} \alpha_o = \arctan\left(v_{ox}/v_{oz}\right) \\ \beta_o = \arctan\left(v_{oy}/v_{oz}\right) \end{cases}$$
(8)

Combining equations Eq. (8), Eq. (7) and Eq. (6) defines the analytical model $p_{a,a}$, giving a closed-form expression of α_0 and β_o as a function of α_i and β_i . This expression is somewhat complex, it is not presented here explicitly (see the code in [13] for more details on the exact implementation). For a grating-based SD-CASSI, a similar approach can be derived to define an analytic model for wavelength dependent ray deviation [17].

B.2. M_p : combination of elementary models

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The model M_p describes the propagation from a position (x_d, y_d) at wavelength λ through two lenses and a prism as il-205 lustrated in figure 5. The optical layout is such that the optical axis of the lenses L_1 and L_2 corresponds to the minimum deflection of the prism for the central wavelength of the system. The latter is a design choice made with respect to the spectral range of the instrument. 210

The full model M_p is defined by the composition of the elementary transformations $l_{p,a}$, $p_{a,a}$ and $l_{a,p}$:

$$M_p: (x_d, y_d, \lambda) \to (x_m, y_m, \lambda)$$

$$p = l_{a,p} \circ R_{P_a \to L_1} \circ p_{a,a} \circ R_{L_2 \to P_i} \circ l_{p,a}$$
(9)



Fig. 5. The backward propagation model M_p is the combination of the elementary transformations $l_{p,a}$, $p_{a,a}$ and $l_{a,p}$ and the rotation operators $R_{L_2 \rightarrow P_i}$ and $R_{P_o \rightarrow L_1}$.

with $R_{L_2 \rightarrow P_i}$ and $R_{P_o \rightarrow L_1}$ the rotation operators associated re-256 213 spectively with the change of reference frame between $\{R_{L_2}\}$ 214 257 and $\{R_{P_i}\}$, and between $\{R_{P_o}\}$ and $\{R_{L_1}\}$ (see figure 5). 215 258



Fig. 6. Prism alignment parameters. The angle α_c is the angle of incidence corresponding to the minimum deflection of the prism at the central wavelength. The angles $\delta \alpha_c$ and $\delta \beta_c$ describe small errors of alignment respectively with respect to the minimum deflection and with respect to the verticality of the prism.

These two operators are parametrized according to the op-216 tomechanics on which the prism is fixed. To allow the align-217 ment of the optical system, it is generally necessary to have at 218 least two degrees of freedom on this component. A rotation 219 around the y-axis, illustrated in figure 6, allows the positioning 220 around the minimum of deflection, and a rotation around the 221 x-axis guarantees the verticality of the prism. To account for 222 alignment errors, we have parametrized these two rotations by 223 $\delta \alpha_c$ and $\delta \beta_c$, which represent respectively the error from the in-224 cidence α_c corresponding to the minimum of deviation and the 225 268 error from the vertical (see figure 6). To summarize, our model 226 269 parameters are the focal length F, the apex angle A, the angle of $_{270}$ 227 incidence on the prism α_c ensuring minimum deflection at $\lambda_{c, 271}$ 228 and the two angles $\delta \alpha_c$ and $\delta \beta_c$ that define the alignment errors 272 229 of the prism. 230 273

C. Discussions on dispersive elements 231

In the literature, there has been several discussions on which 232 233 optical component to use for spectral dispersion.

276 One commonly employed element is the grating, in trans-234 277 mission or reflection, renowned for its linear dispersion capa-235 bilities. However, its utility is somewhat constrained by a rel-236 atively narrow spectral range, beyond which its efficiency no-237 tably diminishes. Moreover, going beyond one octave requires 278 238 special care to avoid diffraction orders overlapping. 239 279

Conversely, prisms offer a broader spectral range with high 280 240 efficiency, but at the expense of introducing distortions. These 281 241 include smile and stretching effects, alongside inherent non- 282 242

linear dispersion, which can complicate the interpretation of raw spectral images.

An alternative is the Amici prism [18], a composite of three prisms arranged to combine the benefits of both grating and prism. It achieves linear dispersion over an extensive spectral range, akin to gratings, while retaining the wide-range applicability of prisms [19]. Despite its merits, the Amici prism is not without drawbacks, as it still manifests smile and stretching distortions.

In this article, we chose to focus on the prism as it induces various optical distortions that need to be addressed, and is also the basic building block of the Amici prisms. Note that a grating model is also included in our associated simulator [13].

3. NUMERICAL VALIDATION

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To quantify the advantages of the model M_p over the classical model M_c , both were compared to simulations performed by the optical design software Zemax. Zemax was chosen as the reference for the evaluation because it is widely used in the optical systems industry.

The simulated system is an SD-CASSI as depicted in figure 5. We first assume that there is no misalignment ($\delta \alpha_c = \delta \beta_c = 0$) and simulate in Zemax the propagation of a grid of 19×19 points, forming a field of view of $8 \times 8 \text{ mm}^2$, from the detector plane to the mask plane (see figure 7). The spectral range



Fig. 7. Typical comparison: a regular grid of 19×19 points in the detector plane (top) is propagated to the mask plane (bottom) with both the M_c model (black dots) and Zemax (green dots).

evaluated is the visible and near-infrared [450 - 780] nm. The lenses are perfect, with focal length F = 165 mm, and the prism is equilateral ($A = 60^{\circ}$) in BK7 (these parameters are those of the prototype presented in section A, except the field of view which is larger, so as to better exhibit the differences between the models). The central wavelength is fixed at $\lambda_c = 532 \,\text{nm}$, and the associated minimum deviation D_m is calculated by direct application of the Snell-Descartes law [20]:

$$D_m = 2 \arcsin(n(\lambda_c)\sin(A/2)) - A$$
 (10)

where $n(\lambda_c)$ is the refractive index of the prism at the central wavelength and A the apex angle of the prism. We finally get the angle of incidence on the prism α_c :

$$\alpha_c = \frac{A + D_m}{2} \tag{11}$$

In our case, $\alpha_c = 49.44^\circ$. Since the misalignments cannot be taken into account by the classical model we first assume that they are non-existent. In the last validation step, we perform a comparison between our model and the Zemax simulations with increasing misalignments.

Research Article

²⁸³ A. Evaluation of the classical model M_c

325 Preliminary methodological note: The M_c model assumes iden-284 tical spectral dispersion for all positions in the field of view 326 285 (assumption H1, section A). The Ω function related to the dis-327 286 persion thus depends only on the wavelength λ . Different wavs ³²⁸ 287 of evaluating this dispersion have been proposed in the litera-288 ture [7, 11, 14]. To avoid any bias due to the use of an inade-280 quate model for Ω , we extrapolate this function from the Zemax 290 reference simulation with a polynomial of degree 10. We obtain 29 292 a dispersion function for the point at the centre of the detector $(x_d, y_d) = (0, 0)$. This function faithfully reproduces the spatial 293 shifts observed at the mask for the different wavelengths. 294



Fig. 8. Comparison between Zemax and M_c propagation models for the central wavelength $\lambda = 532$ nm: (a) points in the mask plane (x_m, y_m) predicted by M_c (black) and by Zemax (green); (b) zoom on the stretching distortion along the horizontal axis x; (c) zoom on the smile distortion along the vertical axis y; (d) geometric distortions mapped on detector points: Euclidean distance D between points predicted by Zemax and M_c .

Figure 8 (a) shows the points propagated by Zemax (black 345 dots) and by the M_c model (green dots) for the wavelength $\lambda = 346$ 532 nm. The M_c model is accurate around the centre of the field 347 of view but the error increases with the distance from the centre. 348 One can see different geometric distortions along the *x* and *y* 349 axes.

Let us observe more precisely what happens along the *x* axis 301 with the zoom in figure 8 (b): all the black points are to the right 302 of the corresponding green points. This is due to the non-linear 303 deviation of the prism around its minimum of deviation, which 304 induces a geometric distortion that we call stretching distortion 305 as the horizontal magnification G_x changes with the position. 306 Since the classical model M_c assumes a constant magnification 307 G_{x} across the field of view (assumption H2), it does not prop-308 erly account for this stretching distortion. 309

Along the vertical axis y, after propagation, the vertical lines are bowed, as illustrated in figure 8 (c). This is the *smile distortion*, well known in spectroscopy. It couples the horizontal position x_m in the mask plane to the vertical position y_d in the detector plane. As the classical model M_c treats both directions independently (see eq. Eq. (2)), it cannot account for smile distortion.

For a more quantitative analysis of the differences between Zemax and M_c , we present in figure 8 (d) the distortion map, i.e. the Euclidean distance D between the corresponding black and green points. Note that distortion values D are plotted as a function of the originating position in the detector plane (x_d, y_d) to allow easier comparison between considered wavelengths. We can see that this distance D grows with the radial distance and the set of the se

to the centre of the field of view, reaching up to $190 \,\mu\text{m}$ in the corners. For comparison, the typical size of a mirror of a digital micro-mirror device (DMD) typically used as a mask is of the order of $5 \,\mu\text{m}$. This distance thus corresponds to a maximum error of around 38 micro-mirrors on a typical DMD.

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Fig. 9. Distance maps comparing the classical M_c model to Zemax for (left) upper, (centre) central and (right) lower wavelengths.

Let us now observe the wavelength dependence of these geometric distortions. As a reminder, the prism is positioned so that the centre of the field of view is at the minimum of deviation for $\lambda_c = 532$ nm. The distortion maps presented in figure 9 correspond to this configuration for three wavelengths: 450, 532 and 780 nm. One can see that they are off-centred and reach higher values for wavelengths lower (450 nm) and higher (780 nm) than the central wavelength. Indeed, for both these wavelengths, we are no longer at the minimum of deviation where the geometric distortions are minimal.

This can also be observed in figure 10, in which histograms of the distances *D* for the 19 × 19 points of the field of view are plotted for three wavelengths: 450 nm, 532 nm and 780 nm. For each of these wavelengths, the dotted lines indicate the 95th percentile of *D*. By definition, 95% of the values of *D* lie below the 95th percentile, which is therefore a relevant scalar metric to assess the differences between the two models. We observe a higher 95th percentile of *D* for $\lambda = 450$ and 780 nm (respectively 136 and 133 µm) than for the central wavelength $\lambda_c = 532$ (116 µm). The classical model tends here to produce significant positioning errors, of the order of 25 micro-mirrors for a typical DMD.



Fig. 10. Histogram of Euclidean distance *D* between points propagated into the mask plane with Zemax and the with classical model M_c for lower, central, and upper wavelengths. For each wavelength, the dashed line corresponds to the distance of the 95th percentile (i.e. 95% of the distances are below this limit).

B. Evaluation of the proposed model M_p

 M_p gives the position (x_m, y_m) in the mask plane as a function of the position in the detector plane (x_d, y_d) and the considered

wavelength λ . Contrary to the classical model, the spectral dispersion induced by the prism is not the same for all positions in the field of view (no **assumption H1**). Furthermore, we make no assumptions about the magnification in the *x* and *y* directions (no **assumptions H2** and **H3**).



Fig. 11. Distance maps comparing the proposed M_p model to Zemax for (left) upper, (centre) central and (right) lower wavelengths.

Figure 11 presents the distortion maps obtained with the M_p 359 model for three wavelengths, with the same metric as in sec-360 tion A (Euclidean distance D between image points from the 361 M_{v} model and those from the Zemax simulation). Comparing 362 the distortion maps in figures 9 and 11, one can observe a size-363 able difference in the scale of the distortion in the field of view: 364 the maximum D distance decreases from $190 \,\mu\text{m}$ to $0.015 \,\mu\text{m}$, 365 i.e. a reduction of about 4 orders of magnitude. Figure 11 also 366 shows that the accuracy of the model does not depend on the 367 wavelength. 368

The same trend can be observed by comparing the histograms of *D* presented in figure 12 and the values of the 95th percentile of *D*. These values are more than two orders of magnitude below the typical size of a micro-mirror (5 μ m) and therefore correspond to negligible positioning errors.



Fig. 12. Histogram of Euclidean distance *D* between Zemax and the proposed model M_p in the mask plane for lower, central, and upper wavelengths. For each wavelength, the dashed line corresponds to the distance of the 95th percentile (i.e. 95% of the distances are below this limit).

The M_{ν} model also accounts for prism misalignments (non-401 374 zero values of $\delta \alpha_c$ and $\delta \beta_c$, see figure 6). Since they cannot be 402 375 modelled by the classical M_c model, we can only compare to 403 376 the Zemax simulations. Figure 13 represents the histograms of 404 377 *D* for a series of misalignment values, starting with typical val-378 405 ues compatible with an expected experimental alignment, and 406 379 multiplying them by a factor of 3 and 10. The model M_p esti-407 380 mates the distortions with an accuracy similar to that obtained 408 381 without misalignments (compare to figure 12), even for large 409 382 misalignments: the slight variations observed between the val- 410 383 ues of the 95th percentiles for the various misalignment values 411 384 385 are not significant. 412



Fig. 13. Histograms of Euclidean distances D between the Zemax simulations and the M_p model, with increasing misalignment, for three wavelengths. In dotted line, the 95th percentile of D.

4. EXPERIMENTAL VALIDATION

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399 400 We present in this section a validation of the proposed model M_p with a prototype SD-CASSI system. The model parameters are first determined with a calibration process, and the spectral sampling defined by the model is validated by observing a known spectrum.

A. Geometric spatio-spectral calibration of an SD-CASSI system

One of the benefits of a parametric model of the image formation is that it allows a quick calibration, using only a few images of a known geometric pattern and an optimisation technique. This is more straightforward than building an experimental LUT for each individual pixel, which is reckoned as a laborious process in the literature [11, 12, 21], even though efficient approaches have been proposed [22]. Also, while building experimental LUTs is tractable for low spectral and/or spatial resolution, the process becomes more difficult and time consuming for high resolution imagers.

The proposed model-based calibration approach pertains to the *geometry* of an SD-CASSI system, as expressed by the model M_p : we do not consider purely photometric phenomena (quantum efficiency and vignetting).

The principle of the calibration is to fit the M_p model parameters to real measurements with a non-linear least-square optimisation. For this purpose, we rely on the association of features of known positions (x_m , y_m) in the scene to pixel coordinates (x_d , y_d) in the detector, for a series of wavelengths

 λ . We use the DMD as the scene, displaying a chequerboard 437 413 pattern, which defines easily detectable corners on the image 438 414 formed in the detector plane (figure 14). The corners are 439 415 detected with the OpenCV image processing library. 416

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Fig. 14. Calibration pattern on the DMD at the scene (a), and extracted corners in the detector plane (b).

Our SD-CASSI prototype is composed of two lenses, two 418 folding mirrors, one prism, a matrix of micro-mirrors acting as 419 a programmable mask, and a CMOS camera (figure 15). The



Fig. 15. 3D model of the prototype.

420 folding mirrors have no contribution to the propagation model, 421 they just ensure easier alignment and reduced volume of the 422 optical system. The input scene in the object plane of the sys-423 458 tem is $3.5 \times 3.5 \text{ mm}^2$ wide, and spectrally ranges from 450 to 424 750 nm. The components of the prototype are described table 1. 425 460

<u> </u>	T 1.1 () ('
Component	Type and characteristics
Lenses (L_1, L_2)	Thorlabs TTL165-A,
	achromatic over $[450 - 780]$ nm, F = 165 mm
Prism (P_1)	BK7 material,
	equilateral,
	apex angle $A = 60^{\circ}$
DMD	Texas Instruments DLP3010,
	720×1280 square 5.4 µm micro-mirrors,
	defining a $3.9 \times 6.9 \text{ mm}^2 \text{ mask}$
Detector	The Imaging Source,
	33UP2000,
	1920×1200 4.8 μ m pixels

Table 1. Characteristics of the prototype components

We experimentally estimated the system point spread func-426 tion (PSF): it slightly changes with the position in the field of 427 view and wavelength, and its diameter is bounded by 15 µm. 428

The calibration chequerboard pattern is made up by 17×17 429 square regions of 30×30 mirrors, which defines 289 corners. 430 Three lasers are used to illuminate the chequerboard pattern, 43 with respective wavelengths 450, 532 and 780 nm. In total, 432 $3 \times 289 = 867 (x_d, y_d, x_m, y_m, \lambda)$ quintuplets are provided to 433 the non-linear least-square regression process to estimate the 434 parameters that define the M_p model. As a reminder, these pa-435 rameters are the focal length F, the apex angle A, the angle of 436

incidence on the prism α_c ensuring minimum deflection at λ_c , and the two angles $\delta \alpha_c$ and $\delta \beta_c$ that define the alignment errors of the prism.

The estimated parameters are given in Table 2 with the associated 95% confidence interval obtained with a jackknife resampling. According to the components' specifications, the focal lengths F and apex angle A may respectively vary by 1% and 0.1 % around their specified values of 165 mm and 60°: the estimated values are within these bounds. The minimum of deviation is also extremely close to the expected value. As for $\delta \alpha_c$ and $\delta\beta_c$, the estimated values are coherent with the precision of the experimental alignment. In particular, as verticality of the prism is easier to assess than its orientation with respect to the minimum of deviation, it is not surprising that $|\delta\beta_c|$ is smaller than $|\delta \alpha_c|$.

	F [µm]	<i>A</i> [°]	$\alpha_c [^\circ]$	$\delta \alpha_c [^\circ]$	δβ _c [°]
Value	165 449	60.0005	49.4416	0.5778	-0.1407
C.I.	26	0.0098	0.0134	0.312	0.092

Table 2. Calibration results: value estimated for the parameters and associated 95 % confidence interval (C.I.).

An alternate way to assess the quality of the calibration is to calculate the reprojection error, *i.e.* the Euclidean distance D between the extracted corner positions and the ones predicted using the model with the estimated parameters. The values of D plotted as a function of the original position in the image plane are reported in figure 16, and the associated histograms are plotted in figure 17, with the distance of the 95th percentile for the three reference wavelengths. For 95% of the points in the field of view, the reprojection errors do not exceed 6.9 µm for the upper wavelength, 10.7 µm for the central wavelength and 8.4 µm for the lower wavelength.

These errors are significantly larger than the ones obtained with the numerical projections presented in section 3. This comes from various experimental causes. In particular, the lasers used for the calibration have spectral FWHM of the order of 0.5 nm, are not thermally regulated and exhibit a temperature drift of the order of 0.2 nm/° C, resulting in an uncertainty on the exact position on the detector of more than one pixel $(\gtrsim 4.8 \,\mu\text{m})$ Additionally, the detector $4.8 \,\mu\text{m}$ sampling step and the experimentally measured PSF of 15 µm diameter affects the precision of the corners detection. Nevertheless, these errors are sub-PSF and correspond to a rather accurate spatio-spectral calibration of the SD-CASSI.



Fig. 16. Experimental reprojection error D for the 3 reference wavelengths.



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Fig. 17. Histogram of the reprojection error *D* for the 3 reference wavelengths. For each wavelength, the color dashed line corresponds to the distance of the $95^{\ensuremath{\check{t}h}}$ percentile.



Fig. 18. The spectral sampling at each pixel in the detector plane depends on the back-propagation to the mask micromirror centres.

B. Estimation of the spectral sampling 475

To evaluate the spectral sampling of an SD-CASSI or a DD-476 CASSI system, one usually resort to a an experimental LUT-477 based calibration [5, 11, 12, 23, 24]. The proposed method relies 478 on the 5-parameter model estimated in section A to calculate 479 the spectral sampling specific to each pixel, without any addi-480 tional measurement. 481

482 As illustrated in figure 18, for any given image pixel (x_d, y_d) , with the analytical model M_p one can back propagate light 483 into the mask plane to determine which micro-mirrors can send 484 light at that pixel position. In the mask plane, the light from the 485 pixel would form an horizontal rainbow line, spread over sev-486 eral adjacent micro-mirrors. The wavelengths corresponding 487 to the centre of each of these micro-mirrors define the spectral 488 sampling that the DMD is imposing on the considered pixel. 489 Due to the non-linear dispersion, the smile distortion and the 490 stretch distortion, this sampling differs for each pixel in the de-491 tector plane. We thus call it the local spectral sampling. The entire 492 process to determine the local spectral sampling can be summa-493 rized in three steps: 494

- 536 1. from the position (x_d, y_d) of the pixel in the detector plane, 495 537 one determines the angles (α_o, β_o) on the back side of the 496 538 prism using the lens model $l_{p,a}$; 497 539
- 540 2. from the position (x_m, y_m) of a given micro-mirror in the 498 541 mask plane, one determines the angles (α_i, β_i) on the front 499 542 side of the prism using the lens model $l_{a,p}$; 500 543
- 3. using the prism model $p_{a,a}$, one determines whether there 544 501 is a wavelength λ such that (α_i, β_i) corresponds to (α_o, β_o) 545 502 thanks to the deflection induced by the prism. This corre-546 503 sponds to finding the value of $n(\lambda)$ that solves the equation 547 504 7. 505 548

If such a wavelength exists, it is part of the local spectral sampling of the considered pixel. If it doesn't, it means this particular micro-mirror does not influence the light impinging 508 on the pixel. The local spectral sampling for this pixel is fully defined when all the micro-mirrors have been tested. Once this 510 procedure has been applied to all the pixels of the detector, we have the local spectral sampling for the whole detector, together with the link between each micro-mirror and the spectral samples it controls over various pixels in the detector.

Of course, in practice, we do not test every single micromirror and every possible wavelength in the spectral range of 516 interest for each pixel of the detector, as it is prohibitively time-517 consuming. Some considerations can be used to significantly reduce the micro-mirrors to test for any given pixel. In particular, as the device is mostly line-independent, for each pixel, we can limit the test to a single line of micro-mirrors. Moreover, for each pixel, it is easy and fast to back propagate the two extreme wavelengths of the spectral range of interest, thus bounding the lateral positions of the micro-mirrors to test on the DMD. Note that once the 5 physical parameters of the model are known, all these pixel-dependant spectral samplings can be pre-computed for spectral reconstruction or any other interpretation or processing of the encoded images.

C. Experimental validation of the local spectral sampling

The spectral calibration obtained using the local spectral sampling is tested using a fluorescent lamp that illuminates the object and mask planes homogeneously. The DMD mask pattern consists of a single vertical slit, turning the SD-CASSI into a prism-based spectrometer.



Fig. 19. Spectral calibration validation: (a) detector image for a single slit DMD mask and a uniform scene lighted by a fluorescent lamp; (b) fluorescent lamp spectrum measured with a reference spectrometer (grey line) and with the SD-CASSI using line $y_d = 600$ of the detector (orange line). Two spectral peaks at 546 nm and 611.5 nm are used for comparison.

The image formed on the detector (see figure 19(a)) shows the different spectral lines and bands of the fluorescent lamp. Note that the dispersion is such that wavelength λ decreases with increasing x_d position on the detector (longer wavelengths are imaged on the left). Using our local spectral sampling, the intensity along any horizontal line can be calibrated in wavelength to retrieve the spectrum. Figure 19 (b) shows the spectrum obtained from the line $y_d = 600$ of the detector (orange line), together with an independent reference measurement of the spectrum of the scene (black line, obtained using a UV-VIS Ocean Optics Flame spectrometer).

We can see from this comparison that the spectral local sampling is accurate enough to precisely locate the features of the spectrum (like the peaks at 546 and 611.5 nm for ex-

ample). However, we can see significant intensity dissimilar- 593 549 ities between the retrieved and reference spectra: this is due to 594 550 the absence of intensity (or quantum efficiency) calibration on 595 551 our prototype, whereas the quantum efficiency is properly ac- 596 552 553 counted for in the signal from the reference spectrometer. More- 597 554 over, as it consists of a single line of pixels, the retrieved spec- 598 555 trum is significantly noisier than the reference spectrum ob-599 tained by collecting the light for the whole scene. 556 600

In order to ascertain the validity of the local spectral sam- 601 557 pling, we retrieved the calibrated spectrum using several lines 602 558 across the field of view. Figure 20 (a) shows typical spectra re- 603 559 trieved for $y_d = [310, 600, 890]$. As these spectra are almost per-560 fectly overlapped, we shifted them vertically for better read-605 56 ability of the figure. 562 606

A close up on the main peak at 611.5 nm of the spectrum is 563 shown in figure 20 (b): it reveals that although the measured 564 spectra are perfectly consistent and correspond to the same 565 spectral shape (give or take the acquisition noise), they are 566 based on different spectral samples. 56

568 In this simple example of a single slit DMD pattern, recom-569 bining accurately the different lines of the detector in one single spectrum with higher signal to noise ratio requires to take this 570 change of the spectral sampling into account. 571



Fig. 20. Spectral calibration validation across the field of view: (a) spectra measured from three different lines of the detector $(y_d = [310, 600, 890])$ give consistent results; (b) close inspection of the spectra reveal the local spectral sampling that changes across the detector.

5. SPECTRAL SUPER-RESOLUTION 572

In most papers [5, 14, 15] authors assume that the data 573 610 collected by CASSI systems can be interpreted with a uni-574 611 form spectral sampling, thus neglecting the change observed 612 575 in figure 20(b). This assumption can simplify hyperspec-613 576 tral cube reconstruction, as uniform spectral sampling implies 577 614 that acquired raw data conveniently consists of well-defined 578 615 monochromatic planes or images. But it leads to various issues, 616 579 580 like moiré patterns, that are particularly detrimental for high 617 resolution CASSI imagers [7]. There has been some discussion 618 58 regarding these issues [7-10, 12], but even the most effective 619 582 methods, i.e correction of the filtering function *H* [16], result in 583 620 a loss of spectral resolution by a factor of 2. Conversely, model-584 621 based calibration gives an easy access to the spectral local sam-585 622 pling, with no resolution losses and can be harnessed to achieve 586 623 super-resolution. 58

Indeed, considering local spectral sampling gives the abil-625 588 ity to achieve spectral super-resolution when several adjacent 626 589 pixels share the same spectrum. Such groups of pixels, dubbed 627 590 spectrally homogeneous spatial regions are fairly common and 628 59 can be harnessed to speed-up hyperspectral imaging [7, 25]. To 629 592

demonstrate spectral super-resolution we compare the hyperspectral measurements achieved with our SD-CASSI prototype with either a uniform or local spectral sampling, on a spectrally homogeneous scene.

The acquisition scheme is a simple moving-slit sequence: we successively open the columns of micro-mirrors on the DMD mask and record the corresponding image on the detector. The mask plane is illuminated with a large single-mode He-Ne laser beam ensuring an homogeneous spectrum across many pixels. The raw data-cube is then interpreted to retrieve the spectrum at every point in the mask plane.

The figure 21 shows the spectral sampling ((a) and (c)) and measured spectral intensities ((b) and (d)) across 7 neighbour pixels (yellow to blue color code) when using either the uniform ((a) and (b)) or local ((c) and (d)) spectral sampling.

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Fig. 21. Spectral super resolution of a He-Ne laser spectrum: the uniform spectral sampling ((a) & (b)) implies that adjacent pixels provide several intensity values for a single wavelength whereas the proposed approach ((c) & (d)) properly describes the minute changes in sampling and adjacent pixels provide spectral super-resolution.

With a uniform spectral sampling (see figure 21 (a)), each of the 7 pixels provides an independent measurement over the same spectral samples (see the dotted red line in figure 21 (a)). We thus obtain one spectrum per pixel (see for example the dark blue and yellow spectra in figure 21 (b), measured 6 pixels apart). We can observe a large intensity variation across these spectra (dotted red line in figure 21(b)) and a large, regular, spectral sampling of about 0.65 nm. Similar intensity variations, forming moiré patterns, have been observed in DD-CASSI systems and require cumbersome mitigation post-processing that degrade significantly either spatial of spectral resolution of the reconstructed hyperspectral data [16], hindering the development of high resolution DD-CASSI.

Conversely, with a pixel-specific spectral sampling, the same raw data can be interpreted in a more precise and faithful way. The spectral sampling is slightly different for each pixel (see the slanted dotted red line in figure 21 (c)). We thus have access to interleaved spectral samples (see the dotted red curve in figure 21 (d)) and we can reconstruct a super-resolved spectrum for the whole region, with a sampling of about 0.09 nm. This super-resolved spectrum is irregularly but finely sampled, and measured accurately in intensity.

It is noteworthy that harnessing spectral sampling effects 630 691 has been previously explored for a color-coded aperture spec-631 tral imager [26], exploiting the difference of spectral sampling 632 692 between the response of the color filters and the prism disper-633 sion. Interestingly, these are two different but complementary 634 635 ways of using sampling effects: our method relies on the op-636 tical system distortions and discretization of mask and detector, while [26] harnesses the difference of spectral sampling in-637 duced by spectral filters and prism dispersion. Both approaches 638 698 could in principle be combined. 639

6. CONCLUSION 640

The proposed analytical model for back-propagation in prism 641 703 SD-CASSI is based on ray-tracing and simple rotation matri-642 704 ces, and relies on a few physical parameters directly related to 705 643 644 the optical components properties and their alignment. It mod-706 707 645 els accurately systems under design, and allows fast and easy 708 calibration of existing prototypes. This model is part of a DD-646 CASSI simulator, available online as Free Open Source Software 647 (FOSS)[13]. 648

711 This model offers positioning accuracy of the propagated 649 712 principal rays comparable to that of an industry standard like 650 713 Zemax. However, contrary to Zemax, it is perfectly fit to be 651 714 integrated into larger software ecosystems, e.g. for optimiza-652 715 tion, data processing or co-design. This enables the complete 653 study of co-designed prototypes and companion algorithms in 717 654 718 one unified framework. Such studies could provide meaning-655 719 ful insights on the interplay between opto-geometrical and al-656 720 657 gorithmic trade-offs and their impact on the overall efficiency 658 of a particular hyperspectral measurement strategy.

Using this model to calibrate an SD-CASSI prototype, we 659 accurately describe minute changes in the spectral sampling 660 724 across the field of view, without resorting to the building of 725 661 experimental LUTs. These changes are inducing spectral mea-662 726 surement artefacts, particularly detrimental in high-resolution 727 663 CASSI systems. Access to the local spectral sampling allows 728 664 to eliminate the spectral artefacts and even to exploit them to 729 665 achieve super-resolved measurements. 666

731 667 As presented here, this model is limited to prism-based 732 single disperser (SD) CASSI systems. However, it could be 668 trivially extended to grating-based SD-CASSI by replacing the 669 734 prism wavelength-dependent angular dispersion by that of 670 735 transmission or reflection gratings. Moreover, this model is also 671 736 extremely valuable for dual-disperser (DD) CASSI systems that 737 672 either rely on two chained SD-CASSI (two arms) or on a double-738 673 pass in one SD-CASSI (one arm in double-pass). A key require- 739 674 ment with DD-CASSI is the perfect symmetry between each 740 675 pass or arm: exploiting the model to simulate or calibrate in-741 676 742 dependently half of the DD-CASSI system would provide hints 677 743 on the impact of misalignment and dissymmetry on the achiev-678 744 able performance. 679 745

In addition, this model could be complemented by a an esti-746 mate of the spectral quantum efficiency over the field of view, as 68 747 well as precise estimate of the spectrally-dependant PSF, as pro-682 748 posed in [6]. Moreover, it could also account for the diffraction 683 749 induced by the DMD, as was done in [27]. Finally, in order to 750 684 take into account defocussing at the different relay and imaging 751 685 planes together with non-paraxial lens aberrations, the perfect 752 686 753 lens model could be replaced by a proxy model of some specific 687 real lens or lens assembly, as proposed in [28]. 688

Data availability. Data underlying the results presented in this pa-689 756 per are not publicly available at this time but may be obtained from the 757 690

authors upon reasonable request.

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