Exploring robust structured static output feedback design

Dimitri Peaucelle - LAAS-CNRS - Université de Toulouse
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Abstract

Many design problems in control can be recast as the search for static structured (diagonal) output-feedback gains stabilizing an augmented and rearranged dynamical plant model. Moreover, in first approximation that plant may be considered as linear, or at least, one will usually request the linearized plant to be stable before considering the more complicated non-linear version. Because of the approximations leading to the linearized version, the plant parameters are usually uncertain and time-varying. In the presentation we discuss the possibility to design such static diagonal output-feedback gains for uncertain linear systems using a recently proposed matrix inequality based formulation. As expected for this hard problem, the methodology does not provide a guaranteed to succeed result, but provides some interesting promising paths for an efficient algorithm. If we have time we also mention an adaptive control strategy for updating (learning) the structured static gains.

Structured Robust Static Output Feedback

Stabilize $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$ with a static output feedback $u = Ky$

Find a gain $K \in \mathbb{R}^{m \times p}$ such that $\dot{x} = (A + BKC)x$ is stable

- ➡ Without having an initial guess

  $\neq$ Knowing $A + BK_oC$ stable, find a better gain (with some criterion)
- ➡ Without having indications of a range of admissible values

  no possibility to test on a grid
- ➡ Robust w.r.t. uncertainties in A_δ, B_δ and C_δ

  eg. matrices in a polytope
- ➡ Structured : eg. K diagonal (decentralized control)

"From Static Output Feedback to Structured Robust Static Output Feedback: A Survey", M.S. Sadabadi, D. Peaucelle Annual Reviews in Control Volume 42, 2016

Structured Static Output Feedback - a generic problem

Assume a linear plant $\dot{x} = A(K_1, \ldots, K_{\bar{k}})x$ rational in the design parameters $K_{k=1\ldots\bar{k}}$

- ➡ Linear plant : before considering nonlinear dynamics, or local properties of a NL control
- ➡ K_k : gains of a dynamic control, filter param., decentralized control ... or plant parameters
- ➡ Rational in the parameters : or in 'gains' with one-to-one NL maps to true design parameters

It can always be reformulated (Linear Fractional Transformation) as a feedback control loop

$$\left\{ \begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx + Du \end{array} \right. \qquad u = Ky = \begin{bmatrix} I_{r_1} \otimes K_1 & & 0 \\ 0 & \ddots & 0 \\ 0 & & I_{r_{\bar{k}}} \otimes K_{\bar{k}} \end{bmatrix} y$$

where K is structured (block-diagonal), parameters may be repeated: $I_{r_k} \otimes K_k$.

- ➡ with $y := y - Du$ one may consider $D = 0$
- ➡ All these design problems look 'simple': find a (structured) K s.t. $A + BKC$ is Hurwitz

Optimization based approaches

Find a (structured) K s.t. $A+BKC$ is Hurwitz $\Leftarrow$ minimization of a non-linear, non-smooth function

➡ [Apkarian, Noll 2003] optimize an H_∞ gain (and more) - hinfstruct - Clark's sub-gradient
➡ [Overton et al. 2006] optimize an H_∞, H_2 gains, spectral abscissa - Hifoo - gradient sampling
➡ [Peretz 2013] optimize the spectral abscissa - randomized approximation

▲ ▲ Very efficient in practice
▼ Randomized flavour (random initial conditions, randomization in the algorithm)
▼ No extensions to robust design

▲ Lyapunov based approches with matrix inequalities may handle robustness

$$P \succ 0 \quad , \quad (A+BKC)^* P + P(A+BKC) = \{P(A+BKC)\}^{\mathcal{H}} \prec 0$$

$$Q \succ 0 \quad , \quad \{(A+BKC)Q\}^{\mathcal{H}} \prec 0$$

▼ Bilinear matrix inequalities (not convex)

LMI cases

Find a (structured) K s.t. $A+BKC$ is Hurwitz is sometimes convex (up to a transformation)

SI ➡ $B=C=I$ State-Injection : $\{PA\}^{\mathcal{H}} \prec Z$ gives K s.t. $\{PK\}^{\mathcal{H}} = Z$
OI ➡ $B=I$ Output-Injection : $\{PA+LC\}^{\mathcal{H}} \prec 0$ gives $K=P^{-1}L$
SF ➡ $C=I$ State-Feedback : $\{AQ+BF\}^{\mathcal{H}} \prec 0$ gives $K=FQ^{-1}$
➡ Almost commutative on the input : $\{PA+BLC\}^{\mathcal{H}} \prec 0,\ PB=B\hat{P}$ gives $K=\hat{P}^{-1}L$
➡ Almost commutative on the output : $\{AQ+BFC\}^{\mathcal{H}} \prec 0,\ CQ=\hat{Q}C$ gives $K=F\hat{Q}^{-1}$

▼ Applies only to special cases
▲ Robustness can be considered
(eg. test on all vertices with common decision variables proves stability of the polytope)
▼ Structured K cannot be considered
unless one considers structured P and Q (very conservative)

Iterative LMI method

Simple P-K-iterative algorithm

Initialization	Choose a positive definite P
K-iteration	For fixed P find $K = \arg\min \alpha$ under $\{P(A+BKC)\}^{\mathcal{H}} \prec \alpha I$
P-iteration	For fixed K find $P = \arg\min \alpha$ under $\{P(A+BKC)\}^{\mathcal{H}} \prec \alpha I$
Stop	Repeat until $\alpha < 0$ (success) or varies too slowly (failure)

- ▲ Easy to implement
- ▲ Strictly decreasing sequence of α
- ▼ Very sensitive to initialization
- ▼ Little progress after very few steps
- ▼ Not effective in practice

Finsler (elimination) lemma

$$\exists S \ : \ M \prec \{X_1^* S X_2\}^{\mathcal{H}} \quad \Leftrightarrow \quad \left\{ \begin{array}{l} N_{X_1}^* M N_{X_1} \prec 0 \\ N_{X_2}^* M N_{X_2} \prec 0 \end{array} \right. \ : \ X N_X = 0 \ , \quad \mathrm{Rank}(N_X) = \dim(\mathrm{Ker}(X))$$

Applied to the SOF problem [Scherer, Iwasaki...] it gives

$$\exists K \ : \ \left\{ \begin{array}{l} \{P(A+BKC)\}^{\mathcal{H}} \prec 0 \\ \{(A+BKC)Q\}^{\mathcal{H}} \prec 0 \\ PQ = I \end{array} \right. \quad \Leftrightarrow \quad \left\{ \begin{array}{l} N_C^* \{PA\}^{\mathcal{H}} N_C \prec 0 \\ N_{B^*}^* \{AQ\}^{\mathcal{H}} N_{B^*} \prec 0 \\ PQ = I \end{array} \right.$$

- ▲ May be converted to pure LMIs (of small dimensions) ... ▼ with a rank constraint
- ▲ Many dedicated iterative algorithms
- ▼ Sensitive to initial guesses, not very effective in practice
- ▼ Cannot take into account structured K

Converse Finsler lemma : S-variable approach

$$\exists S \ : \ M \prec \{X_1^* S X_2\}^{\mathcal{H}} \quad \Leftrightarrow \quad \left\{ \begin{array}{l} N_{X_1}^* M N_{X_1} \prec 0 \\ N_{X_2}^* M N_{X_2} \prec 0 \end{array} \right. \quad : \quad X N_X = 0 \ \ \mathrm{Rank}(N_X) = \dim(\mathrm{Ker}(X))$$

Assume P proves stability for both an output-feedback gain K_{of} and a state-feedback gain K_{sf}
(always true with $K_{sf} = K_{of} C$)

$$\{P(A + BK_{of}C)\}^{\mathcal{H}} = \begin{bmatrix} I \\ K_{of}C \end{bmatrix}^* \begin{bmatrix} \{PA\}^{\mathcal{H}} & PB \\ B^*P & 0 \end{bmatrix} \begin{bmatrix} I \\ K_{of}C \end{bmatrix} \prec 0$$
$$\{P(A + BK_{sf})\}^{\mathcal{H}} = \begin{bmatrix} I \\ K_{sf} \end{bmatrix}^* \begin{bmatrix} \{PA\}^{\mathcal{H}} & PB \\ B^*P & 0 \end{bmatrix} \begin{bmatrix} I \\ K_{sf} \end{bmatrix} \prec 0$$

Equivalent to the existence of S such that

$$\begin{bmatrix} \{PA\}^{\mathcal{H}} & PB \\ B^*P & 0 \end{bmatrix} \prec \left\{ \begin{bmatrix} K_{sf}{}^* \\ -I \end{bmatrix} S \begin{bmatrix} K_{of}C & -I \end{bmatrix} \right\}^{\mathcal{H}}$$

S-variable approach

$$\exists P \succ 0, S, K_{sf}, K_{of} \quad : \quad \begin{bmatrix} \{PA\}^{\mathcal{H}} & PB \\ B^* P & 0 \end{bmatrix} \prec \left\{ \begin{bmatrix} K_{sf}{}^* \\ -I \end{bmatrix} S \begin{bmatrix} K_{of} C & -I \end{bmatrix} \right\}^{\mathcal{H}}$$

- ▼ Still not LMI, matrix inequalities of larger size, more decision variables
- ▲ More degrees of freedom, P and K_{of} separated one for the other
- ▲ Simple to code K_{sf}-K_{of}-iterative algorithm

$$K_{sf}\text{-iteration} \quad K_{of} = S^{-1} L_{of} \quad \arg\min \alpha \quad : \quad \begin{bmatrix} \{PA\}^{\mathcal{H}} - \alpha I & PB \\ B^* P & 0 \end{bmatrix} \prec \left\{ \begin{bmatrix} K_{sf}{}^* \\ -I \end{bmatrix} \begin{bmatrix} L_{of} C & -S \end{bmatrix} \right\}^{\mathcal{H}}$$

$$K_{of}\text{-iteration} \quad K_{sf} = S^{-1} L_{sf} \quad \arg\min \alpha \quad : \quad \begin{bmatrix} \{PA\}^{\mathcal{H}} - \alpha I & PB \\ B^* P & 0 \end{bmatrix} \prec \left\{ \begin{bmatrix} L_{sf}{}^* \\ -S \end{bmatrix} \begin{bmatrix} K_{of} C & -I \end{bmatrix} \right\}^{\mathcal{H}}$$

- ▲ Smart initial guess of K_{sf} (finding K_{sf} is a convex problem)
- ▲ Strictly decreasing sequence of α
- ▲ Much more efficient than the P-K-iterative algorithm (P is free at each step)

  Implicitly the algorithm searches for $K_{sf} \to K_{of} C$.

S-variable approach - variant (dual)

Variant of the same result based on output-injection gain K_{oi}

$$\exists Q \succ 0, S, K_{oi}, K_{of} \quad : \quad \begin{bmatrix} \{AQ\}^{\mathcal{H}} & QC^* \\ CQ & 0 \end{bmatrix} \prec \left\{ \begin{bmatrix} BK_{of} \\ -I \end{bmatrix} S \begin{bmatrix} K_{oi}{}^* & -I \end{bmatrix} \right\}^{\mathcal{H}}$$

- ▲ Simple to code K_{oi}-K_{of}-iterative algorithm
- ▲ Smart initial guess of K_{oi} (finding K_{oi} is a convex problem)
- ▲ Strictly decreasing sequence of α
- ▲ Much more efficient than the P-K-iterative algorithm (P is free at each step)
  Implicitly the algorithm searches for $K_{oi} \to BK_{of}$.

- ▲ Robustness can be dealt with easily (eg. solve the constraints for all vertices of a polytope)
  - ▼ yet conservative (common Lyapunov certificate P or Q for all uncertainties)
- ▲ Structured SOF : achievable with constraints on S, not on P of Q
  - ▼ yet conservative

S-variable approach - variant (Lyapunov certificate)

Variant [Peres et al. 2020] assuming two Lyapunov certificates P_1 and P_2 for $A+BKC$

$$\exists P_1 \succ 0, P_2 \succ 0, S = S^*, K \quad : \quad \begin{bmatrix} 0 & (A+BKC)^* \\ A+BKC & 0 \end{bmatrix} \prec \left\{ \begin{bmatrix} P_2 \\ -I \end{bmatrix} S \begin{bmatrix} P_1 & -I \end{bmatrix} \right\}^{\mathcal{H}}$$

- ▲ Matrix inequalities of larger size
- ▲ Simple to code P_1-P_2-iterative algorithm
- ▼ No smart initial guess of P
- ▲ Robustness can be dealt with easily (eg. solve the constraints for all vertices of a polytope)
- ▲ No difficulty to include structure constraints on K
- ▼ Seems less efficient than the K_{sf}-K_{of}-iterative algorithm

2S-variable approach - In Honor of Roberto Tempo

"Robust static output feedback design with deterministic and probabilistic certificates",
D. Arzelier, F. Dabbene, S. Formentin, D. Peaucelle, L. Zaccarian
Birkhäuser Mathematics - Springer Nature Chapter for "Uncertainty in Networked Systems", 2019

$$P \succ 0 \qquad \begin{bmatrix} 0 & 0 & P \\ 0 & 0 & 0 \\ P & 0 & 0 \end{bmatrix} \prec \left\{ S_1 \begin{bmatrix} -I & (BF+M) & A \end{bmatrix} + \begin{bmatrix} 0 \\ -I \\ H^* \end{bmatrix} \begin{bmatrix} 0 & -S_2 & LC \end{bmatrix} \right\}^{\mathcal{H}}$$

$$Q \succ 0 \qquad \begin{bmatrix} 0 & 0 & Q \\ 0 & 0 & 0 \\ Q & 0 & 0 \end{bmatrix} \prec \left\{ \begin{bmatrix} -I \\ LC+M \\ A \end{bmatrix} S_{1d} + \begin{bmatrix} 0 \\ -S_{2d} \\ BF \end{bmatrix} \begin{bmatrix} 0 & -I & H^* \end{bmatrix} \right\}^{\mathcal{H}}$$

New conditions with interesting properties for initialization, robustness, and structure.

2S-variable approach

$$P \succ 0 \qquad \begin{bmatrix} 0 & 0 & P \\ 0 & 0 & 0 \\ P & 0 & 0 \end{bmatrix} \prec \left\{ S_1 \begin{bmatrix} -I & (BF+M) & A \end{bmatrix} + \begin{bmatrix} 0 \\ -I \\ H^* \end{bmatrix} \begin{bmatrix} 0 & -S_2 & LC \end{bmatrix} \right\}^{\mathcal{H}} \qquad (1)$$

➡ If $F = 0$ then $K_{si} = MH$ is a stabilizing state-injection gain ($A + K_{si}$ is stable)

Let $\eta = \begin{pmatrix} \dot{x} \\ Hx \\ x \end{pmatrix}$ then: $\begin{aligned} &\begin{bmatrix} -I & M & A \end{bmatrix} \eta = -\dot{x} + (A + K_{si})x = 0 \\ &\begin{bmatrix} 0 & -I & H \end{bmatrix} \eta = 0 \\ &\eta^*(1)\eta \quad \Rightarrow \quad \dot{x}^* P x + x^* P \dot{x} < 0 \end{aligned}$

2S-variable approach

$$P \succ 0 \quad \begin{bmatrix} 0 & 0 & P \\ 0 & 0 & 0 \\ P & 0 & 0 \end{bmatrix} \prec \left\{ S_1 \begin{bmatrix} -I & (BF+M) & A \end{bmatrix} + \begin{bmatrix} 0 \\ -I \\ H^* \end{bmatrix} \begin{bmatrix} 0 & -S_2 & LC \end{bmatrix} \right\}^{\mathcal{H}} \quad (1)$$

➡ If $F = 0$ then $K_{oi} = MS_2^{-1}L$ is a stabilizing output-injection gain ($A + K_{oi}C$ is stable)

Let $\eta = \begin{pmatrix} \dot{x} \\ S_2^{-1}LCx \\ x \end{pmatrix}$ then:

$$\begin{bmatrix} -I & M & A \end{bmatrix} \eta = -\dot{x} + (A + K_{oi}C)x = 0$$

$$\begin{bmatrix} 0 & -S_2 & LC \end{bmatrix} \eta = 0$$

$$\eta^*(1)\eta \quad \Rightarrow \quad \dot{x}^*Px + x^*P\dot{x} < 0$$

2S-variable approach

$$P \succ 0 \qquad \begin{bmatrix} 0 & 0 & P \\ 0 & 0 & 0 \\ P & 0 & 0 \end{bmatrix} \prec \left\{ S_1 \begin{bmatrix} -I & (BF+M) & A \end{bmatrix} + \begin{bmatrix} 0 \\ -I \\ H^* \end{bmatrix} \begin{bmatrix} 0 & -S_2 & LC \end{bmatrix} \right\}^{\mathcal{H}} \tag{1}$$

➡ If $M = 0$ then $K_{sf} = FH$ is a stabilizing state-feedback gain ($A + BK_{sf}$ is stable)

$$\text{Let } \eta = \begin{pmatrix} \dot{x} \\ Hx \\ x \end{pmatrix} \text{ then: } \begin{array}{l} \begin{bmatrix} -I & BF & A \end{bmatrix} \eta = -\dot{x} + (A + BK_{sf})x = 0 \\ \begin{bmatrix} 0 & -I & H \end{bmatrix} \eta = 0 \\ \eta^*(1)\eta \quad \Rightarrow \quad \dot{x}^* P x + x^* P \dot{x} < 0 \end{array}$$

2S-variable approach

$$P \succ 0 \qquad \begin{bmatrix} 0 & 0 & P \\ 0 & 0 & 0 \\ P & 0 & 0 \end{bmatrix} \prec \left\{ S_1 \begin{bmatrix} -I & (BF+M) & A \end{bmatrix} + \begin{bmatrix} 0 \\ -I \\ H^* \end{bmatrix} \begin{bmatrix} 0 & -S_2 & LC \end{bmatrix} \right\}^{\mathcal{H}} \qquad (1)$$

➡ If $M = 0$ then $K_{of} = FS_2^{-1}L$ is a stabilizing output-feedback gain ($A + BK_{of}C$ is stable)

Let $\eta = \begin{pmatrix} \dot{x} \\ S_2^{-1}Lx \\ x \end{pmatrix}$ then:

$$\begin{aligned} \begin{bmatrix} -I & BF & A \end{bmatrix}\eta &= -\dot{x} + (A + BK_{of}C)x = 0 \\ \begin{bmatrix} 0 & -S_2 & LC \end{bmatrix}\eta &= 0 \\ \eta^*(1)\eta \quad &\Rightarrow \quad \dot{x}^*Px + x^*P\dot{x} < 0 \end{aligned}$$

2S-variable approach - robustness

$$\begin{bmatrix} 0 & 0 & P_\delta \\ 0 & 0 & 0 \\ P_\delta & 0 & 0 \end{bmatrix} \prec \left\{ S_1 \begin{bmatrix} -I & (B_\delta F + M_\delta) & A_\delta \end{bmatrix} + \begin{bmatrix} 0 \\ -I \\ H^* \end{bmatrix} \begin{bmatrix} 0 & -S_2 & LC_\delta \end{bmatrix} \right\}^{\mathcal{H}}$$

➡ If $F = 0$ then $K_{si\delta} = M_\delta H$ is a stabilizing state-injection gain ($A_\delta + K_{si\delta}$ is stable)
➡ If $F = 0$ then $K_{oi\delta} = M_\delta S_2^{-1} L$ is a stabilizing output-injection gain ($A_\delta + K_{oi\delta} C_\delta$ is stable)
➡ If $M = 0$ then $K_{sf} = FH$ is a stabilizing state-feedback gain ($A_\delta + B_\delta K_{sf}$ is stable)
➡ If $M = 0$ then $K_{of} = FS_2^{-1} L$ is a stabilizing output-feedback gain ($A_\delta + B_\delta K_{of} C_\delta$ is stable)

- In case of polytopic uncertainties: sufficient to test on vertices
- Robust stability proved with parameter-dependent Lyapunov certificate $x^T P_\delta x$
- Result is also new for robust state-feedback design

2S-variable approach - Proposed iterative algorithm

$$\begin{bmatrix} 0 & 0 & P_\delta \\ 0 & 0 & 0 \\ P_\delta & 0 & 0 \end{bmatrix} \prec \left\{ S_1 \begin{bmatrix} -I & (B_\delta F + M_\delta) & A_\delta \end{bmatrix} + \begin{bmatrix} 0 \\ -I \\ H^* \end{bmatrix} \begin{bmatrix} 0 & -S_2 & LC_\delta \end{bmatrix} \right\}^{\mathcal{H}}$$

- ▼ Bilinear matrix inequalities in the decision variables
- ▲ Initialization: rather easy to find $K_{si\delta} = M_\delta H$ s.t. $A_\delta + K_{si\delta}$ stable (eg. $M_\delta = -A_\delta - \lambda I$, $H = I$)
- ➡ Iteratively freeze (F, M_δ, H) then $(S_1, S_2^{-1}L)$
- ➡ During the algorithm $M_\delta \to 0$, $K_{si\delta} \to B_\delta K_{sf}$ and $K_{oi\delta} \to B_\delta K_{of}$

- ▲ Lyapunov certificate P_δ and matrices defining $K_{of} = FS_2^{-1}L$ optimized at all steps
- ▲ Rather efficient even on non "hard" CompLib problems
- ▼ Includes a line search
- ▼ Non trivial tuning parameters (related to stopping criteria)

2S-variable approach - Structured OF case

$$\begin{bmatrix} 0 & 0 & P_\delta \\ 0 & 0 & 0 \\ P_\delta & 0 & 0 \end{bmatrix} \prec \left\{ S_1 \begin{bmatrix} -I & (B_\delta F + M_\delta) & A_\delta \end{bmatrix} + \begin{bmatrix} 0 \\ -I \\ H^* \end{bmatrix} \begin{bmatrix} 0 & -S_2 & LC_\delta \end{bmatrix} \right\}^{\mathcal{H}}$$

▼ The structured case: initialization requires rank constrained state-injection gain

➡ Diagonal $K_{of} = FS_2^{-1}L \in \mathbb{D}^{m\times m}$ requires diagonal $F \in \mathbb{D}^{m\times m}$, $S_2 \in \mathbb{D}^{m\times m}$, $L \in \mathbb{D}^{m\times m}$

➡ Consequence: $M_\delta \in \mathbb{R}^{n\times m}$ and $H \in \mathbb{R}^{m\times n}$, i.e. $K_{si\delta} = M_\delta H$ is rank m

▲ Rather efficient heuristic:

❶ $\min \mathrm{Tr}(Z) \quad : \quad \{PA_\delta\}^{\mathcal{H}} \prec Z, \quad P \succ 0, \quad Z \succ 0$

❷ If Z is rank m (or less) solve the Lyap. equality $\{PG\}^{\mathcal{S}} = Z$ and let $G = G_1 H$ with $H \in \mathbb{R}^{m\times n}$

❸ For given H $\min \|PB_\delta - \hat{M}_\delta\| \quad : \quad \{PA_\delta + \hat{M}_\delta H\}^{\mathcal{S}} \prec 0, \quad P \succ 0$

❹ Choose $M_\delta = P^{-1}\hat{M}_\delta$.

2S-variable approach - Summary

$$P \succ 0 \qquad \begin{bmatrix} 0 & 0 & P \\ 0 & 0 & 0 \\ P & 0 & 0 \end{bmatrix} \prec \left\{ S_1 \begin{bmatrix} -I & (BF+M) & A \end{bmatrix} + \begin{bmatrix} 0 \\ -I \\ H^* \end{bmatrix} \begin{bmatrix} 0 & -S_2 & LC \end{bmatrix} \right\}^{\mathcal{H}}$$

- ▲ Stabilizing state-injection property : Good for initialization
- ▲ All matrices P, A, B and C decoupled:
  OK for robustness with parameter-dependent Lyapunov certificates
- ▲ Results are new (and efficient) even for robust K_{sf} and K_{oi} design
- ▲ No need to structure the Lyapunov certificate for structured SOF
- ▼ Iterative algorithm is less trivial than for previous conditions
- ▼ Needs to be tested on more examples

Conclusions

➡ Several results for SOF using the S-variable approach
▲ Rather efficient to deal with robust structured SOF
▼ No guarantee of success
▲ ▼ Results with more or less intuitive initialization

➡ Latest result - 2S-varaible
▲ Promising numerical experiments
▲ Elegant combination of the SI, OI, SF, OF problems
▲ Variations of the algorithms for

Deterministic robust design
Probabilistic robust design
With comparisons of the two

"Robust static output feedback design with deterministic and probabilistic certificates",
D. Arzelier, F. Dabbene, S. Formentin, D. Peaucelle, L. Zaccarian
Birkhäuser Mathematics - Springer Nature Chapter for "Uncertainty in Networked Systems"

About Adaptive Control

Static OF: Stabilize $\begin{array}{l}\dot{x} = Ax + Bu \\ y = Cx\end{array}$ with a static output feedback $u = Ky$, $\dot{K} = 0$

Direct Adaptive OF: Stabilize $\begin{array}{l}\dot{x} = Ax + Bu \\ y = Cx\end{array}$ with $u = Ky$ where $\dot{K} = f(y, K)$

➡ Adaptation $\equiv$ learning rule

➡ Assumes that there exists a Static OF solution - guaranteed by finding one

Why implementing the non-linear adaptive control ?

▲ Can deal with non-linearities of the plant away from the equilibrium

See presentation at IFAC WC where adaptation helps to avoid saturation of actuators

Could improve robustness compared to parameter-independent Static OF $u = Ky$

▲ Matrix inequality conditions proving robust Adaptive OF assuming parameter-dependent $u = K_\delta y$

▼ No example found: $\nexists u = Ky$ stabilizing $\begin{array}{l}\dot{x} = A_\delta x + B_\delta u \\ y = C_\delta x\end{array}$ but $\exists$ stabilizing adaptive control