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Actuator fault diagnose for interconnected system via invertibility

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Abstract: This paper deals with the problem of actuator fault diagnosis for a class of interconnected invertible nonlinear systems. For that, the actuator is viewed as an independent dynamic subsystem in series with the plant dynamic subsystem, thus forming an interconnected system. The invertibility of the interconnected system in both normal and fault models is investigated. An interconnected observer is proposed to monitor the performance of the interconnected system and provide fault information of the actuator subsystem. Then a local fault filtering algorithm is triggered to identify the root faulty parameter causing the detected actuator fault. According to the real situation of the industry, it is assumed that the output of the actuator is not available for measurement and should be reconstructed by the global output of the interconnected system. A method capable of supervising the actuator subsystem at both local and global levels is provided.

Keywords: interconnected system; invertibility; FDD; fault detection and diagnosis; actuator fault diagnosis; fault distinguishability; root faulty parameter.

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1 Introduction

With the development of technology, each component of a modern control system is becoming increasingly integrated. These complexities bring difficulties to the supervision of the system. Therefore, the safe and reliable operation of the system is an important part of the whole system performance and sustainability. It can be achieved by advanced fault detect and diagnose (FDD) methods.

Actuator is the basic part of a control system, and a failure of an actuator may result in the failure of the whole system. In the past several decades, there have been important research activities for nonlinear dynamic systems that suffer actuator fault, such as Li and Dahhou (2008), Fragkoulis et al. (2011), Han et al. (2018), Keliris et al. (2015a, 2015b), Buciakowski et al. (2017), Guo and Zhu (2017), Yan and Edwards (2008), Blesa et al. (2014), Alwi and Edwards (2013), Bartyś et al. (2006), Ma et al. (2017), Sharifi et al. (2018), Zakharov and Tikkala (2013) and Tang et al. (2016). One of the main categories is system level diagnosis method, in which the internal dynamics of actuator is ignored. Its purpose is to detect and identify the existence and location of an actuator fault from the perspective of global system, such as Li and Dahhou (2008), Fragkoulis et al. (2011), Han et al. (2018), Keliris et al. (2015), Buciakowski et al. (2017), Guo and Zhu (2017), Yan and Edwards (2008), Blesa et al. (2014) and Alwi and Edwards (2013). Faulty actuator can be identified, but specific faulty parameter of the actuator is not capable of recognising. Another common method is to recognise the internal dynamics of a specific actuator from the perspective of the field device without considering the global system dynamics, such as Bartyś et al. (2006), Ma et al. (2017), Sharifi et al. (2018), Zakharov and Tikkala (2013) and Tang et al. (2016).

The most common method of actuator fault diagnosis in the literature is usually focused on the system level, in which the main purpose is focused on the performance supervision of the global system. In order to obtain satisfactory results, there is generally an important assumption in these methods, that is to ignore the internal

dynamics of the actuator, although this is usually not true in real situations. To achieve the system level based FDD purpose, one of the key methods is related to residual generation (Besançon, 2000; Keliris et al., 2015); another important method is based on fault reconstruction or fault estimation (Persis and Isidori, 2001; Tan and Edwards, 2002; Zhu and Cen, 2010).

For the above mentioned methods, the research mainly focuses on recognition of faulty devices, while the research on the root faulty parameter analysis within the faulty device is less. This is because the control inputs in these methods are assumed to be linear, and the fault feature is represented by the element change of the input matrix or function. This assumption may obtain applicable results in the traditional dynamic system. However, the progress of technology makes the actuators more and more integrated and complex. Each actuator may be composed of multiple components connected by any configuration, as a result, the actuator is regarded as a dynamic system. Therefore, it is of great importance to recognise the internal dynamics of the failed actuator, so as to find out the root cause of abnormal operation of the process. For example, Raimondi et al. (2015) indicates that the decrease of fluid flow results in the decrease of temperature in the heat exchange reactor, this decrease implies a fault of the actuator. With the help of the above mentioned FDD algorithm, the faulty actuator can be isolated, but cannot identify the root faulty parameter internal to the actuator. The potential root cause of this failure could be a blocked valve, a stopped utility liquid pump, or a leak.

In order to study the potential relationship between the cause and the effect of actuator failure, some researches focus on locating the failure from the perspective of component level. Such analysis is realised by local intelligence. The proposed FDD method is shown in Ma et al. (2017). In Bartyś et al. (2006), a passive fault detection method of control valve based on interval observer is proposed.

However, few existing FDD methods can provide a means to monitor and diagnose the whole system both locally and globally in the reliable operation of

interconnected dynamic systems. The main challenge of field device level based FDD approaches is due to the lack of dynamic information concerning the global system. In addition, when studying FDD method locally, another challenge is to obtain reliable measurements from the observed subsystem. This is because, due to the distance or harsh environment, it is usually impossible or difficult to directly measure the actuator output. In addition, even if the output of the actuator can be measured, there will be too much noise in the numerical differentiation due to the noise output of the actuator sensor. The control input under the influence of noise signal will not only damage the actuator, but also reduce the tracking accuracy, resulting in the instability of FDD scheme.

Therefore, how to use both local and global information to isolate different faulty devices and find out the root causes of the fault is a very important and challenge topic in the field of actuator fault diagnosis. In fact, what is needed is the combination of local intelligence with advanced diagnosis ability, so as to perform FDD function on the global process and single subcomponent.

Based on the above considerations, this paper proposes an actuator fault diagnosis method, and aims at solving the problem of fault propagation and local faulty parameter identification globally for a class of invertible interconnected systems. The purpose is to investigate rules to use global output to locate the root faulty parameter in the local actuator subsystem. An interconnected invertible nonlinear system structure is proposed, in which the actuator is regarded as an independent subsystem cascaded with the controlled plant subsystem. In order to guarantee that the faults in the actuator subsystem will generate unique and distinguishable impacts on the global system, the interconnected system is required to be invertible. According to the actual situation, the actuator output is assumed to be immeasurable and needs to be reconstructed from the terminal measurable output of the global system. It is designed a local fault filter to recognise the root abnormal parameter of the actuator, the filter is assumed to implement in the actuator subsystem with the known input and estimated output. The proposed actuator FDD algorithm is an attempt to combine global and component level based fault diagnosing intelligence.

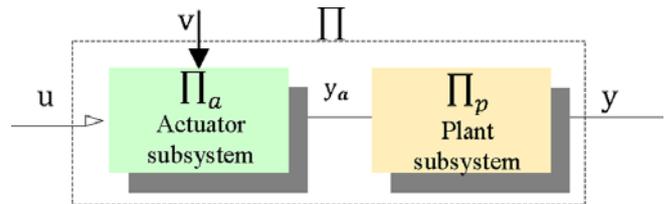
The following is the structure of the paper. In Section 2, the proposed strategy is described, and the main problem is formulated. Section 3 introduces the proposed actuator FDD architecture. In the fourth section, the conditions of fault distinguishability locally and globally are analysed, which are to guarantee the implementation of this method. Then, in Sections 5 and 6, an interconnected observer is proposed, aimed at supervising the performance of the interconnected system and isolating the faulty actuator. Once a fault is detected, a local fault filter is triggered to recognise the root abnormal parameter of the isolated faulty actuator. Numerical simulations are given and discussed in Section 7. Finally, in Section 8, the conclusion is made.

2 Problem formulation

The modern control system can be seen as a cascade of dynamic subsystems. Therefore, when estimating the reliability of the whole system, it can be analysed in a decentralised way at different levels from the view of global system or individual subsystems. For example, a typical control system can be considered as composed of controlled plant, actuator and sensor cascaded subsystems. All three subsystems must work normally to ensure the normal operation of the overall system, and the fault may be caused by the abnormal internal parameters of any subsystem. This paper mainly studies the influence of the internal dynamic characteristics of the actuator on the global system and the propagation of faults, so the interconnected system of this paper is considered of consisted of the controlled plant and the actuator subsystems.

As shown in Figure 1, the interconnected system Π is composed of controlled plant Π_p and actuator Π_a subsystems. The initial motivation is to recognise the local fault vector V with the global measurement y , aims at indicating the potential root abnormal parameters of the actuator.

Figure 1 Structure of the interconnected system (see online version for colours)



Assumed that the plant subsystem Π_p is an input affine nonlinear system described by equation (1):

$$\Pi_p \begin{cases} \dot{x}_p = f_p(x_p) + g_p(x_p)y_a, & x_p(t_0) = x_{p0} \\ y_p = h_p(x_p) \end{cases} \quad (1)$$

where $x_p \in \mathfrak{X}^n$ represents state of Π_p , $y_p \in \mathfrak{R}^p$ represents the output of Π_p , as well as the output of Π . $y_a \in \mathfrak{R}^m$ refers to input of Π_p , as well as the output of Π_a . g_p and f_p are both smooth vectors on \mathfrak{X}^n and h_p is smooth vector field on \mathfrak{X}^n .

Similarly, suppose that the actuator subsystem Π_a can be described by a nonlinear dynamic system in equation (2):

$$\Pi_a \begin{cases} \dot{x}_a = f_a(x_a, u, \theta_a) \\ y_a = h_a(x_a, u, \theta_s) \end{cases} \quad (2)$$

where $x_a \in \mathfrak{X}^n$ is the state, $u \in \mathfrak{R}^l$ is the input, $y_a \in \mathfrak{R}^m$ is the output of the actuator subsystem Π_a , as well as the input of the plant subsystem Π_p , $\theta_a \in \mathfrak{R}^k$ and $\theta_s \in \mathfrak{R}^k$ represent the internal parameters that may impact dynamics of the actuator subsystem.

Therefore, the two subsystems (Π_a and Π_p) described in (1) and (2) constitute an interconnected cascade system Π , in which the input vector is $u \in \mathfrak{R}^l$ and the output vector is $y = y_p \in \mathfrak{R}^n$, interconnection y_a is supposed to be inaccessible for measurement.

In general, the fault vector $V = (v_1, \dots, v_k)$ can be regarded as integration of either parameters faults in θ_a, θ_s or other disturbance signals. Therefore the actuator subsystem in fault mode becomes:

$$\Pi_a \begin{cases} \dot{x}_a = f_a(x_a, u) + g_a(x_a, u)V \\ y_a = h_a(x_a, u) + l_a(x_a, u)V \end{cases} \quad (3)$$

where g_a, l_a are analytic functions, the system may suffer possible multiple and simultaneously faults.

The initial purpose of this work is to describe the causal relationship between the subsystem variable v_i and the global system output y , so as to provide advanced predictive maintenance technology in the operating equipment. Considering the interconnected system shown in Figure 1, the main purpose is then to independently recognise the existence of fault v_i in equation (3) by using the information y of the global plant. This can be regarded as the problem of fault identifiability. Given two faults, their identifiability refers to the capability of generating distinguishable outputs of the system.

According to Li and Dahhou (2008), the identifiability of a fault in nonlinear system (3) can be described as:

Definition 1 : For any fault $v_i (i=1, \dots, k) \neq 0$, if the following relation

$$y(x_{p0}, x_p; x_{a0}, x_a; u; 0) = y(x_{p0}, x_p; x_{a0}, x_a; u; 0, \dots, v_i, \dots, 0)$$

is satisfied, then the fault v_i cannot be identified; if not, the fault v_i is said to be identified.

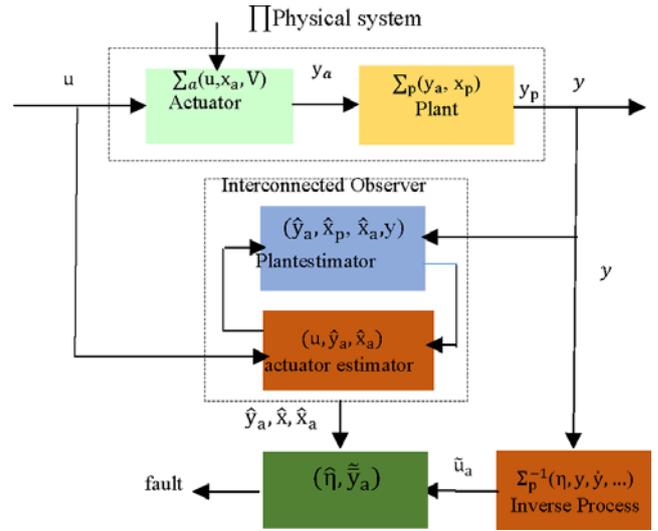
3 Actuator fault diagnosis and root faulty parameter analysis architecture

3.1 Performance supervision and fault detection

In order to achieve the purpose of monitoring and fault diagnosis of the system at deferent levels, one effective way is to design observers for individual subsystems, as well as the whole system. However, the main challenge here is that the connection between subsystems is not measurable. The connection is in fact the actuator subsystem output. This is due to that online measurement of output of an actuator is not easy and reliable. Therefore, the available measurements in this paper can only depend on the input and output of the global system.

Considering interconnected systems depicted by equations (1) and (3), the particular aim of this paper is to estimate the state vector x_p, x_a of the subsystems and the unmeasured interconnection vector y_a online accurately, as well as indicating occurrences of fault V.

Figure 2 Interconnected observer for performance supervision (see online version for colours)



In order to achieve this purpose, the main ideas are described as follows. As shown in Figure 2, in order to monitor the performance of the whole system and each subsystem, a nominal interconnected observer is proposed for the nominal system. To achieve this purpose, a nominal observer is first proposed for the nonlinear actuator subsystem Π_a with output y_a , it is assumed that there is no fault existed; then the output y_a is replaced by its estimation \hat{y}_a to implement the observer. In order to obtain the estimate \hat{y}_a , the states of the controlled plant subsystem Π_p is expanded to include y_a as an additional state. By calculating the derivative of y_a in the actuator subsystem, the expression of y_a is obtained which is a function of u , derivatives of u and x_a ; and then the observer of the extended controlled plant subsystem is constructed. In this way, a nominal observer for the interconnected system based on state estimators of actuator and controlled plant subsystems is proposed.

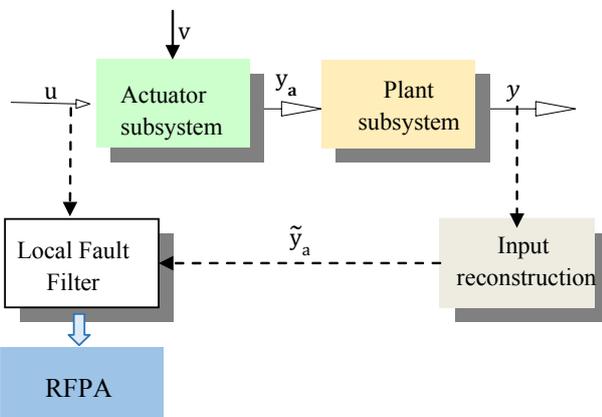
The nominal interconnected observer is proposed under the assumption that no fault existed, thus once the observer converges, the estimated value of y_a is reliable, in other words, the estimated \hat{y}_a can be used as the real signal y_a of the controlled plant subsystem for input reconstruction to realise fault occurrence. An inverse observer is constructed, aims at eliminating the impact of drift, initials and etc. The implementation of the inverse observer ensures the effectiveness of subsequent root faulty parameter analysis. It can be seen from Figure 2 that the inverse observer input is the output y , and the estimation output of the observer is \tilde{y}_a, \hat{y}_a is the reference output. Once the observer converges, the estimation meets $\tilde{y}_a = \hat{y}_a = y_a$, which means that once the fault occurs, the estimation error will be higher than the threshold. When a sudden fault is detected, a local filter is triggered for root cause analysis.

3.2 Fault identification and root faulty parameter analysis

After a faulty actuator is recognised, the following task is to determine the root faulty parameter in the actuator subsystem. For that, a local fault filtering scheme is proposed. Each fault filter is assigned to supervise a particular actuator and provide information regarding of its health for final decision-making. This process is referred to root faulty parameter analysis (RFPA), as shown in Figure 3. The initial purpose is to monitor the whole system at both local and global level according to the need of reliable operation of the interconnected system. At the same time, the requirement to recognise the existence of fault candidate v_i in equation (3) is satisfied. The fault v_i relates to the fault sources of an actuator fault, which is related to specific physical factor. For instance, v_i corresponds to valve leakage or blockage. A local FDD strategy of actuator subsystem based on local filter is proposed to recognise these actuator fault candidates ($v_i, i = 1, \dots, k$).

The local fault filter is designed to implement within the local actuator, thus the input and output of the filter should be u, y_a , as shown in Figure 3. However, an important challenge is that only the global system output y is available here. This is due to that, generally speaking, the actuator fault diagnosis system is realised remotely, so the direct measurement of y_a is usually not realistic under the actual industrial conditions. Therefore, in this work, y_a is assumed to be unmeasured. In addition, when performing FDD function in the local subsystem, the global output information should also be included to monitor the plant globally. In a sum, the above two considerations can be settled if y_a can be uniquely reconstructed by the global output y . In that way, advanced fault diagnose strategies can be adopted to identify the internal faulty parameters in the actuator subsystem, utilising estimated output \tilde{y}_a and input u . As a result, a means of actuator FDD is achieved which is capable of supervising and diagnosing of the interconnected system locally and globally simultaneously.

Figure 3 Main idea of the proposed root faulty parameter analysis (RFPA) algorithm (see online version for colours)



In conclusion, to achieve the purpose, the critical issue is to put forward guarantee conditions for verifying the causal

relationship between the output vector Y and the fault vector V , and at the same time, to ensure that there is a one-to-one correspondence between the reconstructed \tilde{y}_a and the fault vector V . In this case, the local fault filter can be implemented in the actuator subsystem to recognise the failure parameter v_i , so as to achieve the purpose of root faulty parameter analysis of the detected faulty actuator. There are four main tasks to be completed:

- To provide condition so as to guarantee that the faults $V = (v_1, \dots)$ occurred in actuator subsystem can generate unique and distinguishable outputs on the global system.
- To estimate the unknown output y_a of the actuator subsystem, denoted as \tilde{y}_a , by using global measurement output y .
- To put forward a condition to guarantee one to one correspondence between reconstructed \tilde{y}_a and fault V .
- To propose a root faulty parameter analysis (RFPA) algorithm to identify fault candidate $V(v_1, \dots)$ at local level.

4 On condition of fault distinguishability locally and globally

4.1 On condition of fault distinguishability

For a given system, fault distinguishability refers to the capability of generating different outputs of the system. If the corresponding outputs of any initial state vector and control input (not all zero) of the system are different, then two faults are said to be strictly distinguishable. The study of distinguishability is to put forward sufficient and/or necessary conditions to verify whether two different fault signals can be distinguished outputs.

To sum up, one of the key features, opportunities and technical challenges of this scheme is to guarantee that fault information $V = (v_1, v_2, \dots, v_k)$ in the actuator subsystem (3) can generate obvious different outputs y in (1). If the whole system is regarded as a black box, in which the input of the system is the fault vector V and the output is the measurable output Y , then this problem can be regarded as the input observability problem of the dynamic system. As it can be seen in reference (Edelmayer et al., 2004; Tanwani and Liberzon, 2010; Martínez-Guerra et al., 2013), the problem of input observability of dynamic systems can be equivalent to the problem of left invertibility of systems. According to Edelmayer et al. (2004), if the fault model of a dynamic system is fault affine and the fault affine model is invertible, then the fault vector can be calculated from the output vector and their derivatives of the system. As a result, it can be concluded that if the fault model of the interconnected system is fault affine and the fault affine model is invertible, the fault vector V can be obtained by output vector Y .

Definition 2: (Algebraic observability condition). For system depicted by (1) and (3), if the fault $v_i (i=1, \dots, k)$ is algebraically observable, then it can be diagnosable.

Proposition 1: If the fault $v_i (i=1, \dots, k)$ can be estimated from the available measures of the interconnected system in fault mode depicted by (1) and (3), then it can be diagnosable

Property 1: If the interconnected system in fault mode depicted by (1) and (3) is invertible, it can be obtained the fault $v_i (i=1, \dots, k)$ by means of the global output y .

Theorem 1: If the interconnected system in fault mode depicted by (1) and (3) is invertible, the fault $v_i (i=1, \dots, k)$ is capable of generating distinguishable effects on the global output y of the system.

4.2 On condition of invertibility of the nonlinear interconnected system

From the above analysis, the key problem of the proposed method is to put forward conditions to verify the invertibility of the interconnected systems. Here, we regard the fault candidates as the unknown inputs of the local actuator subsystem, the objective is therefore to provide invertibility checking algorithm for the fault model of the system, where the input and output refer to $V (v_i, i=1, \dots, k)$ and Y in (1) and (3), respectively. It is obviously that classical inversion technology cannot meet the needs. For that we present a new tool to guarantee the invertibility of interconnected dynamic systems.

Theorem 2: Consider that the interconnected system Π composes of controlled plant Π_p and actuator Π_a subsystems depicted by (1) and (3), output set is \mathcal{Y} . Iff both controlled plant Π_p and actuator Π_a subsystems are invertible at x_{a0} over \mathcal{U}_a , and x_{p0} over \mathcal{Y} respectively, the interconnected system is said to be invertible at (x_{p0}, x_{a0}) over \mathcal{Y} .

Proof. Supposed M_a and M_p are the input-output mappings of subsystem Π_a and Π_p respectively, the input-output mapping of the interconnected system Π is their composition $M_a \circ M_p$.

- (Sufficiency): On one hand, composition map $M_a \circ M_p$ is the input output mapping of the cascade system. Bijective of the input output mapping is equal to invertibility of a dynamic system, therefore we need to confirm that map $M_a \circ M_p$ is bijective. On the other hand, since a composition map is bijective if and only if the involving two mappings are bijective. In our case, mappings M_a and M_p correspond to the maps of two invertible subsystems, therefore M_a and M_p are bijective mappings. Thus, their composition map $M_a \circ M_p$ is

bijective, it means invertibility of the interconnected system.

- (Necessity): Next, we have to prove the necessity of the condition. For that, it can be achieved by proving that non-invertibility of any subsystems will result in non-invertibility of the interconnected system.

First, let's suppose that the actuator subsystem Π_a is invertible at x_{a0} over \mathcal{U}_a while the plant subsystem Π_p is not. That is to say, for actuator subsystem Π_a in equation (2), fix an output set \mathcal{U}_a and an arbitrary interval $[t_0, T)$, there have two different inputs for $\exists \varepsilon > 0$ $v_1 \neq v_2$ on $[t_0, t_0 + \varepsilon)$, that generate two different outputs $M_{a(x_{a0})}(v_{1[t_0, T)}) = y_{a1[t_0, T)}$, $M_{a(x_{a0})}(v_{2[t_0, T)}) = y_{a2[t_0, T)}$, $y_{a1[t_0, T)} \neq y_{a2[t_0, T)}$. However, for the controlled plant subsystem (1), fix an output set \mathcal{Y} , these two different inputs $y_{a1} \neq y_{a2}$ on $[t_0, t_0 + \varepsilon)$ may produce two equal outputs $M_{p(x_{p0})}(y_{a1[t_0, T)}) = M_{p(x_{p0})}(y_{a2[t_0, T)}) = y_{[t_0, T)}$. In a sum, for the cascade system, these two different inputs $v_1 \neq v_2$ on $[t_0, t_0 + \varepsilon)$ may result in two equal outputs:

$$\begin{aligned} (M_a \circ M_p)_{(x_{a0}, x_{p0})}(v_{1[t_0, T)}) &= (M_a \circ M_p)_{(x_{a0}, x_{p0})}(v_{2[t_0, T)}) \\ &= y_{[t_0, T)} \end{aligned}$$

In other words, it indicates that the interconnected system Π cannot be invertible at (x_{p0}, x_{a0}) over $(\mathcal{U}_a, \mathcal{Y})$.

Similarly, assume that controlled plant subsystem Π_p is invertible, the while actuator subsystem Σ_a is not. Then for Π_a in (2), fix an output set \mathcal{U}_a and consider an arbitrary interval $[t_0, T)$, there have two different inputs for $\exists \varepsilon > 0$ $v_1 \neq v_2$ on $[t_0, t_0 + \varepsilon)$, that may yield two equal outputs $M_{a(x_{a0})}(v_{1[t_0, T)}) = y_{a1[t_0, T)}$, $M_{a(x_{a0})}(v_{2[t_0, T)}) = y_{a2[t_0, T)}$, $y_{a1[t_0, T)} = y_{a2[t_0, T)}$. thus even if Π_p in (1) is invertible, these two distinct inputs $y_{a1} = y_{a2}$ on $[t_0, t_0 + \varepsilon)$ can only produce one output $M_{p(x_{p0})}(y_{a1[t_0, T)}) = M_{p(x_{p0})}(y_{a2[t_0, T)}) = y_{[t_0, T)}$. As a result, for the interconnected system, these two distinct inputs $v_1 \neq v_2$ on $[t_0, t_0 + \varepsilon)$ result in two equal outputs:

$$\begin{aligned} (M_a \circ M_p)_{(x_{a0}, x_{p0})}(v_{1[t_0, T)}) & \\ = (M_a \circ M_p)_{(x_{a0}, x_{p0})}(v_{2[t_0, T)}) &= y_{[t_0, T)} \end{aligned}$$

Again, it means that the interconnected system Π cannot be invertible at (x_{p0}, x_{a0}) over $(\mathcal{U}_a, \mathcal{Y})$. ■

4.3 Number of faults that can be identified simultaneously

A sufficient and necessary condition capable of verifying invertibility of nonlinear dynamic system based on differential output rank is given in Martínez-Guerra et al. (2013). A system is invertible under the condition that the differential output rank is equal to the number of inputs. In other words, this means that, in order to ensure invertibility of the system, the number of faults cannot be greater than the number of outputs in our case.

Remark 1: Iff the differential output rank ρ is equal with fault numbers, the actuator Π_a subsystem in fault mode is left invertible, e.g., $\rho = k$ in equation (3).

Remark 2: For the actuator subsystem Π_a in fault mode, if fault number is larger than available output number, its left invertibility cannot be ensured, e.g., $k > m$ in equation (3).

Remark 3: The controlled plant Π_p subsystem is said to be left invertible iff the differential output rank ρ is equal to input number, e.g., $\rho = m$ in equation (1).

Remark 4: For the controlled plant Π_p subsystem, if input number is larger than output number, it may not be left invertible, e.g., $m > n$ in equation (1).

Therefore, in order to ensure invertibility of the interconnected system depicted by equations (1) and (3), the number of fault k cannot be larger than the number of global output n .

4.4 The condition of one to one relationship between fault vector V and estimated \tilde{u}_a

In addition, one of the basic requirements for the combination of single actuator and advanced diagnosis capability is the feasibility of measurement of output y_a of the actuator subsystem and the reliability of the measurement value. Since for controlled plant subsystem, y_a is the input, thus this problem can be regarded as the problem of input reconstruction of controlled plant subsystem. It can be viewed as the problem of system invertibility. For the related concepts of invertibility and reconfigurability of the system, please refer to references like Hirschorn (1979) and Daoutidis and Kravaris (1991).

Definition 4 : If left invertibility of controlled plant subsystem (1) can be ensured, it can be obtained the input vector y_a via the output vector y , represented by \tilde{y}_a .

Definition 5: If left invertibility of actuator subsystem (3) in fault mode can be ensured, then it can be obtained the fault vector $V (v_i, i = 1, \dots, k)$ via the output vector y_a .

Theorem 3: If the reconstructed \tilde{y}_a is converged, then it can be obtained the fault vector $V (v_i, i = 1, \dots, k)$ via the estimated output vector \tilde{y}_a by substituting y_a with \tilde{y}_a .

Proposition 3: If the reconstructed \tilde{y}_a is converged, then it can be concluded that \tilde{y}_a and the fault vector $V (v_i, i = 1, \dots, k)$ has one to one correspondence relationship.

From above analysis, under condition that both the independent subsystem and the interconnected system are invertible, there is a one-to-one correspondence relationship between the reconstructed \tilde{y}_a and the fault vector V . In this case, the local fault filter strategy can perform on the actuator subsystem to recognise fault candidate V_i , so as to realise the root faulty parameter of the actuator fault.

5 Interconnected observer design and analysis

If it can be ensured that both individual subsystems and the interconnected system are invertible, the next step is then to propose an interconnected observer, aims at supervising the performance of the system both locally and globally. In addition, once an actuator fault is detected, the local fault filter is triggered to identify the failure parameter V_i in the actuator subsystem (3).

5.1 Interconnected observer design

According to Gauthier et al. (1992), one can transform the actuator subsystem in (3) into the form (4) without losing generality, as follows:

$$\begin{cases} \dot{\xi}_a = A_a \xi_a + \Omega_a (\xi_a, u) \\ y_a = C_a \xi_a \end{cases} \quad (4)$$

$$\text{Where } \xi_a^T = (\xi_{a1} \ \dots \ \xi_{an})^T, \quad A_a = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 \\ 0 & \dots & 0 & 0 \end{bmatrix},$$

$$\Omega (\xi_a, u) = \begin{bmatrix} \Omega_{a1} (\xi_{a1}, u) \\ \vdots \\ \Omega_{an} (\xi_{a1}, \dots, \xi_{an}, u) \end{bmatrix}, \quad C_a = [1 \ 0 \ \dots \ 0]$$

It can be seen from reference (Farza et al., 1998) that the system with structure (4) indicates the availability of designing a high gain observer as follows.

$$\begin{cases} \dot{\hat{\xi}}_a = A_a \hat{\xi}_a + \Omega_a (\hat{\xi}_a, u) - S_{\theta_a}^{-1} C_a^T (C_a \hat{\xi}_a - y_a) \\ 0 = \theta_a S_{\theta_a} + A_a^T S_{\theta_a} + S_{\theta_a} A_a - C_a^T C_a \end{cases} \quad (5)$$

Denote the state estimation error as $e_a := \xi_a - \hat{\xi}_a$. Then, the error dynamics equation (6) can be obtained by subtracting the corresponding terms of (4) and (5):

$$\begin{aligned} \dot{e}_a(t, e_a) &= A_a \xi_a + \Omega_a (\xi_a, u) - A_a \hat{\xi}_a - \Omega_a (\hat{\xi}_a, u) \\ &\quad - S_{\theta_a}^{-1} C_a^T (C_a \hat{\xi}_a - y_a) \end{aligned} \quad (6)$$

Denote $(u, \hat{\xi}_a, y_a) = S_{\theta_a}^{-1} C_a^T (C_a \hat{\xi}_a - y_a)$,

The observer (5) can be accomplished under the assumption that y_a is measurable. However, the actuator output y_a in our design is assumed to be inaccessible, and it needs to be replaced with its estimate \tilde{y}_a .

Assumed that estimation \tilde{y}_a is already obtained, by replacing y_a as \tilde{y}_a in equation (5), one can transform the observer for actuator subsystem as follows:

$$\begin{cases} \dot{\hat{\xi}}_a = A_a \hat{\xi}_a + \Omega_a(\hat{\chi}_a, u) - S_{\theta_a}^{-1} C_a^T (C_a \hat{\xi}_a - \tilde{y}_a) \\ 0 = \theta_a S_{\theta_a} + A_a^T S_{\theta_a} + S_{\theta_a} A_a - C_a^T C_a \end{cases} \quad (7)$$

denote $(u, \hat{\xi}_a, \tilde{y}_a) = k_a(\mathcal{G}_a, \hat{\xi}_a) \left(h_a(\hat{\xi}_a) - \tilde{y}_a \right)$.

By subtracting corresponding equation in (4) and (7), the new error dynamics is obtained as the following form:

$$\begin{aligned} \dot{e}_a(t, \tilde{e}_a) &= A_a \xi_a + \Omega_a(\xi_a, u) - A_a \hat{\xi}_a \\ &\quad - \Omega_a(\hat{\xi}_a, u) - (u, \hat{\xi}_a, \tilde{y}_a) \\ &= A_a \xi_a + \Omega_a(\xi_a, u) - A_a \hat{\xi}_a - \Omega_a(\hat{\xi}_a, u) - (u, \hat{\xi}_a, y_a) \\ &\quad + (u, \hat{\xi}_a, y_a) - (u, \hat{\xi}_a, \tilde{y}_a) \\ &= \dot{e}_a(t, e_a) + (u, \hat{\xi}_a, y_a) - (u, \hat{\xi}_a, \tilde{y}_a) \end{aligned} \quad (8)$$

From the error dynamics equation (8), if the sensitivity of $(u, \hat{\xi}_a, y_a)$ with respect to changes of y_a can be guaranteed, then exponential stability of (8) can also be ensured. Assumption 1 could provide a sufficient condition for achieving this purpose.

Assumption 1: For any $u \in \mathcal{U}, (t, \hat{\xi}_a, \tilde{y}_a) \in \mathcal{A.C}(\mathfrak{R}^+, \mathfrak{R})$, there can always find a real constant γ_1 satisfies that:

$$\left\| (u, \hat{\xi}_a, y_a) - (u, \hat{\xi}_a, \tilde{y}_a) \right\| \leq \gamma_1 \|y_a - \tilde{y}_a\|$$

According to Farza et al. (2012), Assumption 1 means that function is global Lipchitz. Thanks to physically bounded of u and y_a , the global Lipchitz condition can be replaced by the local smooth condition. And if Assumption 1 holds, $\dot{e}_a(t, \tilde{e}_a)$ in equation (8) will not be impacted.

In particularly, since \tilde{y}_a used in observer (7) is estimated by available measured output of the plant subsystem, another significant issue addressed in this design is to give the estimate for y_a in equation (1). For that, an observer is proposed in the following way.

Define $\xi_p = [\xi_{p1} \quad \xi_{p2}]^T = [x_p \quad u_a]^T$, then system (1) can be extended as:

$$\begin{cases} \dot{\xi}_{p1} = f_1(\xi_{p1}, u) + g_1(\xi_{p1}, u) \xi_{p2} \\ \dot{\xi}_{p2} = \omega(u, \dot{u}, \xi_a) \\ y = \xi_{p1} \end{cases} \quad (9)$$

Inspired by Farza et al. (2012), function $\omega(u, \dot{u}, \xi_a)$ is defined with regards to the time derivative of the output y_a in (3).

$$\omega(u, \dot{u}, \xi_a) = \frac{\partial h_a}{\partial u}(u, \xi_a) \dot{u} + \frac{\partial h_a}{\partial \xi_a}(u, \xi_a) f_a(u, \xi_a) \quad (10)$$

Assumption 2: For any $u \in \mathcal{U}, (t, \hat{\xi}_a) \in (\mathfrak{R}^+, \mathfrak{R})$, there can always find a real constant γ_2 satisfies:

$$\left\| \omega(u, \dot{u}, \hat{\xi}_a) - \omega(u, \dot{u}, \xi_a) \right\| \leq \gamma_2 \left\| \xi_a - \hat{\xi}_a \right\|$$

Similar to Assumption 1, Assumption 2 also means that function ω is of global Lipchitz-type, and since u, \dot{u}, ξ_a are physically bounded, the global Lipchitz condition can also be substituted by the local smooth condition.

To rewrite system (9) in the following condensed form:

$$\begin{cases} \dot{\xi}_p = l(\xi_{p1}) G(\xi_{p1}) \xi_p + \Theta(\xi_p, u) + \bar{\omega}(u, \dot{u}, \xi_a) \\ y = \xi_p \end{cases} \quad (11)$$

Where: $G(\chi_{p1}) = \begin{pmatrix} 0 & g_1(\xi_{p1}) \\ 0 & 0 \end{pmatrix}$, $\Theta(\chi_{p1}) = \begin{pmatrix} f_1(\xi_{p1}) \\ 0 \end{pmatrix}$,

$C(I_n \ 0), \bar{\omega}(u, \dot{u}, \xi_a) = [0 \ \omega(u, \dot{u}, \xi_a)]^T$, $l(\xi_{p1})$ is a scalar real function on their arguments, I_n is $n \times n$ identity matrix, $\alpha_l \leq |l(\xi_{p1})| \leq \beta_l$.

Then an extended high gain observer can be obtained according to Farza et al. (1998):

$$\begin{cases} \dot{\hat{\xi}}_p = l(\hat{\xi}_{p1}) G(\hat{\xi}_{p1}) \hat{\xi}_p + \Theta(\hat{\xi}_{p1}) + \bar{\omega}(u, \dot{u}, \hat{\xi}_a) \\ \quad + K(\hat{\xi}_{p1})(\hat{y} - y) \\ \hat{y} = C \hat{\xi}_p \end{cases} \quad (12)$$

Where: $K = [K_{\xi_{p1}} \quad K_{\xi_{p2}}]^T = \Lambda^{-1}(\hat{\xi}_{p1}) S_{\theta}^{-1} C^T$,

$\Lambda(\hat{\xi}_{p1}) = \begin{bmatrix} I & 0 \\ 0 & G_1(\hat{\xi}_{p1}) \end{bmatrix}$, S_{θ} is the unique symmetric positive definite matrix which is satisfying the algebraic Lyapunov equation as follows:

$$\theta S_{\theta} + \Xi^T S_{\theta} + S_{\theta} \Xi - C^T C = 0$$

Where $\Xi = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$, $\theta > 0$ is a parameter defined by equation (11).

Then, the gain of the observer can be determined as:

$$K = \Lambda^{-1}(\hat{\xi}_{p1}) S_{\theta}^{-1} C^T = \Lambda(\hat{\xi}_{p1}) \begin{bmatrix} 2\theta I \\ \theta^2 G_1^{-1}(\hat{\xi}_{p1}) \end{bmatrix}$$

Therefore, an interconnected observer is proposed which is composed by system (7) and (12):

$$\begin{cases} \dot{\hat{\xi}}_a = A_a \hat{\xi}_a + \Omega_a(\hat{\xi}_a, u) - S_{\theta}^{-1} C_a^T (C_a \hat{\xi}_a - \tilde{y}_a) \\ \dot{\hat{\xi}}_p = l(\hat{\xi}_{p1}) G(\hat{\xi}_{p1}) \hat{\xi}_p + \Theta(\hat{\xi}_{p1}) + \bar{\omega}(u, \dot{u}, \hat{\xi}_a) \\ \quad + K(\hat{\xi}_{p1})(\hat{y} - y) \end{cases} \quad (13)$$

5.2 Inverse observer design and fault detection

(a) Modelling of inverse system

With the help of rules of system inversion and input reconstruction, like in Edelmayer et al. (2004), a dynamic inverse system for the invertible system (1) can be constructed.

$$\begin{cases} \dot{\eta} = \bar{f}(\bar{\xi}, \eta) \\ y_a = H(\varphi^{-1}(\bar{\xi}, \eta))^{-1} \begin{bmatrix} \bar{\xi}_1^{(\varepsilon_1)} \\ \vdots \\ \bar{\xi}_m^{(\varepsilon_m)} \end{bmatrix} - \begin{bmatrix} L_f^{\varepsilon_1} h_1(\varphi^{-1}(\bar{\xi}, \eta)) \\ \vdots \\ L_f^{\varepsilon_m} h_m(\varphi^{-1}(\bar{\xi}, \eta)) \end{bmatrix} \end{cases} \quad (14)$$

(b) Inverse Observer design

To design observer for (14), the input and output of the observer is the measured y and the estimated \tilde{y}_a respectively. the reference output is \tilde{y}_a which is provided by the interconnected observer.

Denote estimation error as $e_0(t) = \hat{y}_a - \tilde{y}_a$, once the observer converges, it yields $\tilde{y}_a = \hat{y}_a = y_a$.

$$\begin{cases} \dot{\hat{\eta}} = \bar{f}(\bar{\xi}, \hat{\eta}) - k_{ob}(\hat{y}_a - \tilde{y}_a) \\ \tilde{y}_a = H(\varphi^{-1}(\bar{\xi}, \hat{\eta}))^{-1} \begin{bmatrix} \bar{\xi}_1^{(\varepsilon_1)} \\ \vdots \\ \bar{\xi}_m^{(\varepsilon_m)} \end{bmatrix} - \begin{bmatrix} L_f^{\varepsilon_1} h_1(\varphi^{-1}(\bar{\xi}, \hat{\eta})) \\ \vdots \\ L_f^{\varepsilon_m} h_m(\varphi^{-1}(\bar{\xi}, \hat{\eta})) \end{bmatrix} \end{cases} \quad (15)$$

k_{ob} is the gain of the observer.

(c) Residual generation

The observer error meets the following equation:

$$\begin{cases} \tilde{e}_a = \hat{\xi}_a - \xi_a \\ e_p = \hat{\xi}_p - \xi_p \end{cases} \quad (16)$$

Let $\gamma_i(t)$ represents fault detection residual:

$$\gamma_i(t) = d \|\tilde{e}_{ai}(t)\| / dt \quad (17)$$

Then, it can conclude:

$$\begin{cases} \gamma_i < \delta; \text{ fault free} \\ \gamma_i \geq \delta; \text{ exist fault} \end{cases} \quad (18)$$

where δ is the threshold that is prespecified.

6 Local fault filter design for root faulty parameter analysis

Once an actuator fault is diagnosed, the next job relates to determining the abnormal parameter caused the fault. In fact, this is to realise the occurrences of candidate v_i in equation (3) independently. Assuming that the task is implemented locally in the actuator subsystem where the output is not available for measurement, then the initial problem is to estimate the actuator system output y_a with the global output y of the system.

6.1 Input estimation

Input estimation procedure can be related to references like in Edelmayer et al. (2004). If the controlled plant subsystem shown in equation (1) is differential left invertible, then the input of the controlled plant subsystem can be reconstructed from the output y by finite ordinary differential equations.

By system inverse algorithm, a differential algebraic polynomial of the input vector y_a can be obtained firstly by means of the output vector y .

Given the finite relative order r_1, \dots, r_m for equation (1) with corresponding to the output y , by calculating their derivatives, one can obtain:

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(x_p) \\ \vdots \\ L_f^{r_m} h_m(x_p) \end{bmatrix} + \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x_p) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x_p) \\ \cdots & \cdots & \cdots \\ L_{g_1} L_f^{r_m-1} h_m(x_p) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x_p) \end{bmatrix} y_a \quad (19)$$

Let the matrix:

$$\dot{A}(x_p) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x_p) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x_p) \\ \cdots & \cdots & \cdots \\ L_{g_1} L_f^{r_m-1} h_m(x_p) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x_p) \end{bmatrix}$$

Define the following change of the coordinates:

$$\begin{aligned} \vartheta_i &= [\vartheta_i^1, \vartheta_i^2, \dots, \vartheta_i^{r_i}] = [\phi_i^1(x_p), \phi_i^2(x_p), \dots, \phi_i^{r_i}(x_p)] \\ &= [h_i(x_p), L_f h_i(x_p), \dots, L_f^{r_i-1} h_i(x_p)] \quad i=1, \dots, m \\ \vartheta &= [\vartheta_1, \vartheta_2, \dots, \vartheta_m] = [\phi_1(x_p), \phi_2(x_p), \dots, \phi_m(x_p)] \\ \zeta &= [\phi_{r+1}(x_p), \phi_{r+2}(x_p), \dots, \phi_n(x_p)] \\ y &= [\vartheta_1^1, \vartheta_2^1, \dots, \vartheta_m^1] \end{aligned}$$

Applying the new local coordinate introduced in Edelmayer et al. (2004), one can always find the functions $\phi_{r+1}(x_p), \phi_{r+2}(x_p), \dots, \phi_n(x_p)$, thus

$$\Phi(x_p) = [\phi_1(x_p), \phi_2(x_p), \dots, \phi_m(x_p), \phi_{r+1}(x_p), \dots, \phi_n(x_p)]$$

$$x_p = \Phi^{-1}(\vartheta, \zeta)$$

Then it is capable of obtaining input vector y_a with the output vector y and their derivatives.

$$y_a = \dot{A}(\Phi^{-1}(\vartheta, \zeta))^{-1} \left(\begin{bmatrix} \vartheta_1^{(r_1)} \\ \vdots \\ \vartheta_m^{(r_m)} \end{bmatrix} - \begin{bmatrix} L_f^{r_1} h_1(\Phi^{-1}(\vartheta, \zeta)) \\ \vdots \\ L_f^{r_m} h_m(\Phi^{-1}(\vartheta, \zeta)) \end{bmatrix} \right) \quad (20)$$

However, the algebraic polynomials (20) based on the inverse of the system needs to calculate the successive derivatives of the output, which may not be practical in the real applications. It is because measurements in real application often suffer to noise and interference.

A second-order high gain sliding mode observer is adopted to accurately estimate derivative information $\vartheta_i^j, 1 \leq i \leq m, 1 \leq j \leq r_i$ in equation (20) in a finite time. In this way, one can avoid directly using these derivative information.

By constructing:

$$\begin{aligned} y_i &= \vartheta_i^1 \\ \dot{\vartheta}_i^j &= \vartheta_i^{j+1}; \quad 1 \leq j \leq r_i - 1 \\ \dot{\vartheta}_i^{r_i} &= L_f^{r_i} h_i(\Phi^{-1}(\vartheta, \zeta)) + \sum_{j=1}^m L_{g_i} L_f^{r_i-1} h_i(\Phi^{-1}(\vartheta, \zeta)) y_{aj}; \quad j = r_i \end{aligned}$$

Following is structure of the observer:

$$\begin{aligned} \hat{y}_i &= \hat{\vartheta}_i^1 \\ \dot{\hat{\vartheta}}_i^j &= \hat{\vartheta}_i^{j+1} + \lambda_i^j |\hat{y}_i - y_i|^{1/2} \operatorname{sgn}(\hat{y}_i - y_i); \quad 1 \leq j \leq r_i - 1 \\ \dot{\hat{\vartheta}}_i^{r_i} &= \lambda_i^{r_i} |\hat{y}_i - y_i|^{1/2} \operatorname{sgn}(\hat{y}_i - y_i); \quad j = r_i \end{aligned}$$

An algebraic method of unknown input reconstruction is proposed with the help of the estimation of these derivatives.

$$\tilde{y}_a = \dot{A}(\Phi^{-1}(\hat{\vartheta}, \hat{\zeta}))^{-1} \left(\begin{bmatrix} \hat{\vartheta}_1^{(r_1)} \\ \vdots \\ \hat{\vartheta}_m^{(r_m)} \end{bmatrix} - \begin{bmatrix} L_f^{r_1} h_1(\Phi^{-1}(\hat{\vartheta}, \hat{\zeta})) \\ \vdots \\ L_f^{r_m} h_m(\Phi^{-1}(\hat{\vartheta}, \hat{\zeta})) \end{bmatrix} \right) \quad (21)$$

6.2 Design of local fault filter

Considering the actuator subsystem shown in equation (2) and fault model shown in equation (3), the root faulty parameter analysis of detected actuator fault, that is to recognise fault candidates $v_i, i=1, \dots, k$ independently, can be achieved by utilising the information of reconstructed \tilde{y}_a . In order to achieve this purpose, by adopting adaptive observer introduced in Li and Dahhou (2008), m banks observers are constructed and extended, each bank includes k observers and each observer is corresponding to a particular faulty parameter candidates:

$$\begin{cases} 1 \leq i \leq k, 1 \leq j \leq m, t \geq t_f \\ \dot{\hat{x}}_a^{ij} = f_a^j(\hat{x}_a^{ij}, u_j) + \sum_{l \neq i} g_{al}^j(\hat{x}_a^{ij}, u_j) \theta_l^j \\ \quad + g_{ai}^j(\hat{x}_a^{ij}, u_j) \hat{v}_i^j + H_{i_j}(\tilde{y}_a^j - \hat{y}_a^{ij}) \\ \hat{v}_i^j = 2\gamma_{i_j}(\tilde{y}_a^j - \hat{y}_a^{ij})^T P_{i_j} g_{ai}^j \\ \hat{y}_a^{ij} = h_a^j(\hat{x}_a^{ij}, u_j) \end{cases} \quad (22)$$

where j denotes the number of actuators, and i is related to the number of possible fault resource candidates. For the i th observer of the j th actuator, \hat{x}_a^{ij} is the estimated state, \hat{v}_i^j is the estimated fault, and \hat{y}_a^{ij} is the estimated output. \tilde{y}_a^j is reconstructed output. $\theta_l^j (l \neq i)$ refers to the nominal parameter value. H_{i_j} is a Hurwitz matrix, it should be chosen to increase the dynamic of the observer. The matrix P_{i_j} can be calculated with the help of equation (23); where Q_{i_j} is a positive definite matrix that can be freely selected.

$$H_{i_j}^T P_{i_j} + P_{i_j} H_{i_j} = -Q_{i_j} \quad (23)$$

Denote the tracking error $e_y^{ij}(t)$ as:

$$e_y^{ij}(t) = \tilde{y}_a^j - \hat{y}_a^{ij}$$

which corresponds to the i th observer for j th actuator.

The root faulty parameter residual is defined as:

$$s_{i_j}(t) = \|e_y^{ij}(t)\|, \quad 1 \leq i \leq k, 1 \leq j \leq m \quad (24)$$

The principle of the design of the residual is as follows: a specific residual is only 'insensitive' to a specific failure parameter of an actuator, and sensitive to all remaining failure parameter candidates. For example, consider the j th actuator, if the actuator fault is caused by the i th faulty parameter candidate, then the i th root faulty parameter residual will break its threshold and will not return, while the rest $(k - 1)$ residuals will remain below their thresholds.

In this way, root faulty parameter analysis of a detected actuator fault is achieved. The classes of faulty parameters under consideration are those directly affect the dynamics of the actuator, including both abrupt and incipient faults. In addition, according to real operation environment, a reasonable assumption is made, that is the number of faults occurred simultaneously will not more than the number of available measurement.

7 Numerical simulations

Numerical simulation results are presented in this section to validate the effectiveness and robustness of the proposed method. A case study is considered on a heat exchanger. Details of the heat exchanger could be referred to Raimondi et al. (2015). In general, the simulation aim is to verify that the internal dynamics of the actuator is available to be recognised by the outlet temperatures of the heat exchanger which is available for measurement.

7.1 Modelling of the interconnected system

1) Modelling of the actuator subsystem

The actuator in this system is considered as pneumatic control valve. By applying Bernoulli's law for continuous fluids that is incompressible, the fluid flow rate is obtained by:

$$F_c = \lambda_v f(O) \sqrt{\frac{\Delta P}{sg}}$$

where F_c , λ_v , O , sg denote the fluid flow rate, valve coefficient, the valve opening percentage and specific gravity respectively, ΔP is the fluid pressure drop across the valve, $f(O)$ represents the relationship between fluid travel through the valve and the valve capacity. In Han et al. (2018), a dynamic model of a pneumatic control valve is defined as:

$$p_c A_a = m \frac{d^2 O}{dt} + \mu \frac{dO}{dt} + kO$$

where p_c , A_a , m , μ and k are the pneumatic pressure, the diaphragm area, the stem mass, the friction, the spring compliance respectively.

As introduced in Bartyś et al. (2006), the actuator system model can be described as:

$$x_a^T = [x_{a1} \ x_{a2} \ x_{a3} \ x_{a4}] = \left[O_1 \ \frac{dO_1}{dt} \ O_2 \ \frac{dO_2}{dt} \right],$$

$$u^T = [u_1 \ u_2] = [p_{c1} \ p_{c2}],$$

$$u_a^T = [F_1 \ F_2] = \left[C_v \sqrt{\frac{\Delta P_1}{sg}} O_1 \ C_v \sqrt{\frac{\Delta P_2}{sg}} O_2 \right],$$

$$C = [c_1 \ c_2 \ c_3 \ c_4]$$

$$= \left[C_v \sqrt{\frac{\Delta P_1}{sg}} \ 0 \ C_v \sqrt{\frac{\Delta P_2}{sg}} \ 0 \right]$$

Then the actuator subsystem can be expressed in equation (25), which is composed of two inputs, four states and two outputs as:

$$\left\{ \begin{array}{l} \dot{x}_a = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m} & -\frac{\mu_1}{m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k_2}{m} & -\frac{\mu_2}{m} \end{bmatrix} x_a + \begin{bmatrix} \frac{A_a}{m} & 0 \\ 0 & 0 \\ 0 & \frac{A_a}{m} \\ 0 & 0 \end{bmatrix} u \\ u_a = \left[C_v \sqrt{\frac{\Delta P_1}{sg}} \ 0 \ C_v \sqrt{\frac{\Delta P_2}{sg}} \ 0 \right] x_a \end{array} \right. \quad (25)$$

In this paper, four kinds of faults affecting the dynamic characteristics of valves are considered. Fault $f1$ is related to unexpected changes of μ , the reasons that can cause the change may due to valve clogging. Fault $f2$ represents pressure drop change, and it results in changes of ΔP . Fault $f3$ may be resulted by bellow-seal leakage due to leak, it can be expressed as changes $p_c A_a$. Fault $f4$ refers to perforation of valve diaphragm, and resulting in change of k .

According to the above description, internal parameter $\mu, k, u, \Delta p$ of the control valve are the root causes of an actuator fault. When comes back to the two fluid flowrate, there are then eight related parameters. The process of root faulty parameter analysis is in fact to recognise abnormal changes of these parameters. For this particular purpose, the above actuator model in fault model is rewritten in a failure affine format as follows:

$$x_a^T = [x_{a1} \ x_{a2} \ x_{a3} \ x_{a4} \ x_{a5} \ x_{a6}] \\ = \left[O_1 \ \frac{dO_1}{dt} \ O_2 \ \frac{dO_2}{dt} \ C_v \sqrt{\frac{\Delta P_1}{sg}} O_1 \ C_v \sqrt{\frac{\Delta P_2}{sg}} O_2 \right]$$

$$V = [v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6 \ v_7 \ v_8]^T \\ = [k_1 \ \mu_1 \ k_2 \ \mu_2 \ p_{c1} \ p_{c2} \ \Delta P_1 \ \Delta P_2]^T$$

$$u_a^T = [u_{a1} \ u_{a2}] = [F_1 \ F_2] = [x_{a5} \ x_{a6}]$$

$$= \left[C_v \sqrt{\frac{\Delta P_1}{sg}} O_1 \ C_v \sqrt{\frac{\Delta P_2}{sg}} O_2 \right]$$

$$\tilde{C} = [0 \ 0 \ 0 \ 0 \ 1 \ 1]$$

As a result, the actuator subsystem has eight unknown inputs, six states and two outputs. Then the transformed form of the actuator subsystem is:

$$\begin{cases} \dot{x}_a = f_a(x_a) + \sum_{i=1}^8 g_{ai}(x_a)v_i \\ u_a = \tilde{C}x_a \end{cases} \quad (26)$$

where $f_a(x_a) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$,

$$g_a(x_a) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{x_{a1}}{m} & -\frac{x_{a2}}{m} & 0 & 0 & \frac{A_a}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{x_{a3}}{m} & -\frac{x_{a4}}{m} & 0 & \frac{A_a}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & C_v \frac{1}{sg} x_{a1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_v \frac{1}{sg} x_{a2} \end{bmatrix}$$

2) Modelling of the controlled Plant subsystem

The heat exchanger subsystem can be modelled based on the mass and energy balances. The dynamic equation governing the heat balance of the fluids are given by:

$$\dot{T}_p = \frac{UA}{\rho_p c_{pp} V_p} (T_u - T_p) + \frac{1}{V_p} (T_{pi} - T_p) F_p$$

$$\dot{T}_u = \frac{UA}{\rho_u c_{pu} V_u} (T_p - T_u) + \frac{1}{V_u} (T_{ui} - T_u) F_u$$

where ρ_p, ρ_u are density of the fluids (in $\text{kg}\cdot\text{m}^{-3}$), V_p, V_u are volume of fluids (in m^3), c_{pp}, c_{pu} are specific heat of the fluids (in $\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$), A is the reaction area (in m^2), U is the overall heat transfer coefficient (in $\text{J}\cdot\text{m}^{-2}\cdot\text{K}^{-1}\cdot\text{s}^{-1}$). F_p, F_u are mass flowrate of the fluids (in $\text{kg}\cdot\text{s}^{-1}$). T_{pi}, T_{ui} are the inlet temperature of the fluids.

Define the state vector as $x^T = [x_1, x_2]^T = [T_p, T_u]^T$, the control input $u_a^T = [u_{a1}, u_{a2}]^T = [F_p, F_u]^T$, the output vector as $y^T = [y_1, y_2]^T = [T_p, T_u]^T$, then the controlled plant subsystem can be described by the following state-space form:

$$\begin{cases} \dot{x} = f(x) + \sum_{i=1}^2 g_i(x)u_a \\ y = h(x, u_a) \end{cases} \quad (27)$$

where

$$f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} \frac{h_p A}{\rho_p C_{pp} V_p} (T_p - T_u) \\ \frac{h_u A}{\rho_u C_{pu} V_u} (T_u - T_p) \end{pmatrix}, \text{ and}$$

$$g(x) = (g_1, g_2) = \begin{pmatrix} \frac{(T_{pi} - T_p)}{V_p} & 0 \\ 0 & \frac{(T_{ui} - T_u)}{V_u} \end{pmatrix},$$

$y_1 = x_1, y_2 = x_2, T_{pi}, T_{ui}$ are the inlet temperature of process and utility fluid respectively.

By using (10), the derivatives function of u_a can be obtained:

$$\begin{aligned} \dot{u}_a &= \varepsilon(u, \dot{u}, x_a) \\ &= \frac{\partial h_a}{\partial u}(u, x_a) \dot{u} + \frac{\partial h_a}{\partial x_a}(u, x_a) f_a(u, x_a) \\ &= \left(C_v \sqrt{\frac{\Delta P_1}{sg}} \quad 0 \quad C_v \sqrt{\frac{\Delta P_2}{sg}} \quad 0 \right) x_a \\ &\quad + \left(\frac{A_a}{m} C_v \sqrt{\frac{\Delta P_1}{sg}} \quad \frac{A_a}{m} C_v \sqrt{\frac{\Delta P_2}{sg}} \right) u \end{aligned} \quad (28)$$

Define the state vector as $x_1^T = [x_{11}, x_{12}]^T = [T_p, T_u]^T$, unmeasured state as $x_2^T = [x_{21}, x_{22}]^T = [u_{a1}, u_{a2}]^T = [F_p, F_u]^T$, the measured output vector as $y^T = [y_1, y_2]^T = [T_p, T_u]^T$, then the system (27) can be rewritten in the following form:

$$\begin{cases} \dot{x}_1 = G_1(x_1)x_2 + g_1(x_1, u) \\ \dot{x}_2 = \varepsilon(u, \dot{u}, x_a) \\ y = x_1 \end{cases} \quad (29)$$

where,

$$G_1(x_1) = \begin{pmatrix} \frac{(T_{pi} - x_{11})}{V_p} & 0 \\ 0 & \frac{(T_{ui} - x_{12})}{V_u} \end{pmatrix}, \text{ and}$$

$$f_1(x) = \begin{pmatrix} \frac{h_p A}{\rho_p C_{pp} V_p} (x_{11} - x_{12}) \\ \frac{h_u A}{\rho_u C_{pu} V_u} (x_{12} - x_{11}) \end{pmatrix}.$$

7.2 Simulation results and discussion

The simulation results verifies the effectiveness and robustness of the proposed strategy. Observers in (13) and (15) are designed to estimate unmeasured fluid flowrates F_p, F_u as well as monitor performance of final product T_p, T_u fault detection residuals are generated by (18) to alert existence of actuator faults, and root faulty parameter analysis observers (22) are formed to recognised fault candidates f1-f4. The available measurements include the inlet pneumatic pressure p_{c1}, p_{c2} of the actuators and the inlet and outlet temperatures of the heat exchanger.

The operating conditions of the simulation are as follows. Details can be found in Raimondi et al. (2015). The inlet temperatures T_{pi} is 76°C and T_{ui} is 15°C respectively. The flow rate of the utility fluid F_u and process fluid F_p are $4.22e^{-5}m^3s^{-1}$, and $4.17e^{-6}m^3s^{-1}$ respectively. Initial condition for observers are supposed to be 0.

As described in Bartyś et al. (2006), in the real operation, for a specific actuator, the most possibility is that only one parameter fault may occur at one time, while it is very rare that multiple parameters fall in fault simultaneously. Therefore, it is considered that each actuator will suffer only one parameter fault at the same time, and two parameters may fail simultaneously in the studied system. In addition, in order to prove the robustness, it is achieved by considering external interference or measurement noise in the simulation.

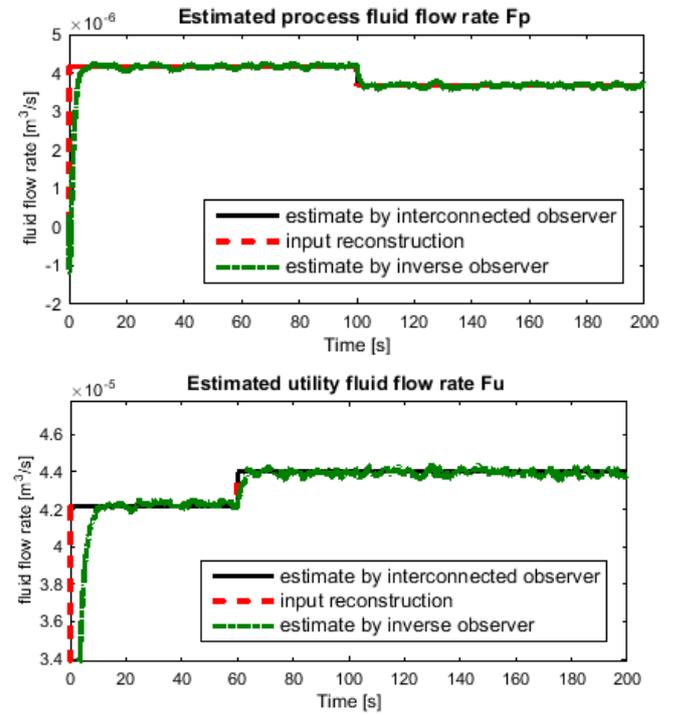
Two parameter faults are considered. Each actuator suffers one. The actuator of process fluid subjects to a fault caused by fault candidate $f2$ at $t = 100s$, ΔP drops 0.2 MPa. The actuator of utility fluid fails due to the influence of fault parameter candidate $F4$ at 60s, the value of k_2 increases 1000 nm^{-1} .

Simulation results are demonstrated in Figures 4–8. Fault detection residual $r = [r_1 \ r_2]$ are generated using residual detection rule in equation (18). r_1 indicates fault occurrences in F_p , and r_2 implies fault of F_u . In order to identify root faulty parameters, the local fault filters will generate two banks of four residuals, aims at identifying the root faulty parameter candidates. One bank of residuals $s_{11}, s_{12}, s_{13}, s_{14}$ are with the purpose to identify fault candidates $f1-f4$ in F_p . while with the help of the other bank of residuals $s_{21}, s_{22}, s_{23}, s_{24}$, the recognition of fault parameter candidates $f1-f4$ in F_u can be achieved. If any of the residual exceeds its threshold, it can be concluded that the fault is caused by the corresponding faulty parameter.

It can be seen from Figure 4 that for the process fluid flow rate, the estimation \hat{F}_p in solid line by interconnected observer, dash line via input construction rule and dash dot line by inverse observer converge after short transient response. After that, at 100s, an unexpected decrease happens, fortunately, three estimated values give a quick

response to the variation, while compares to other two curves, curve of inverse observer takes more time to converge. The sudden falls implies abnormal parameter changes of process fluid actuator, as a result, it satisfies the assumption that k_2 changes at $t = 100s$. For the utility fluid, the same results are obtained, at time 60s, as expected, all the three curves jump due to the change of ΔP_1 . The estimated \hat{F}_u in solid line by interconnected observer and dash dot line by input reconstruction rule well simulated expected F_u . Just as the situation in process fluid, the convergence speed of inverse observer is slower. All though measurements are corrupted by noise, the curves are not obviously influenced.

Figure 4 Estimated fluids flow rate F_p, F_u (see online version for colours)



As shown in Figure 5, detection residual r_1 means that a fault is occurred in F_p at 100s, and it takes 1.1s to detect the fault. At time 60s, fault detection residual r_2 break through its threshold, it implies a fault in F_u . By adopting larger gain or smaller threshold, one can obtain smaller detection time and can detect smaller faults. However, a large gain or a small threshold may cause the fault is unable to be diagnosed correctly. This is because the observer with a large gain may be too sensitive to noise, while a small threshold may cause the noise to be unable to be well distinguished. Therefore, in order to detect faults correctly, a trade-off must be made between detectability and sensitivity. In summary, in the presence of noise, it may not be possible to detect small faults. Thirdly, after detecting the fault, we must find out the root causes.

Figure 5 Detection residual

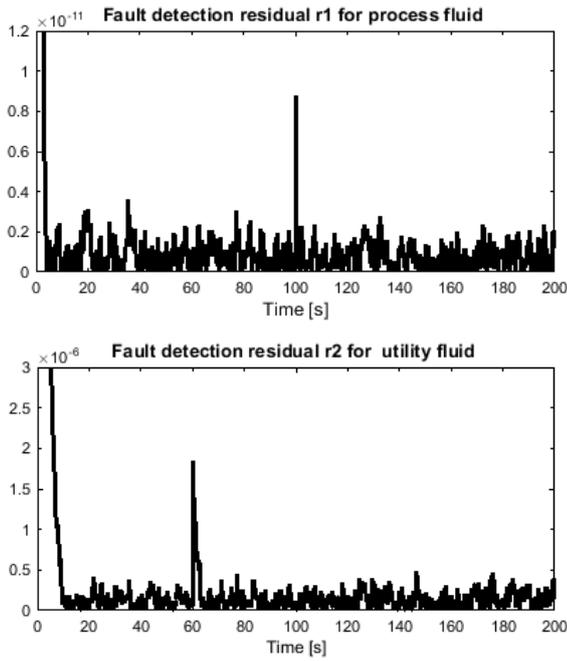
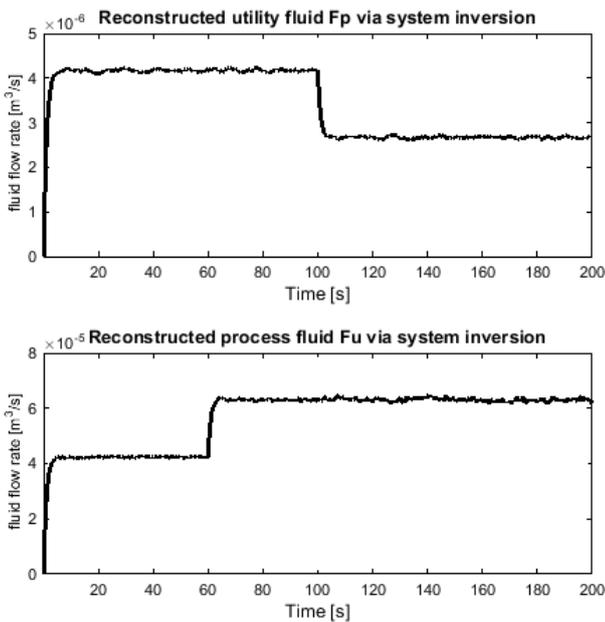
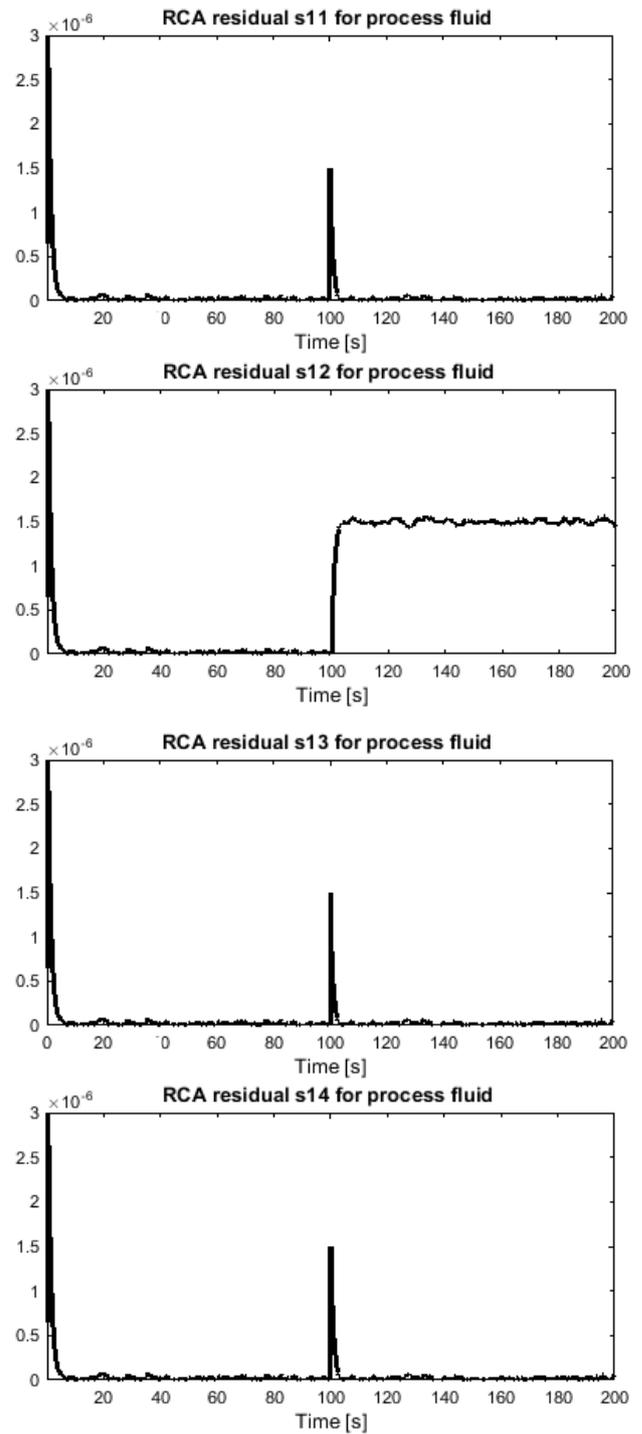


Figure 6 Reconstructed input \tilde{F}_u, \tilde{F}_p from output T_p, T_u

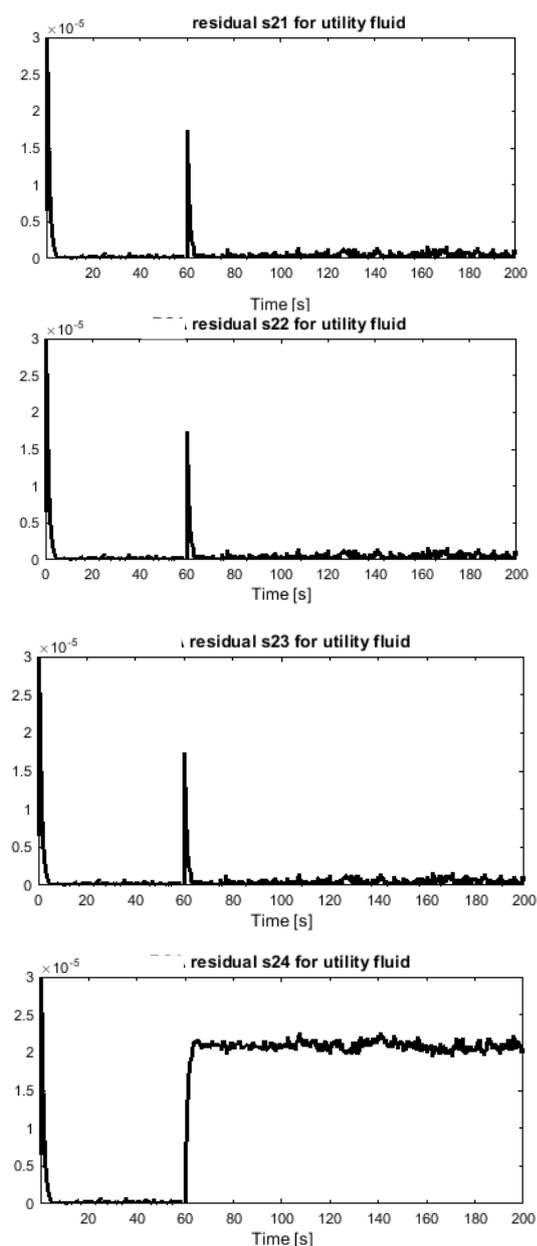


As illustrated in Figure 6, the proposed input reconstruction scheme can provide information of the fluid flowrates with a good accuracy even if there is noise existed. For actuator of process fluid F_p , it suddenly decreases at 100s and takes about 4s to be stable at a new level. It implies occurrence of a fault. When it comes to F_u , the reconstructed \tilde{F}_u grows at 60s, and steadies again after about 3s. It is then indicated a fault. These reconstructed values are then fed to local fault filter to identify the root causing parameters of these two faults.

Figure 7 Residuals for recognising root faulty parameters in process fluid



It is shown in Figure 7 that only root faulty parameter residual s_{12} breaks through its threshold, while the remaining three root faulty parameter residuals are not, then it can be concluded that the fault candidate f_2 of actuator of F_p is recognised. Similarly, it can be seen from Figure 8, for actuator of F_u , only root faulty parameter residuals s_{24} is exceeded its threshold, it implies the occurrence of fault parameter f_4 which verifies the assumption.

Figure 8 Residuals for recognising root faulty parameters in utility fluid

With respect to the simulation results, it can be concluded that the proposed actuator FDD strategy can detect and isolate faulty actuator correctly, as well as analyse the root abnormal parameter of each detected actuator fault. The simulation results verify the accuracy and robustness of the scheme.

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