# Incentive Mechanisms For Cooperation In Delay Tolerant Networks

Thi Thu Hang Nguyen

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**Thi Thu Hang NGUYEN**

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# Les mécanismes d'incitation à la coopération dans les réseaux tolérants aux délais

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Thèse dirigée par
**Olivier BRUN et Balakrishna PRABHU**

Jury
**Mme Sandrine VATON,** Rapporteur
**M. Rachid EL AZOUZI,** Rapporteur
**M. Cyril BRIAND,** Examinateur
**Mme Jocelyne ELIAS,** Examinateur
**M. Olivier BRUN,** Directeur de thèse
**M. Balakrishna PRABHU,** Co-directeur de thèse

# Abstract

Delay-Tolerant Networks (DTNs) were designed to provide a sustainable means of communication between mobile terminals in regions without cellular infrastructure. In such networks, the set of neighbors of every node changes over time due to the mobility of nodes, resulting in intermittent connectivity and unstable routes in the network. We analyze the performance of an incentive scheme for two-hop DTNs in which a backlogged source proposes a fixed reward to the relays to deliver a message. Only one message at a time is proposed by the source. For a given message, only the first relay to deliver it gets the reward corresponding to this message thereby inducing a competition between the relays. The relays seek to maximize the expected reward for each message whereas the objective of the source is to satisfy a given constraint on the probability of message delivery. We consider two different settings: one in which the source tells the relays for how long a message is in circulation, and one in which the source does not give this information.

In the first setting, we show that the optimal policy of a relay is of threshold type: it accepts a message until a first threshold and then keeps the message until it either meets the destination or reaches the second threshold. Formulas for computing the thresholds as well as probability of message delivery are derived for a backlogged source. We then investigate the asymptotic performance of this setting in the mean field limit. When the second threshold in infinite, we give the mean-field ODE and show that all the messages have the same probability of successful delivery. When the second threshold is finite we only give an ODE approximation since in this case the dynamics are not Markovian.

For the second setting, we assume that the source proposes each message for a fixed period of time and that a relay decides to accept a message according to a randomized policy upon encounter with the source. If it accepts the message, a relay keeps it until it reaches the destination. We establish under which condition the acceptance probability of the relays is strictly positive and show that, under this condition, there exists a unique symmetric Nash equilibrium, in which no relay has anything to gain by unilaterally changing

its acceptance probability. Explicit expressions for the probability of message delivery and the mean time to deliver a message at the symmetric Nash equilibrium are derived, as well as an expression of the asymptotic value of message delivery.

Finally, we present numerous simulations results to compare performances of the threshold-type strategy and the randomized strategy, in order to determine under which condition it is profitable for the source to give the information on the age of a message to the relays.

# Résumé en français

Les réseaux tolérants aux retards (DTN) ont été conçus pour fournir un moyen de communication durable entre terminaux mobiles dans les régions dépourvues d'infrastructure cellulaire. Dans de tels réseaux, l'ensemble des voisins de chaque nœud change au fil du temps en raison de la mobilité des nœuds, ce qui entraîne une connectivité intermittente et des routes instables dans le réseau. Nous analysons la performance d'un système d'incitation pour les DTN à deux sauts dans lequel une source en arriéré offre une récompense fixe aux relais pour délivrer un message. Un seul message à la fois est proposé par la source. Pour un message donné, seul le premier relais à le délivrer reçoit la récompense correspondant à ce message, induisant ainsi une compétition entre les relais. Les relais cherchent à maximiser la récompense attendue pour chaque message alors que l'objectif de la source est de satisfaire une contrainte donnée sur la probabilité de livraison du message. Nous considérons deux réglages différents : l'un dans lequel la source indique aux relais pendant combien de temps un message est en circulation, et l'autre dans lequel la source ne donne pas cette information.

Dans le premier paramètre, nous montrons que la politique optimale d'un relais est de type seuil : il accepte un message jusqu'à un premier seuil et le conserve jusqu'à ce qu'il atteigne la destination ou le deuxième seuil. Les formules de calcul des seuils ainsi que de la probabilité de livraison des messages sont dérivées pour une source d'arriérés. Nous étudions ensuite la performance asymptotique de ce réglage dans la limite moyenne du champ. Lorsque le deuxième seuil est infini, nous donnons l'ODE du champ moyen et montrons que tous les messages ont la même probabilité de réussite. Lorsque le deuxième seuil est fini, nous ne donnons qu'une approximation ODE car dans ce cas, la dynamique n'est pas markovienne.

Pour le second réglage, nous supposons que la source propose chaque message pour une période de temps fixe et qu'un relais décide d'accepter un message selon une politique randomisée lors d'une rencontre avec la source. S'il accepte le message, un relais le garde jusqu'à ce qu'il atteigne la destination. Nous établissons dans quelle condition la probabilité d'acceptation des

relais est strictement positive et montrons que, dans cette condition, il existe un équilibre de Nash symétrique unique, dans lequel aucun relais n'a quelque chose à gagner en changeant unilatéralement sa probabilité d'acceptation. Des expressions explicites pour la probabilité de livraison du message et le temps moyen de livraison d'un message à l'équilibre symétrique de Nash sont dérivées, ainsi qu'une expression de la valeur asymptotique de la livraison du message.

Enfin, nous présentons de nombreux résultats de simulations pour comparer les performances de la stratégie de type seuil et de la stratégie randomisée, afin de déterminer dans quelle condition il est rentable pour la source de donner l'information sur l'âge d'un message aux relais.

# Contents

# Chapter 1

# INTRODUCTION

Delay-tolerant networking was designed to enable communications over long distances in environnements with long delays and frequent disconnections. Initially conceived in the 2000s by the NASA for deep-space communications [36], this approach has found many other applications since then, in particular for communication in sparsely populated regions without pre-existing telecommunication infrastructure. Due to the lack of end-to-end connectivity, nodes have to cooperate for message delivery in a delay-tolerant network (DTN). An implicit assumption here is that they are willing to help each other for packet forwarding. Although this assumption is arguably legitimate in certain cases, it is more debatable for some other applications in which the selfish behavior of some nodes may significantly degrade the network performance. Many incentive schemes have been proposed to convince nodes to cooperate in DTNs. In this thesis, we propose reward-based incentive mechanisms in which only the first relay to deliver the message to the destination will get the reward and analyze their performances.

In this chapter, we first introduce the technological background on Mobile Ad-hoc Networks (MANET) in Section 1.1. We then present the technologies used in Delay Tolerant Networks in Section 1.2, explaining why the traditional solutions used in MANETs cannot be applied. In Section 1.2, we also present some of the applications of DTNs. Section 1.3 presents the three main categories of incentive mechanisms that have been proposed for DTNs – reputation-based, barter-based and reward-based mechanisms, discussing in greater details the latter category. Section 1.4 describes the contributions of our research work. Finally, Section 1.5 describes the organization of this manuscript.

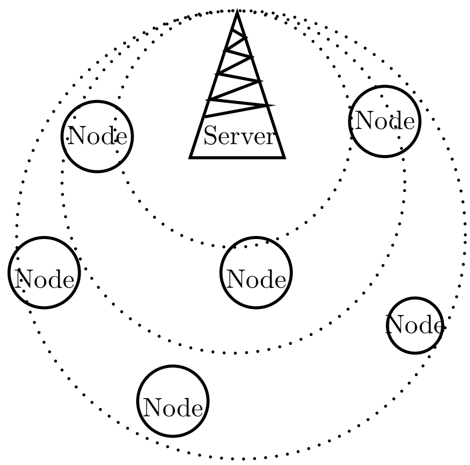



Figure 1.1: In an infrastructure network, nodes communicate through servers, base station or access points.

## 1.1 Mobile Ad-hoc Networks

Traditional networks, such as the Internet, public switched telephone networks or cellular networks are infrastructure-based networks in which hardware and software resources are used to enable network connectivity and communication between users. For instance, as shown in Fig. 1.1, in traditional mobile wireless networks, base stations, access points and servers have to be deployed before the networks can be used.

In contrast, as shown in Fig. 1.2, a mobile ad hoc network (MANET) is a network in which mobile devices can communicate wirelessly without any pre-existing infrastructure [73]. Obviously, this implies that all network activities such as topology discovery and packet delivery are executed by the nodes themselves. In particular, in addition to sending and receiving packets, each node of a MANET has to be able to play the role of a router for relaying packets to their destination or to the next router in the route.

Since they are infrastructure-less networks, MANETs are more easily deployable and more robust to failures than traditional networks. In comparison to an infrastructure network, a MANET can be deployed at low cost and nodes can be quickly added or removed from the network. These advantages make MANETs particularly appealing for environmental monitoring, disaster relief and military communications, among others. However, since mobile devices are often heterogeneous [18], MANETs may contain asymmetric and low capacity links. Moreover, frequent topology updates can lead to a sig-

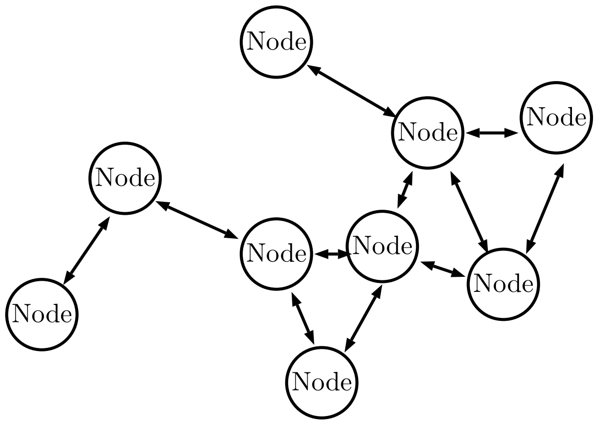



Figure 1.2: A Mobile Ad-hoc Network (MANET) is a self-configuring network of mobile wireless nodes which communicate with each others without any pre-existing infrastructure.

nificant waste of energy for MANET nodes, a critical issue for hand-held battery-powered devices.

A fundamental characteristic of MANETs is that they are self-configuring networks which are able to adapt on the fly to the unpredictable movement of nodes. Since the network topology can change rapidly over time, specific routing protocols have to be used to find alternative paths when a route is no longer valid. Two different types of routing protocols are used in MANETs, as described below (see [39] for a more detailed description).

## Proactive routing protocols

With proactive routing protocols, the list of nodes in the network and the route to them is maintained by periodically distributing routing tables throughout the network. The main advantage is that when a packets needs to be forwarded, the route is already known. Therefore, proactive routing protocols have lower latency than reactive protocols. The downside is that they can result in much greater overhead due to frequent route updates. Note also that proactive protocols consume network bandwidth and node energy for maintaining routes which are never or seldom used. As a consequence, proactive routing protocols are privileged in scenarios with high traffic and low mobility where a low network delay is required. Two well-known examples of proactive routing protocols are OLSR (Optimized Link State Routing) [37], which is a link-state routing protocol, and DSDV (Destination-sequenced Distance

Vector) [47], which is a distance-vector routing protocol.

### Reactive Routing Protocols

As their name suggests, reactive routing protocols, also known as on-demand routing protocols, find a route to a destination only when a message has to be sent to that destination. The route is found by flooding the network with *Route Request* packets. With respect to proactive protocols, on-demand routing protocols have a lower overhead but require a higher latency since routes have to be discovered before sending data. These protocols are therefore mainly used in scenarios with low traffic and high mobility where network delay is not the main concern. Some examples of reactive routing protocols are: DSR (Dynamic Source Routing) [20], AODV (Ad-hoc On-Demand Distance Vector) [54] and TORA (Temporally Ordered Routing Algorithm) [52].

## 1.2 Delay Tolerant Networks

As described in Section 1.1, the main concern in MANETs is to maintain up-to-date routes on all mobile nodes in spite of constant topological changes. Nevertheless, MANET routing protocols assume the existence of an end-to-end path between source and destination pairs, although this path may change over time. These protocols fail to deliver packets if such an end-to-end-path does not exist. In other words, they were not designed to cope with network partitioning.

In delay-tolerant networks, also known as disruption-tolerant networks [27], network partitioning is the rule. Although DTNs are composed of mobile wireless ad-hoc node as are MANETs, the network is most of the time not connected, isolated nodes are common and communication opportunities are short and sporadic. In short, the technology of DTNs has been designed to support communications in environments where end-to-end paths between a source and a destination are only rarely available. It is in particular used to enable the communication between mobile nodes scattered in outermost and sparsely populated regions (see Fig. 1.3). Another natural application scenario is disaster relief in situations where traditional communication infrastructure is incapacitated for a long time [68, 69]. We note however that the use of delay-tolerant networking has been considered for providing an alternative network for smart cities [28, 29] and the Internet of Things [7].

In essence, whereas in MANETs the focus is on keeping up with constant topological changes, the focus in DTNs is on being able to deliver packets to their destination with only limited information about the network. In this

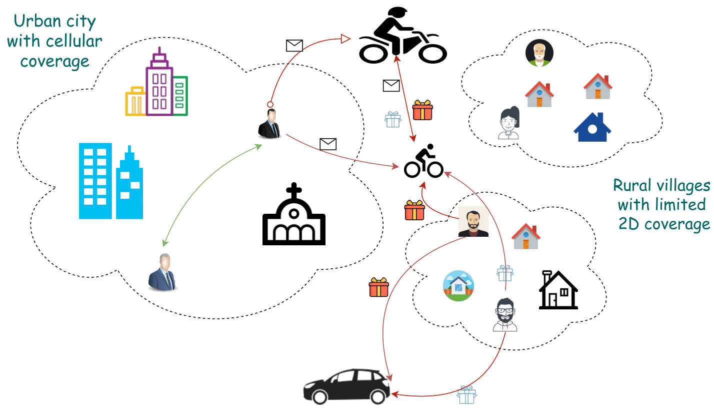



Figure 1.3: Communication problem solved by using DTN.

section, we present the solutions developed for achieving this goal. There are two main ingredients: the first one is the store-carry-and-forward paradigm and the second one is the routing protocol. The store-carry-and-forward paradigm is described in Section 1.2.1. Section 1.2.2 is devoted to DTN routing protocols. Finally, we describe some of the applications of DTN technologies in Section 1.2.3.

### 1.2.1 The store-carry-and-forward paradigm

In a DTN, when a source node wants to transmit a message to a destination node, it can not use the popular ad-hoc routing protocols due to the lack of an end-to-end path. Instead, the approach used in DTN is based on the so-called *store-carry-and-forward* paradigm (see Fig. 1.4). In this approach, the source node transmits its message to some or all the mobile node that it meets. The latter nodes play the role of relays. They store the message and carry it, in the hope that they will eventually reach the destination and be able to deliver the message. Of course, a source can also play the role of a relay and vice versa.

### 1.2.2 Routing protocols in DTNs

We can distinguish two broad routing strategies in DTNs: multi-copy and single copy strategies. We briefly discuss each type of strategy below and refer to Chapter 2 of Ph.D. thesis [15] for a more detailed description of the different routing schemes used in DTNs.

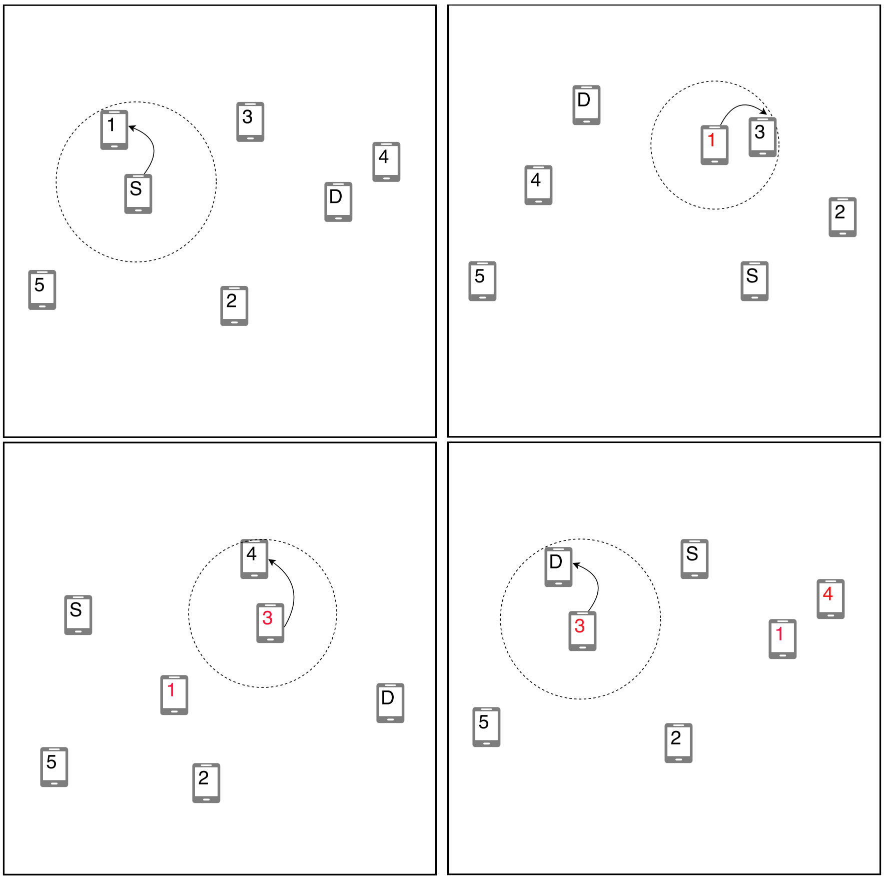



Figure 1.4: DTNs use the store-carry-and-forward paradigm. In this example, node $S$ wants to send a message to node $D$. It forwards its message to the nodes that it meets and hopes that one of them will reach the destination. Nodes in red are the ones having the message. In this example, node 1 meets the source first and receive the message, then it meets node 3 and forward the message to it. Node 3 does the same thing with node 4. Finally, node 3 meets the destination and delivers the message.

Single-copy strategies, also known as forwarding-based strategies, use some heuristic to determine if a node that has come into contact may be part of the path leading to the destination. In these strategies, a single copy of a message exists in the network at any given time since a relay that forwards the message to another relay or to the destination does not keep a copy of the message. Hence, when the message reaches the destination, it is completely deleted from the network. The heuristic used to determine whether the message should be forwarded to a node usually exploits information about future contacts. In some applications, such as for instance the Interplanetary Internet, this information if readily available and can be used to reduce the overhead with respect to replication-based strategies as well as for saving the network's energy. However, it is generally considered that the forwarding-based strategy does not provide a sufficient delivery ratio in applications where future contacts are unpredictable, which is the case of most DTN applications. Examples of DTN routing protocols implementing this strategy are DTLSR (Delay-Tolerant Link State Routing) [21] and Contact Graph Routing [4].

In multi-copy strategies, also know as replication-based strategies, the networks is flooded with copies of the same message in hopes that one of the copies will eventually reach the destination. In these strategies, when a relay forwards the message to another relay, it also keeps a copy for itself, hence increasing the probability that at least one copy reaches the destination. In contrast to forwarding-based strategies, there can be multiple copies of the message the message in circulation when a first copy reaches the destination. This approach usually achieves a higher delivery ratio, but at the expense of a significant waste of resources, and in particular energy. Some well-known replication-based strategies are the followings;

- *Epidemic routing*: in Epidemic routing, relays continuously replicate and transmit messages to newly discovered contacts that do not already possess a copy of the message [78, 35, 71, 45, 81, 40]. The advantage is that it can be guaranteed with high probability that some copy will reach the destination, and with a minimum delivery delay; but the downside is that it floods the network with message copies, leading to a significant energy consumption. The principle of this approach is illustrated in Fig. 1.5.

- *PRoPHET*: The Probabilistic Routing Protocol using History of Encounters and Transitivity (PRoPHET) protocol is a variant of Epidemic routing which exploits the fact that in real-world scenarios contacts between nodes are not purely random [22]. Indeed, in practice there are affinity relations between users, and therefore some users are more

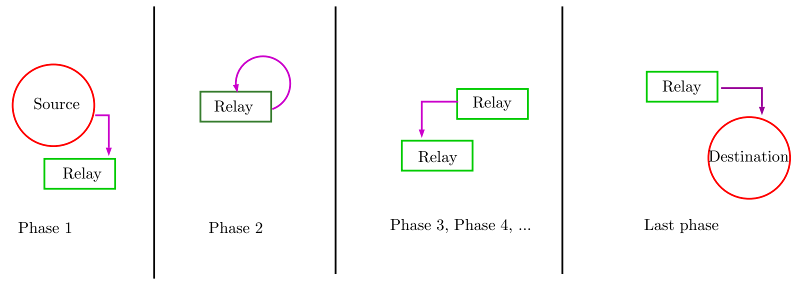



Figure 1.5: In multi-hop network, a node who has the message can forward it to other nodes who have not yet received message from the source or from other nodes.

likely to meet a given destination than others. For instance, if Alice is a friend of Bob, it can be expected that she is more likely to meet him than is Eve who does not know Bob. This observation is exploited by PRoPHET as follows. First, an adaptive algorithm is used to maintain a set of probabilities for successful delivery to known destinations in the DTN. When it meets another node who has not yet received a copy of the message, a relay forwards a copy of the message to it if and only if this node has a greater probability of delivering it.

- *Two-hop routing*: in this protocol, a relay cannot forward the message to another relay, so it stores and carries the message until it is in radio range of the destination. This protocol was shown in [1] to provide a good tradeoff between message delivery time and energy consumption, and has been considered in several recent studies [24, 23, 25, 2, 62, 61]. With respect to Epidemic routing, the number of message copies in circulation in the network is reduced and can be easily controlled by the source. Two-hop routing is illustrated in Fig. 1.6.

In this thesis, we shall only consider Delay Tolerant Networks in which the two-hop routing strategy is used.

### 1.2.3 Applications of Delay Tolerant Networking

As mentioned in the introduction, Delay Tolerant Networking was first developed to support the Interplanetary Internet [36]. Currently, the NASA uses the Delay Tolerant Networking technology as a solution to reliable internetworking for space missions, in which extreme distances and frequent disruptions are commonplace. However, this technology is not limited to

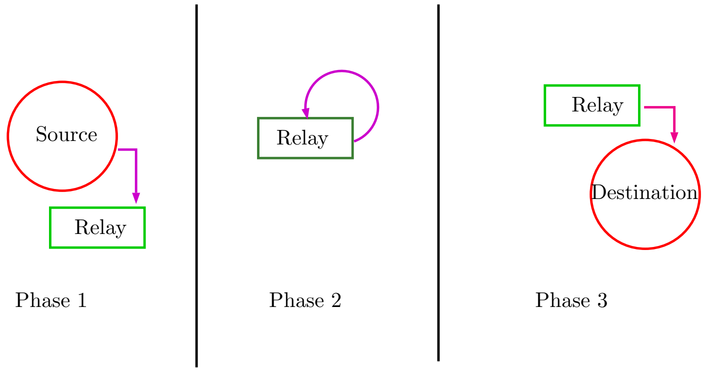



Figure 1.6: In two-hop network, we do not allow a node to forward the message to other nodes. It will store the message until it meets the destination and transmits the message.

deep-space communications and it has found many other applications since its introduction by the NASA, e.g. for wildlife tracking/monitoring [75, 38], emergency communications [57] or even underwater communications [17] (see the book [59] for a more detailed description of these applications). Instead of describing all potential applications of DTNs, we shall focus below on two specific application domains: communication-challenged areas and sparse Vehicular Ad hoc Networks (VANETs).

### Communication-challenged areas

The use of Delay Tolerant Networking technologies has been considered for communication-challenged areas. Although nowadays technology is in its golden age, there are still many places on Earth where people do not have an Internet connection due to high construction costs or security reasons. Traditional telecommunication companies refuse to invest in fixed communication infrastructures since the expected profit does not cover the costs and the risks. One less-expensive solution is to rely on Delay Tolerant Networking technologies where people can use mobile phones, buses, etc to be relays in DTNs.

For instance, the authors of [51] present the implementation of a DTN service called "Bytewalla". This DTN service allows the use of Android phones for the physical transport of data between network nodes in areas where there are no other links available, or where existing links need to be avoided for security reasons. The authors present two applications, one related to store and forward messaging and one related to Sentinel Surveillance health-care

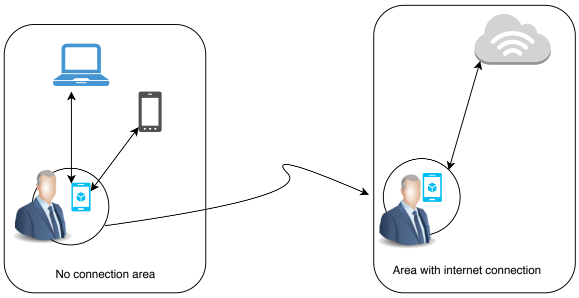



Figure 1.7: In areas where there is no network connectivity, nodes forward data to relays who may move to connected areas. Once a relay has an Internet connection, it sends data to a server.

(for obtaining high-quality data about a particular disease that cannot be obtained through a passive system). The concept is illustrated in Fig. 1.7.

Another application of Delay Tolerant Networking in this context is presented in [19], where the authors address the dissemination of information following the publish-subscribe paradigm in intermittently-connected networks. In the publish-subscribe paradigm, a message is delivered only to those nodes whose subscribed interests match it. The authors of [19] propose *SocialCast*, a routing framework for publish-subscribe that exploits predictions based on metrics of social interaction to identify the best information carriers. The key idea of SocialCast is that socially-related people tend to be co-located quite regularly. There are three phases in SocialCast. The first one is *interest dissemination*, in which all nodes broadcast their message about their interests. The second one is *carrier selection*, in which forecasting techniques are used to identify the best carriers (that is, relays) based on previous observations of the social behaviour of nodes. Finally, *message dissemination* forwards the message to all subscribers and to all interested nodes.

As a final example of the application of DTNs for communication in rural areas, we mention the concept of Pocket Switched Networks (PSNs) which was introduced in [34]. The authors address the design of a networking solution for mobile users moving between connectivity islands (e.g., WIFI at home) and who are not connected outside these islands. The proposed solution is based on Delay Tolerant Networking technologies and make use of both human mobility and local/global connectivity in order to transfer data between mobile users' devices.

**Vehicular Ad hoc Networks**

Communication between vehicles on the road attracts more attention than before. Vehicles can notify others about road traffic, weather reports, accidents, or advertisements, etc.

Data MULEs (Mobile Ubiquitous LAN Extensions)[64] is one example of a Vehicular Delay Tolerant Network (VDTN). A data mule can receive data from sensors which are placed in non-connected areas. The data mule can then transport the data to another place, where there is a wired or wireless connection to a computer server as in Fig. 1.7. This idea saves energy for sensors and extends the range of the network using the store-carry-and-forward principle. To build and analyze the model of data Mules, the authors of [64] assumed the random walk model for the mobility of nodes.

Other authors have also considered using buses as relays for transporting data from kiosks to the Internet. In [58], the authors propose to use buses to carry data from VIC (village Information Center) to Data relay center or to the Internet to solve the communication problem in rural villages in Bangladesh. The same idea is used in [31] where the authors suggests to use buses as gateways from internet kiosk in rural villages to internet access points in towns.

## 1.3 Incentive Mechanism in DTNs

The replication-based routing strategy implicitly assumes that mobile nodes accept to use their scarce energy resources for relaying messages of others out of altruism. In practice, it can be expected that some nodes will act as *free-riders*, that is, that they will use the network to send their own messages without offering their resources in exchange for relaying the messages of others. Clearly, if there are too many selfish nodes in a DTN, the network collapses and mobile nodes can no longer communicate with one another. A central issue in DTNs is therefore to convince mobile nodes to relay messages. Many incentive mechanisms have been proposed to avoid the free-rider problem in DTNs, including reputation-based schemes, barter-based schemes and credit-based schemes. We shall first briefly describe the first two schemes, before describing in more depth the latter scheme, which is the main topic of this thesis.

In reputation-based schemes, all the behaviours of nodes are collected and analyzed by the reputation system [76, 82, 46, 77]. Consequently, each node will have a reputation rating which helps to identify selfish nodes. In [46], the author proposed a watchdog scheme to detect selfish nodes. However, this scheme requires global information sharing between all DTN nodes, which is difficult to implement in DTNs. In [76, 82], the authors propose a reputation architecture for DTNs to record the behaviour of nodes. This architecture requires the communication of reputations between nodes. Based on this rating, they can detect selfish nodes. There is however no analysis of the impact of the dissemination of node reputations on the energy of the network.

Barter-based incentive mechanisms have also been considered to enforce fair cooperation of all nodes. For example, the authors of [67] propose an incentive-aware routing protocol which is based on the Tit-for-Tat (TFT) strategy, in which each node forwards as much traffic for an encountered node as the latter forwards for it. In [10] and [11], Buttyan *et al.* propose a mechanism which is based on the principle of barter: a node relays the message of a neighbor if the latter relays a message of the former in return. One of the issues with this scheme is that a message might be not delivered to its destination if the destination has no message to forward in return.

In reward-based or credit-based schemes, only node(s) who successfully transmit(s) the message receive a reward. The credits earned by nodes from forwarding the messages of other nodes can be used to pay for the delivery of their own ones. As compared to reputation-based schemes, these schemes do not require global information sharing, but they assume the existence of a Trusted Third Party to manage the rewarding procedure. Credit-based incentive schemes are often designed using concepts from Game Theory, such as Vickrey-Clarke-Groves (VCG) auctions [84, 43] or Minority Games [13]. Other examples of credit-based schemes are Mobicent [14], SMART [86], PIS [44], INPAC [16] and FRAME [42], among others. Since it is not possible to share global information between all nodes and the reception of a message cannot be acknowledged, most works consider a deadline or a lifetime $\tau$ for a message. It is assumed that the message is deleted from the network (that is, dropped by relays) after its deadline.

In credit-based schemes, a central question is to determine who should be rewarded for the delivery of a message. Several works have considered rewarding all the nodes who transmit the message on time [72, 14]. This setting is not practical since the source cannot predict how many nodes will reach the destination, and therefore it does not know the amount of reward it will have to pay (the reward paid to each successful relay is known, but the number of relays who will get the reward is unknown). Moreover, the source might have to pay a lot in order to send a message.

Other authors suggest to use a profit-sharing model in which only the intermediate nodes involved in the first successful delivery of a message are rewarded [80]. In this setting, the source node prescribes a maximum number of intermediate nodes and a message is successfully delivered only if it is delivered within its lifetime. Although this mechanism may help the source to estimate the delivery cost of its message and may also increase the delivery ratio, no expression of the main performance metrics (delivery ratio, mean time to deliver a message) are available.

Another option is to reward only the first relay to deliver a message. This option is particularly appealing for two-hop routing DTNs. In [23], the source forwards its message to a relay with probability $u$. This probability may be fixed or be dynamic, and every node can decide to participate when it meets the source. However, it is not allowed to drop the message before time $\tau$ - the lifetime of that message. The authors also assume that the source only proposes its next message after the deadline of the previous one.

In [2], a simple reward-based mechanism was proposed in which the first relay to deliver gets the reward. It is assumed that the relays decide how long they participate in the network and that during this time they accept the message with a certain probability and did not drop it until the lifetime $\tau$ of the message has expired. For a single message, the authors show that the equilibrium policy is of threshold type: relays participate until a certain time after which they are deactivated. A closed-form expression for the probability of successful message delivery is also established. Note that in this work, the lifetime $\tau$ of a message is fixed arbitrarily.

While in [2, 23], the promised reward is fixed and is the same for every relay, the work in [62] considers a setting in which the reward proposed to a relay depends on its meeting time with the source. This reward is the minimum amount that offsets the expected delivery cost, as estimated by the relay from the information given by the source (number of existing copies of the message, age of these copies). The authors assume two-hop routing and that a reward is given only to the relay that is the first one to deliver the message to the destination. It is shown that the expected reward the source pays remains the same irrespective of the information it conveys. The main drawback of this mechanism is that in some cases the source might have to pay a high price for the delivery of its message.

Finally, in [61], the author considers a two-hop routing DTN and a fixed reward scheme in which only the first relay to deliver the message to the destination is rewarded. The competition between relays for the delivery of a message is modelled as a discrete-time stochastic game. It is shown that for two relays, the optimal policy of a relay is threshold-type, that is, a relay accepts a message until a first threshold and then keeps it until it either meets the destination or reaches the second threshold. However, no explicit results on the performance of this mechanism are obtained.

## 1.4 Thesis contributions

One of the main questions in credit-based incentive mechanisms for DTNs is to determine the value of the reward that an agent should propose. From the point of view of the source, the value of the reward should be the minimum amount such that its performance objectives are met. Unfortunately, for most of the proposed mechanisms, it has been difficult to obtain performance measures such as probability of message delivery and mean time to deliver a message. We note however the exceptions of [2] and [62]. The main aim of this thesis is to give the precise relationship between the performance measures and the reward when multiple relays are competing for message delivery. This, in turn, will help the source providing an adequate reward in order to achieve a target delivery probability.

We analyze an incentive mechanism for message delivery in two-hop DTNs assuming a *backlogged* source (a source with infinite number of messages to send) and a *fixed reward* (may depend upon the message). The source and the destination are fixed, whereas the relays move according to a given mobility model. When the source wants to send a message, it proposes a fixed reward to each relay it meets. The reward may vary from message to message but for a given message, the same reward is proposed to each relay. Only one message at a time is proposed by the source and, for a given message, only the first relay to deliver it gets the corresponding reward. The cost of delivering the message for a relay is the sum of a storage cost, which depends upon the duration the relay carries this message, and constant energy costs for receiving the message from the source and transmitting it to the destination. This mechanism induces a competition between relays since each one seeks to maximize its expected reward for each message. For its part, the objective of the source is to satisfy a given constraint on the probability of message delivery.

We consider two different settings: one in which the source tells the relays for how long a message is in circulation, and one in which the source does not give this information.

### First setting

In this setting, a relay is informed of the age of the message when it meets the source. The relay can then decide to accept or to reject the message depending on the time at which it meets the source. There is no cost associated with rejecting a message. Moreover, if it accepts the message, the relay can decide to drop the message at any time in the future at no additional cost. If during this time the relay meets the destination, then it can transmit the message to the destination and claim the reward only if it is the first one to do so for this message.

Assuming independent and exponentially distributed inter-contact times with the source and with the destination, we prove that:

- Any Nash equilibria (NE) policy of a relay is of threshold type: a relay accepts a message until a first threshold and then keeps it until it either meets the destination or reaches the second threshold. Once a message is no longer accepted by the relays, the source starts giving out the following message. The thresholds of a message depend upon its index and the reward proposed.

- The NE may not be unique. A NE could be symmetric, that is, each relay has the same two thresholds for a given message, or asymmetric. We give examples of scenarios with multiple NEs. However, we shall show that any symmetric NE is unique.

- For symmetric NE, for each message, formulas for the thresholds as well as for performance measures such as the probability of delivery and expected delay are derived as a function of the reward proposed for this message.

We then investigate the asymptotic performance of this setting in the mean field limit when the number of relays becomes large. We first study the mean-field limit of this game when the second threshold is infinite and show that in this limit each message is proposed for a duration of $O(1/N)$, where $N$ is the number of relays. We show that the fraction of relays without a message converges in the mean field limit to the solution of an ODE. Based on that limit, we find the formulas to compute various performance metrics such as probability of success and the mean delay. It is also shown that the probability of success is the same for all messages. The main advantage of this analysis is that the formulas obtained for the performance metrics are simpler in the mean-field limit and provide an accurate approximation when $N$ is sufficiently large. When the second threshold is finite, the dynamics are no longer Markovian and we propose an ODE approximation which numerically gives a good match.

## Second setting

In this setting, a relay is not informed of the age of the message when it meets the source. In this case, we assume that the source proposes each message for a fixed period of time $T$ and that a relay decides to accept a message according to a randomized policy upon encounter with the source. If it accepts the message, a relay keeps it until it reaches the destination. We establish under which condition the acceptance probability of the relays is strictly positive and show that, under this condition, there exists a unique symmetric Nash equilibrium, in which no relay has anything to gain by unilaterally changing its acceptance probability. We obtain a closed-form solution for the acceptance probabilities at the symmetric Nash equilibrium. Explicit expressions for the probability of message delivery and the mean time to deliver a message at the symmetric Nash equilibrium are also derived, as well as an expression of the asymptotic value of message delivery.

Finally, we present numerous simulations results to compare performances of the threshold-type strategy and the randomized strategy, in order to determine under which conditions it is profitable for the source to give the information on the age of a message to the relays.

## 1.5 Thesis Organization

In the next chapter, Chapter 2, we give a brief introduction to Markov Decision Processes (MDP), both in discrete and continuous time. The technique to solve a continuous-time MDP by studying an equivalent discrete-time MDP, known as the Uniformization technique, is then presented. The second part of this chapter is a brief introduction to Incomplete information games.

Chapter 3 is devoted to the analysis of the incentive mechanism in the first setting, that is, when the source gives the information on the age of the message to the relays. We prove that any Nash equilibria (NE) policy of a relay is of threshold type. We then show that although the NE may not be unique, there is only one symmetric NE in which each relay has the same two thresholds for a given message. We obtain formulas for the thresholds as well as for the main performance measures.

Chapter 4 is a follow up of the previous chapter. In this chapter, we provide the mean field limit of the symmetric equilibrium when the number of relays and the number of message get large.

Chapter 5 is devoted to the analysis of the incentive mechanism in the second setting, when the source does not tell the relays when the message was generated. In this case, we assume that the relays use a randomized policy for accepting a message. We prove the existence and the uniqueness of the symmetric equilibrium together with the condition under which the relays always accept the messages.

# Chapter 2

# THEORETICAL PRELIMINARIES

In this thesis, we consider a problem which can be modeled as a Markov game - combination of Markov decision process (MDP) and Game. We first construct a Markov decision processes for each player (each stage corresponds to one MDP) and find their best response to others' strategy. Since one player does not know some information related to other players, we then consider an incomplete information game between players and find the equilibrium. However, in the MDP of each player, the rate matrix contains some infinity-value entries which are not suitable for a normal MDP. To deal with this, we use the uniformization technique which is a way to create a discrete time Markov process from a continuous time process. This technique helps us compute the transition probability when the transition rate is large. In this chapter we recall some preliminaries related to continuous-time Markov decision process, Incomplete information games, Mean-field interaction models for communication systems. In the first part, we state a model of Markov decision process and define a Markov policy. We then discuss about the optimal equation of the process and Bellman equation. Uniformization technique which is to approximate a continuous process by a discrete one is stated in the next part. Beside, we present the definition of Incomplete games, and their equivalent concepts.

## 2.1 Markov Decision Processes

Markov Decision Processes (MDPs) is a stochastic dynamic programming which is used to model decision making problem. The problem contains multiple periods with the dependency follow Markov property which is the future event depends only on the present state. In general, people study discrete-time and continuous-time MDPs.

### 2.1.1 Discrete-time Markov Decision Processes

A Discrete-time Markov decision process is a stochastic process whose evolution does not depend on the history but the present state. That is

$$\mathcal{P}(S_n = s_n | S_{n-1} = s_{n-1}, S_{n-2} = s_{n-2}, ..., S_0 = s_0) = \mathcal{P}(S_n = s_n | S_{n-1} = s_{n-1}) \tag{2.1}$$

A discrete time Markov process includes [56]:

$$\{\mathcal{S}, (\mathcal{A}(s), s \in \mathcal{S}), P_{ij}(a), r(s,a), \mathcal{V}\} \tag{2.2}$$

where $\mathcal{S}$ is the state space of the model, $\mathcal{A}(s)$ is the action space available to be picked when the system is in state $s$. The $P_{ij}$ presents the probability that the next state is $j$ given the current state is $i$ and the action taken is $a$. At any decision epoch, if the action $a$ is picked at state $s$, then the system gets a reward/cost $r(s,a)$. The last term defines the objective of the process, for example, to maximize/minimise the expected reward/cost. In the sequel, we let the objective is to maximize. In other cases, we just need to replace the max by the objective we want.

A policy is a decision map from state space to action space,

$$\pi : \mathcal{S} \longrightarrow \mathcal{A} \tag{2.3}$$

$$s \longmapsto a \in \mathcal{A}(s) \tag{2.4}$$

**Definition 2.1.1.** *An optimal policy or a solution of a MDP is a policy $\pi^*$ such that it satisfies the objective of the process,*

$$\pi^* := \underset{\pi}{argmax} \quad V(\pi, s_0) \tag{2.5}$$

*where the objective function $V$ is*

$$V(\pi, s_0) = \lim_{N\to\infty} V_N(\pi, s_0) = \lim_{N\to\infty} \sum_{n=1}^{N} \beta^n \mathbb{E}_\pi(r(s_n, a_n)|s_0). \tag{2.6}$$

*where $0 \leq \beta \leq 1$ is discount factor.*

In some problems, the discount factor $\beta$ can be 1, for instance, in stochastic shortest path problem [8] with a proper stationary policy, i.e starting from any state, the system can reach the destination (node D) after finite $T$ steps. More precisely, if we let

$$\rho_\pi := \max_{i=1,\dots,S} \mathbb{P}(s_T \neq D | s_0 = i) < 1, \tag{2.7}$$

where $S$ is the number of states and $\pi$ is a policy, we have

$$\mathbb{P}_\pi(s_k \neq D | s_0 = i) \quad \leq \quad \mathbb{P}_\pi(s_{\lfloor k/T \rfloor T} \neq D | s_0 = i) \leq \rho_\pi^{\lfloor k/T \rfloor}. \tag{2.8}$$

Then sum in the limit of the Eq. (2.6) can be rewritten as

$$\sum_{n=1}^{N} \mathbb{E}_\pi(r(s_n, a_n)|s_0) = \sum_{n=1}^{N} r(s_n, a_n|s_0)\mathbb{P}_\pi(s_n|s_0) \leq \sum_{n=1}^{N} \rho_\pi^{\lfloor n/T \rfloor} r(s_n, a_n|s_0). \tag{2.9}$$

This means the number $\rho_\pi^{1/T}$ plays a similar role as the discount factor $\beta$.

### Bellman Equation

Bellman equation is designed for dynamic programming where the optimal equation is written in term of the initial action and the value function of the rest choices. By doing this, it divides the problem into sub-problem which may be easier to solve. The Bellman equation is as follows [55],

$$V(s_0) = \max_{a \in \mathcal{A}(s_0)} \quad r(s_0, a) + \sum_{s \in \mathcal{S}} V(s) P(s|s_0, a) \tag{2.10}$$

Or in another way, with the policy dependence,

$$V(\pi, s_0) = r(s_0, \pi(s_0)) + \sum_{s \in \mathcal{S}} V(\pi, s) P(s|s_0, \pi(s_0)) \tag{2.11}$$

## 2.1.2 Continuous-time Markov Decision Processes

### Continuous-time Markov Chain

Continuous-time Markov decision processes is based on Continuous-time Markov chain [70] in which we need to define a rate matrix $Q$ at which transitions occur. Let's denote $P_{ij}(t) := \mathbb{P}(S(t) = j | S(0) = i)$ that is the probability that the state at time $t$ is $j$ given the original state is $i$. This probability satisfies Markov property:

$$\mathcal{P}(S(t+s) = s | S(t) = s_t) = \mathcal{P}(S(s) = s) \tag{2.12}$$

When $t = 0$, we define

$$P_{ij}(0) = \begin{cases} 1 & \text{when} \quad i = j \\ 0 & \text{otherwise.} \end{cases} \tag{2.13}$$

Let $P(t) = (P_{ij}(t)), \forall t$ be the transition probability matrix at time $t$. Then the rate matrix $Q = (q_{ij})$ is defined as

$$q_{ij} = \left.\frac{dP_{ij}(t)}{dt}\right|_{t=0} \tag{2.14}$$

Therefore, $q_{ij}$ satisfies: $q_{ij} \geq 0$ for $i \neq j$ and $q_{ij} \leq 0$ for $i = j$. Moreover, for small $h$, we have

$$P_{ij}(h) = \begin{cases} 1 + hq_{ij} + o(h) & \text{when} \quad i = j \\ hq_{ij} + o(h) & \text{otherwise.} \end{cases} \tag{2.15}$$

The reason we have the sum of $q_{ij}$ equal 0, that is $\sum_j q_{ij} = 0$, is that the sum of all probability that the next state is $j$ for all $j$ is 1: $\sum_j P_{ij} = 1$, therefore the sum of its derivative is 0. Moreover, it is not surprising that $q_{ii} \leq 0$ because the probability that the system stays in state $i$ is decreasing in time, so that its derivative is negative. The rate $Q$ does not depend on time since it is an instantaneous transition rate.

For a given $Q$, we also can find $P(t)$ by the forward or backward equations

$$\frac{dP(t)}{dt} = QP(t) \tag{2.16}$$

$$\frac{dP(t)}{dt} = P(t)Q. \tag{2.17}$$

These above equations come from the Chapman–Kolmogorov equation: $P_{ij}(t+h) = P_{ij}(t)P_{ij}(h)$ together with the following

$$\frac{P(t+h) - P(h)}{h} = P(t)\left(\frac{P(h) - P(0)}{h}\right). \tag{2.18}$$

Let $h$ tend to 0 we have

$$\frac{dP(t)}{dt} = P(t)\left.\frac{dP(s)}{ds}\right|_{s=0} = P(t)Q. \tag{2.19}$$

The solution of Eq. (2.17, 2.16) with the initial condition $P(0) = I$ ($I$ is identity matrix) is

$$P(t) = e^{tQ}, \tag{2.20}$$

where $e^{tQ}$ is matrix exponential defined as

$$e^{tQ} := \sum_{k=0}^{\infty} \frac{t^k Q^k}{k!} \tag{2.21}$$

### Continuous-time Markov Decision Process

We are now ready to define a Continuous-time Markov Decision Process [32]. A Continuous-time MDP is a continuous time stochastic process which satisfies Markov property and it is a four-tuple as in the discrete-time case,

$$\{\mathcal{S}, (\mathcal{A}(s), s \in \mathcal{S}), q(j|i,a), r(s,a)\}, \tag{2.22}$$

where $\mathcal{S}$ is state space, $\mathcal{A}(s)$ is action space when the state is $s$, $q(j|i,a)$ is transition rate from state $i$ to state $j$ when action $a$ was taken and last term, $r(s,a)$ is the reward/cost when the system is at state $s$ and took action $a$.

There are two types of policy, one is stationary policy and the other is randomized policy. The former is defined as, if $\pi_t$ is a policy at time $t$ then

$$a_t = \pi_t(s_t) \tag{2.23}$$

While in the latter, a policy/strategy is a distribution over action given state, that is assuming $\pi_t$ is a policy at time $t$, then

$$\pi_t(a_t|s_t) = \mathbb{P}(Action = a_t|State = s_t), \tag{2.24}$$

In fact, the stationary policy is a special case of randomized policy when all mass is distributed at one action. Therefore, from now we use randomized policy, $\pi = (\pi_t)_t$.

We have the cost at state $s$ follows policy $\pi_t$ in continuous action space is

$$r(s,\pi_t) = \int_{A(s)} r(s,a)\pi_t(da|s), \tag{2.25}$$

and in discrete action space is

$$r(s,\pi_t) = \sum_{a\in A(s)} r(s,a)\pi_t(a|s). \tag{2.26}$$

Hence, the expected cost at time $t$ given state $s(0) = s_0$ and policy $\pi$ is,

$$\mathbb{E}^{\pi}_{s_0} r(s_t,\pi_t) = \sum_{s\in S} r(s,\pi_t)p^{\pi}_{s_0 s}(t), \tag{2.27}$$

where $p^{\pi}_{s_0 s}(t)$ is the probability defined by $q(s|s_0, \pi_t(s_0))$. Then the expected cost of policy $\pi$, given state at time 0 is $s_0$, will be

$$J(s_0,\pi) = \int_0^{\infty} \mathbb{E}^{\pi}_{s_0} r(s_t,\pi_t)dt \tag{2.28}$$

and the optimal cost function is

$$J^*(s_0) := \inf_{\pi \in \Pi} \quad J(s_0, \pi), \tag{2.29}$$

where $\Pi$ is the feasible policy set. Here we take the inf, it is just one example of the objective of MDPs. Depend on each problem, the objective can be max, sup, etc. Then we can define the optimal policy corresponding to the optimal cost function above, i.e. corresponding to the inf optimal cost function.

**Definition 2.1.2.** *A policy $\pi^* \in \Pi$ is a optimal policy if*

$$J(s_0, \pi^*) \leq J^*(s_0) \tag{2.30}$$

### 2.1.3 Uniformization technique

In Discrete time Markov Chain, the system is observed at every time step, while in Continuous time Markov Chain, the observation can be made at anytime. If we observe the continuous time system at the jump-times, we can approximate this continuous time process by a discrete time one. However, the 'time steps' are not the same since they depend on the time the system spends in one state. Uniformization technique [70] is made to create a discrete time process from a continuous time by making all transition rates identical (uniform) in all states. To do that, we add fictitious transitions from state to itself. That is we observe the system more regularly, even when it does not jump to a new state.

Uniformization technique is used to compute the probability when the transition rate $Q$ is large. The technique provides a formula to find $P(t)$ by iterating a sufficient large steps. Let $Q$ be the rate matrix of our continuous time Markov Chain. Then the probability transition matrix of the discrete time embedded Markov Chain is,

$$P = Q\Delta t + I \tag{2.31}$$

where $I$ is identity matrix and $\Delta t \leq \min_i \frac{1}{-q_{ii}}$. Normally, we take $\Delta t = \frac{1}{c}$ with $c = \max -q_{ii}$. We have

$$P = \frac{Q}{c} + I \quad \text{and} \quad Q = c(P - I). \tag{2.32}$$

Plugging this into Eq. (2.20), we have

$$P(t) = e^{tQ} = e^{tc(P-I)} = e^{-tc} \sum_{k=0}^{\infty} \frac{(tc)^k P^k}{k!}. \tag{2.33}$$

Computing the probability $P(t)$ by the multiplication of matrix $P$ instead of matrix $Q$ is easier since matrix $P$ has all positive-less-than-one elements while matrix $Q$ has both negative, positive and large entries.

These two processes have the same distribution since for a small time $\Delta t = \frac{1}{c}$, the transition probability in continuous time (follows Eq. (2.15)) and in discrete time are identical.

## 2.2 Incomplete Information Games

Game is a mathematical model of intelligent decision makers who can be cooperators or rivals. In this section, we introduce Incomplete information games and their equilibrium concepts.

We consider an $N$ players game where players have private and common knowledge. In this game, players do not know some information related to others, for example they do not know others' strategy, utility function, etc. We define an Incomplete information game as follows [41]

**Definition 2.2.1.** *An Incomplete information game or a Bayesian game contains*

- *A set of player types* $\Theta = (\Theta_1 \times \Theta_2 \times ... \times \Theta_N)$*, where* $\Theta_i$ *is player* $i$*'s types.*
- *A set of actions* $\mathcal{A} = (\mathcal{A}_1 \times \mathcal{A}_2 \times ... \times \mathcal{A}_N)$*, where* $\mathcal{A}_i$ *is the actions set set of player* $i$*.*
- *A probability distribution over players' types,* $p(\theta_1, ..., \theta_N)$*.*
- *Payoff function,* $u : \Theta \times \mathcal{A} \longrightarrow \mathbb{R}^N$*. That is the outcome of function* $u$ *is a utility vector whose each entry is utility of one player.*

Let us take an example.

**Example 2.2.1.** *Let consider a two players game where player* 1 *has only one type while player* 2 *has two types, with probability* $p$ *of type* 1 *and* $1-p$ *he will be of type* 2*. Each player has two choice of action: action* 1 *and action* 2 *correspond to the following payoff tables and when the player* 2 *is of type* 2

Before defining the the Bayesian Nash Equilibrium, we present the definition of a strategy.

player 2

| | | action 1 | action 2 |
|---|---|---|---|
| player 1 | action 1 | $4, 1$ | $2, 3$ |
| | action 2 | $3, 2$ | $5, 4$ |

Table 2.1: Player 2 is of type 1

player 2

| | | action 1 | action 2 |
|---|---|---|---|
| player 1 | action 1 | $3, 6$ | $4, 1$ |
| | action 2 | $2, 5$ | $5, 4$ |

Table 2.2: Player 2 is of type 2

**Definition 2.2.2.** *A Bayesian pure strategy is a function from player's types to actions space,*

$$\pi_i : \Theta_i \longrightarrow \mathcal{A}_i \tag{2.34}$$

Let us denote $\pi := (\pi_1, \pi_2, ..., \pi_N), \theta := (\theta_1, ..., \theta_N)$ and

$$\pi(\theta) := (\pi_1(\theta_1), ..., \pi_N(\theta_N)).$$

The expected utility of a strategy $\pi$ is

$$\mathbb{E}(u_\pi) := \sum_{\theta \in \Theta} u(\pi(\theta), \theta) P(\theta), \tag{2.35}$$

where the sum is the sum of vectors. Let $u_i(\pi(\theta), \theta)P(\theta)$ denote the $i^{th}$ entry of the vector $u(\pi(\theta), \theta)P(\theta)$. For convenience we denote

$$\pi_{-i} := (\pi_1, .., \pi_{i-1}, \pi_{i+1}, ..., \pi_N)$$

that is the joint strategy of all players except player $i$.

**Definition 2.2.3.** *A Bayesian Nash Equilibrium of a N players game is $\pi^* = (\pi_1^*, \pi_2^*, ..., \pi_N^*)$ such that, for all i, $\theta_i \in \Theta_i$ and $a_i \in \mathcal{A}_i$ we have*

$$\sum_{\theta_{-i} \in \Theta_{-i}} u_i((\pi_i^*(\theta_i), \pi_{-i}^*(\theta_{-i})), \theta) P(\theta_{-i}|\theta_i) \geq \sum_{\theta_{-i} \in \Theta_{-i}} u_i((a_i, \pi_{-i}^*(\theta_{-i})), \theta) P(\theta_{-i}|\theta_i) \tag{2.36}$$

We now come back to our example. It is easy to see that for player 2, action 2 dominates action 1 in type 1 and action 1 dominates action 2 in type 2. Therefore, player 1 should play action 2 if player 2 is of type 1 and play action 1 otherwise. We have the expected utility for player 1 is: $5p$ for type 1 and $3(1 - p)$ for type 2. This means, if $p \geq \frac{3}{8}$, player 1 best strategy is action 2 and vice-versa.

## 2.3 Mean-field interaction models for communication systems

This section is based on the paper of Michel Benaim and Jean-Yves Le Boudec [6] in which the authors proved the convergence of an $N$ players system in a mean-field model if it satisfies some conditions.

We consider a system of $N$ players and one source. Let $X(t) = (X_i(t))_i$ where $X_i(t)$ is the state of player $i$ at time $t$ and let $R(t)$ be the state of the source at time $t$. We assume that the process $Y(t) = (X(t), R(t))$ is a homogeneous Markov chain with an invariant transition kernel under permutation of players. That is, let $\sigma$ be a permutation of $Id := (1, ..., N)$ then we have,

$$\begin{aligned} &P(X_{Id}(t+1) = I, R(t+1) = j | X_{Id}(t) = I', R(t) = j') \\ &= P(X_\sigma(t+1) = I, R(t+1) = j | X_\sigma(t) = I', R(t) = j') \end{aligned} \tag{2.37}$$

where $X_{Id}(t) = (X_1(t), ..., X_N(t)), X_\sigma(t) = (X_{\sigma_1}(t), ..., X_{\sigma_N}(t))$ and $I = (i_1, ..., i_N), I' = (i'_1, ..., i'_N)$. We denote

$$\mathcal{K}(I, j | I', j') := P(X_{Id}(t+1) = I, R(t+1) = j | X_{Id}(t) = I', R(t) = j'). \tag{2.38}$$

We assume that the state space is finite, that is let $\mathcal{S} = \{s_1, ..., s_I\}$. Let $M(t)$ be the occupancy measure which presents the frequency of the state $s_i$. $M(t) = (M_{s_1}(t), M_{s_2}(t), ..., M_{s_N}(t))$, with $M_{s_i}(t)$ is defined as

$$M_{s_i}(t) = \frac{1}{N} \sum_{n=1}^{N} \mathbb{1}_{\{X_n(t) = s_i\}}. \tag{2.39}$$

We observe that $(M(t), R(t))$ and $(X(t), M(t), R(t))$ are homogeneous Markov chains. For the source, we let $K_{jj'}(\vec{m})$ be the marginal transition,

$$K_{jj'}(\vec{m}) = \mathbb{P}(R(t+1) = j' | M(t) = \vec{m}, R(t) = j). \tag{2.40}$$

We define the expected change of $M(t)$ in one time slot as the drift,

$$\vec{f}(\vec{m}, j) = \mathbb{E}(M(t+1) - M(t) | M(t) = \vec{m}, R(t) = j) \tag{2.41}$$

$$= \sum_{i, i' \in \mathcal{S}, i \neq i'} m_i P_{ii'}(\vec{m}, j)(\vec{e_{i'}} - \vec{e}_i), \tag{2.42}$$

where $P_{ii'}(\vec{m}, j)R$ is the marginal transition probability of a player from state $i$ to state $i'$ given $M(t) = \vec{m}, R(t) = j$. That is

$$P_{ii'}(\vec{m}, j) = \mathbb{P}(X_n(t+1) = i' | M(t) = \vec{m}, R(t) = j, X_n(t) = i). \tag{2.43}$$

We assume that the drift, the marginal transition of the source, and the second moment of number of transitions per time slot satisfied the following conditions.

- The limit of $K_{jj'}(\vec{m})$ exists when the number of players tends to infinity for all $\vec{m}$. Let $K(\vec{m}) = (K_{jj'}(\vec{m}))_{j,j'}$, then this matrix is indecomposable for all $\vec{m}$.

- Let $\epsilon(N)$ be a function of $N$ which tends to zero when $N$ is large. We have for all $\vec{m}$,
$$\lim_{N\to\infty} \frac{\vec{f}(\vec{m}, j)}{\epsilon(N)} \quad \text{exists.} \tag{2.44}$$

- Let $W(t)$ be the upper bound of the transition in one time slot $t$, we have,
$$\mathbb{E}(W(t)^2 | M(t) = \vec{m}, R(t) = j) \leq cN^2\epsilon(N), \tag{2.45}$$
where $c$ is a constant independent of $t, N, \vec{m}, j$.

- $K_{jj'}(\vec{m})$ is a smooth function of $\frac{1}{N}$ and $\vec{m}$.

- $f(\vec{m}, j)$ is a smooth function of $\frac{1}{N}$ and $\vec{m}$.

We now define a re-scaled process $\bar{M}$ as,

$$\begin{cases} \bar{M}(t\epsilon(N)) = M(t) \text{ for all } t \in \mathbb{N} \\ \bar{M}(\tau) \text{ is affine on } \tau \in [t\epsilon(N), (t+1)\epsilon(N)] \end{cases} \tag{2.46}$$

Then the process $\bar{M}(t)$ converges to a process $\vec{\mu}(t)$ which is the unique solution of the ODE,

$$\frac{d\vec{\mu}(t)}{dt} = \vec{F}(\vec{\mu}) \text{ with } \vec{F}(\vec{m}) = \sum_{j=1}^{J} \pi_j(\vec{m})\vec{f}(\vec{m}, j), \tag{2.47}$$

with $\pi(\vec{m})$ is the invariant probability of $K(\vec{m})$.

# Chapter 3

# PERFORMANCE OF A FIXED REWARD INCENTIVE SCHEME FOR TWO-HOP DTNs WITH COMPETING RELAYS

We analyse the performance of an incentive scheme for two-hop DTNs in which a backlogged source proposes a fixed reward to the relays to deliver a message. Only one message at a time is proposed by the source. For a given message, only the first relay to deliver it gets the reward corresponding to this message thereby inducing a competition between the relays. The relays seek to maximize the expected reward for each message whereas the objective of the source is to satisfy a given constraint on the probability of message delivery. We show that the optimal policy of a relay is of threshold type: it accepts a message until a first threshold and then keeps the message until it either meets the destination or reaches the second threshold. Formulas for computing the thresholds as well as probability of message delivery are derived for a backlogged source.

## 3.1 Introduction

We consider an incentive mechanism for message delivery in two-hop DTNs assuming a *backlogged* source (a source with infinite number of messages to send) and a *fixed reward* corresponds to a message. When the source wants to send a message, it proposes a fixed reward to each relay it meets. The first relay to deliver gets the reward. The relay can decide to accept or to reject the message depending on the time at which it meets the source. The cost

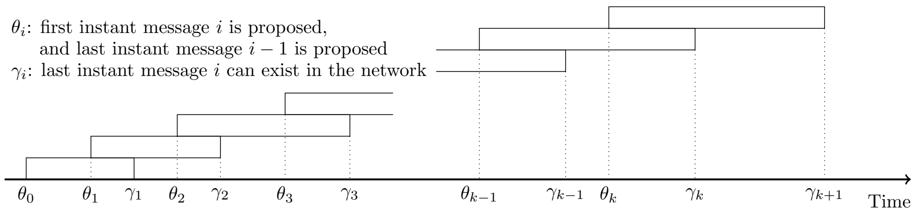



Figure 3.1: Message injection policy of the source at the symmetric Nash equilibrium.

of delivering the message for a relay consists of a linear term which depends upon the duration the relay carries this message as well as constant terms for receiving and transmitting the message.

One of the main questions in incentive mechanisms is to determine the value of reward that an agent should propose. The main aim of this chapter is to give the precise relationship between the performance measures and the reward when multiple relays are competing for message delivery. This, in turn, will help the source providing an adequate reward in order to achieve a target delivery probability. Towards this end:

- For the strategic message delivery based game described above, we show that any Nash equilibria (NE) policy of a relay is of threshold type: a relay accepts a message until a first threshold and then keeps it until it either meets the destination or reaches the second threshold. Once a message is no longer accepted by the relays, the source starts giving out the following message. The thresholds of a message depend upon its index and the reward proposed.

- The NE may not be unique. A NE could be symmetric, that is, each relay has the same two thresholds (depends upon the message) for a given message, or asymmetric. We give examples of scenarios with multiple NEs. However, we shall show that any symmetric NE is unique.

- For symmetric NE, for each message, formulas for the thresholds as well as for performance measures such as the probability of delivery and expected delay are derived as a function of the reward proposed for this message. This analysis will be called *transient* analysis, that is, for message $k$ the quantities will depend upon $k$.

Figure 3.1 illustrates how, at the symmetric NE, messages will be injected by the source into the network. At time $\theta_0$, the source will start proposing message 1, which the relays will accept if they meet the source after $\theta_0$ and before $\theta_1$. At $\theta_1$, the source will stop proposing message 1 because it knows that none of the relays will accept it, and start proposing message 2. A relay which accepted message 1 will keep it until $\gamma_1$ or until it meets the destination, whichever occurs earlier, after which it will drop this message.

**Organization:** The rest of this chapter is organised as follows. Section 3.3 is devoted to model description. In Section 3.4, we show that the structure of the best response policy of a relay is of threshold type. Section 3.5 gives the conditions for the existence and uniqueness of the symmetric Nash equilibrium. In Section 3.6, we present a method for recursively computing the thresholds of the symmetric equilibrium as well as the probabilities of message delivery for a backlogged source. Some conclusions are drawn in Section 3.7.

## 3.2 Related work

There has been a large body of literature on incentive mechanisms for DTNs. These mechanisms can be broadly classified into three categories: reputation-based schemes [46, 82], barter-based schemes [10] and credit-based schemes [86, 14, 83, 62].

Reputation-based schemes, such as SORI [33], MobiGame [79], CONFIDANT [9] and RELICS [77], are based on a simple principle: a node's message is forwarded only if it has forwarded messages originated from others. This however requires each node to monitor the traffic information of all encountered nodes and keep track of their reputation values. In addition, these reputation values should be updated and propagated to all other nodes efficiently and effectively, which is clearly impractical due to the intermittent connectivity between nodes.

Barter-based incentive mechanisms have also been considered to enforce fair cooperation of all nodes. For example, the authors of [67] propose an incentive-aware routing protocol which is based on the Tit-for-Tat (TFT) strategy, in which each node forwards as much traffic for an encountered node as the latter forwards for it. In [10] and [11], Buttyan *et al.* propose a mechanism which is based on the principle of barter: a node relays the message of a neighbor if the latter relays a message of the former in return. One of the issues with this scheme is that a message might be not delivered to its destination if the destination has no message to forward in return.

Finally, in credit-based schemes, the credits earned by nodes from forwarding the messages of other nodes can be used to pay for the delivery of their own ones. As compared to reputation-based schemes, these schemes do not require global information sharing, but they assume the existence of a Trusted Third Party to manage the rewarding procedure. Credit-based incentive schemes are often designed using concepts from Game Theory, such as Vickrey-Clarke-Groves (VCG) auctions [84, 43] or Minority Games [13]. Other examples of credit-based schemes are Mobicent [14], SMART [86], PIS [44], INPAC [16] and FRAME [42], among others.

For most of these schemes, it has been difficult to obtain performance measures such as probability of message delivery and mean time to deliver a message with the exception of [2, 62].

In [2], a simple reward-based mechanism was proposed in which the first relay to deliver gets the reward. The author provided the expression of the success delivery probability of a packet within a fixed time $\tau$ with the assumption that a relay $i$ will participate in delivering the packet until a certain time. It was shown that the equilibrium policy is of threshold type: relays participate until a certain time after which they are deactivated. All the computations and results are for a single message. Our setting is different from [2] in the following ways. In that work, the relays decided how long they participate in the network and during this time they accepted the message with certain probability and did not drop it. In our work, the relays can decide how long they accept they message and then how long they keep it. This gives more freedom to the relays to make their choice. Our cost structure is also different from that in [2]. The linear term in our work depends only upon the duration the relay stores a message whereas in [2] this term depends upon the time the relay is participating. These two are different because in the latter case, relays accrue a cost even if they do not have a message. Furthermore, there is no cost of receiving the message from the source in [2]. The inclusion of this cost leads to a policy with two thresholds in our case as opposed to a single threshold in [2]. We also show how the performance measures depend upon the reward offered by the source thereby giving the source an explicit way to compute the reward so as to achieve its targeted performance. Finally, we consider a backlogged source as opposed to a single message in [2]. This induces dependence between the policies of messages which was not there in that work.

In [62], the source offers rewards that depend upon the meeting time with the condition that only the first one to deliver the message receives its reward. Since the mobility model is random, a relay that meets the source later has lower probability of being the first to deliver the message and hence receiving the reward. The reward proposed to a relay is inversely proportional to its success probability, and is such that a relay always accepts the message. The analysis relies heavily on the assumption that the relays do not discard a message once they accept it from the source. This assumption may be realistic in participative networks in which nodes are altruistic. On the other hand, when nodes are selfish, as is the case in the present paper, they could decide to throw away a message once it is not longer profitable to keep it (because the probability of success is too small) and reduce their costs. This possibility to reject or drop the messages is the main difference of our work with [62], in which no strategic interaction between relays was considered.

A model similar to the one studied in this work was first considered in [63] (which is based on Chapter 5 of [61]) in which the competition was modelled as a stochastic game. That model was in discrete-time, restricted to two relays and a single message, and had partial results on the optimal policy. The model studied in this work is in continuous time, for an arbitrary number of relays, and a backlogged (with possibly infinite number of messages) source which sequentially proposes messages. Preliminary results of this work have appeared in [49]. The results in that paper were limited to a game with just one message for the source. Here, we generalize the results to a backlogged source. Also, we give conditions for the existence and uniqueness of symmetric equilibrium, which were not given in that paper.

## 3.3 Model Description

Consider a network of one source, one destination, and $N$ relays. The source and the destination are assumed to be fixed, whereas the relays move according to a given mobility model. It is assumed that the mobility pattern of any two relays are independent, and that the inter-contact times between a relay and the source (resp. the destination) are independent and identically distributed according to an exponential distribution of rate $\lambda$ (resp. $\mu$). The inter-contact processes of different relays with the source as well as with the destination are assumed to be statistically identical. We note that the assumption of exponentially distributed inter-contact times is satisfied under the Random Waypoint Mobility model [65, 30, 66, 12] and has been observed to hold in real motion traces [85].

When it meets the source, a relay is offered a fixed reward, $R$, to deliver message $k$. The reward is fixed in the sense that, for a given message, each relay is offered the same reward irrespective of their meeting times. The relay has a choice to either accept the message or not. There is no cost associated with rejecting the message. If it accepts the message, the relay can decide to drop the message at any time in the future at no additional cost. If during this time the relay meets the destination, then it can transmit the message to the destination and claim the reward only if it is the first one to do so for this message.

The various costs incurred for accepting and storing a message are assumed to be as follows:

- $C_r$ is the cost of receiving the message from the source;
- $C_d$ is the cost of transmitting the message to the destination;
- $C_s$ is the cost per unit time for storing the message. These costs are all the same for all relays.

We illustrate the cost structure with an example. Suppose that relay $i$ meets the source at time instant $a$, accepts the message $k$ and decides to keep it until time $b$, then the expected total cost of keeping the message in the interval $(a, b)$ will be

$$\begin{aligned} C_r + &\int_a^b \mu e^{-\mu(t-a)}(C_s(t-a) + (C_d - R)p_k^i(t))dt \\ &+ e^{-\mu(b-a)}C_s(b-a) =: C_r + G_k^i(a,b), \end{aligned} \tag{3.1}$$

where $p_k^i(t)$ is the probability that the relay is the first one to deliver this message when it meets the destination at time $t$. A one-time cost of $C_r$ is incurred for accepting the message at $a$. From then on, a storage cost of $C_s$ is incurred per unit of time either until $b$ (that is, for a duration $b - a$) or until it meets the destination. If the relay meets the destination at time $t < b$ and it is the first one to meet the destination with this message then it will transmit the message to the destination and get the reward thereby incurring a net cost of $C_s(t - a) + (C_d - R)p_k^i(t)$. So, the second term is the expected cost incurred if the relay meets the destination before $b$. Finally, the last term is the storage cost incurred if the relay does not meet the destination before $b$. The sum of the last two terms will be denoted by $G_k(a, b)$ which is the expected cost of keeping the message in the interval $(a, b)$. Note that this cost depends on the strategies of the other relays through the success probability $p_k^i(t)$.

We shall sometimes use the notation $\overline{R} = R - C_d$. In addition, we shall assume that $R \geq R_{min} := R_{min} = C_r + \frac{C_s}{\mu} + C_d$. In fact, $R_{min}$ is the result of Eq. (3.1) when $p_k^i(t) = 1$. In the one player case, the probability of being the first one always equals to 1. Thus, $R_{min}$ is the average cost of a relay if it were to be the only one to be competing for the message. It is therefore natural that the reward should be larger than this average cost for any relay to participate in forwarding. Then $R_{min}$ is the minimum required cost in order to have all relays participate in the game.

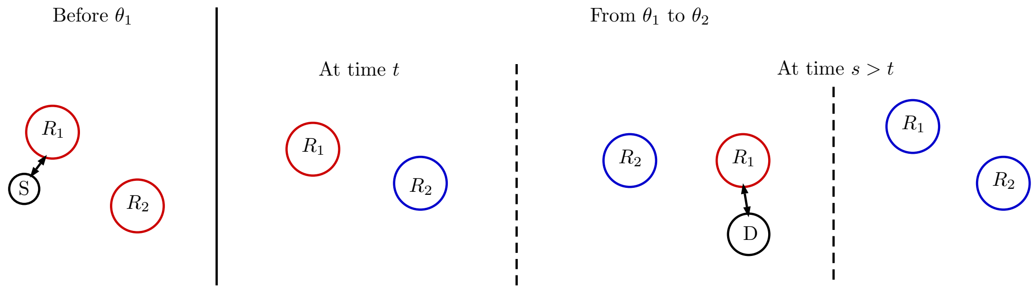



Figure 3.2: Example of two relays and two messages. Red represents the relays competing for message 1 and blue is for relays competing for second message.

It shall be assumed that a relay can store only one message at a time. Further, if a relay already has a message in its buffer, then it does not seek a new (or the same) message until it either meets the destination or drops the message. A message can be dropped only because it is no longer profitable to store this message due to a small probability of success but not because the relay meets the source. Probability of success means the probability of being the first one transmits the message to its destination. Once it has delivered or dropped the message, the relay can seek a new one from the source.

The source has an unlimited number of messages to send to the destination, each of which it proposes sequentially. That is, to each message the source associates an interval of time during which it proposes this message to any relay it meets. We shall denote this interval for message $k$ by $[\theta_{k-1}, \theta_k)$, where $\theta_{k-1}$ is the last time message $k-1$ was proposed.

For a given relay (called tagged relay) when it meets the source, the decision to accept can depend upon its history of contacts and previous decisions but not upon the history of the other relays since the exact history of the other relays is not available to the tagged relay. We assume that it can compute a belief (or a probability distribution) on when the other relays will enter into competition for this message. This belief will be computed based upon the statistics of the mobility model, and will be denoted by $\Phi_k^i(t)$ which is the probability that relay $i$ will enter into competition for message $k$ on or before time $t$. By enter into competition on or before time $t$, we mean that that there was time instant before $t$ at which relay $i$ is free and can accept message $k$ if it meets the source. We shall denote by $\phi_k^i(t)$ the probability density function corresponding to $\Phi_k^i(t)$.

Let us take an example to illustrate the notion of competing for a message as in Fig (3.2). Suppose there are two relays. Relay 1 meets the source for the first time at some instant between $\theta_1$ and $\theta_2$ when the source is proposing message 2. If relay 2 had the message 1 at time $\theta_1$ then we say that it is not in competition for message 2 until it has this message because even if it meets the source it cannot accept message 2. Now suppose that relay 2 meets the destination at some time $t_2 \in [\theta_1, \theta_2)$. At $t_2$, we say that relay 2 enters into competition with relay 1 for message 2. Of course, relay 1 does not have exact knowledge of the contact history of relay 2 but, using the statistics of the mobility model, it can compute the probability that relay 2 entered into competition at time $t \in [\theta_1, \theta_2)$. This probability distribution will be denoted by $\Phi_2^2(t)$.

A further assumption we shall make is that the relays do not know whether there will be any more messages in the future[1]. Hence, they treat each message as though it were the last one. The policy for a message thus does not depend upon the future messages but could depend upon the policy for the previous messages.

## 3.4 Structure of the best-response policy

In this section, we shall show that the optimal strategy for relay $i$ for the delivery of message $k$, given the strategies of the other relays is of threshold type: it accepts a message until a first threshold $\theta_k^i$ and then keeps the message until it either meets the destination or reaches the second threshold $\gamma_k^i$.

We shall refer to this optimal strategy as the best-response of player $i$ to the strategies of the other relays. Without loss of generality, we assume that the relay comes into play for the delivery of message 1 at time 0. Note that all relays are immediately available when the first message is released, which is not necessarily the case for the subsequent messages.

We now describe the model for each relay. Since all relays are homogeneous, their model are all the same. Let $\mathcal{S} = \{0, m_s, 1, m_d, 2\}$ be the set of possible states for relay $i$, where

- State 0 presents when a relay is not busy with any message. At this state, the relay can accept a new message.
- State $m_s$ means the moment when the relay meets the source. At this state, the relay has to decide whether to accept or to reject the message proposed by the source. We note that a relay can be at state $m_s$ when

[1]The source is assumed to be backlogged but this information is not known to the relays.

Table 3.1: State, action sets and costs for a relay for message $k$.

| State | Significance | Action set | Cost |
|---|---|---|---|
| 0 | relay is competing for message $k$ | $\emptyset$ | 0 |
| $m_s$ | relay is in contact with the source | $\{accept,\ reject\}$ | $C_r\, \mathbb{1}_{\{a=accept\}}$ |
| 1 | relay has the packet | $\{drop,\ keep\}$ | $C_s\, \mathbb{1}_{\{a=keep\}}$ |
| $m_d$ | relay is in contact with the destination | $\emptyset$ | $(C_d - R)\, \mathbb{1}_{\{x_j \neq 2, \forall j \neq i\}}$ |
| 2 | relay quits the game | $\emptyset$ | 0 |

it is not busy with any other messages. That is supposing that it meets the source when it is carrying one message, it is not in state $m_s$.

- A relay is in state 1 if it is busy with one message. At this state, the relay can decide to drop or to keep the message.

- Similar to state $m_s$, state $m_d$ is when the relay meets the destination. A relay can only be in this state if it is storing one message. Hence, if it meets the destination and does not have any message, it is still in state 0.

- State 2 means a relay is not in the competition. It also means the state 0 of the next message.

If, at some decision epoch, relay $i$ is in state $x_i \in \mathcal{S}$, it may choose action $a$ from the set of allowable actions in that state, $\mathcal{A}(x_i)$. The interpretation of states as well as the actions available in each state are summarized in Table 3.1. When message $k$ is proposed for the first time by the source, it may happen that relay $i$ still has a previous message. In this case, the relay $i$ is not competing for message $k$ until it either drops the previous message or meets the destination. When this happens, relay $i$ enters state 0 and now has to calculate its optimal policy.

In the following, we shall denote by $x_i(t)$ the state of relay $i$ at time $t$, and by $\mathbf{x}_{-i}(t)$ the state of the other relays. We shall refer to $\mathbf{x}(t) = (x_i(t), \mathbf{x}_{-i}(t))$ as the state of the system at time $t$. We emphasize that relay $i$ does not know the state of the other relays at time 0.

The main difficulty in modelling the decision problem faced by relay $i$ is that some actions (namely, rejecting or dropping the message) lead to an

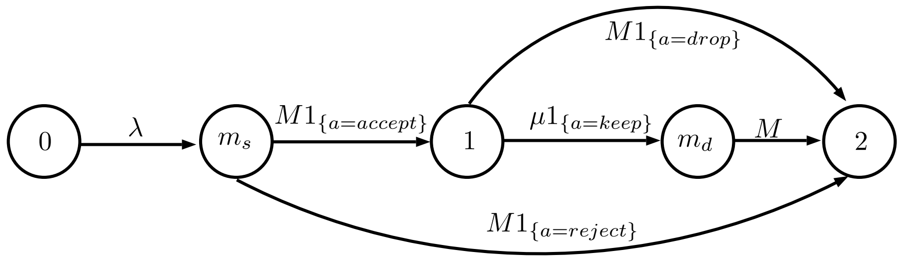



Figure 3.3: Controlled Markov Chain for relay $i$.

immediate change of state, or, in other words, correspond to an infinite transition rate. To circumvent this difficulty, we shall temporarily assume that when the relay makes such a decision, it stays an exponentially distributed amount of time of mean $\frac{1}{M}$ in the same state, where $M$ is some large constant. Under this assumption, it turns out that the optimal decision-making problem of relay $i$ can be cast as an MDP, as we now explain.

It is clear that the stochastic process $x_i(t)$ corresponds to a controlled continuous-time Markov chain, as shown in Figure 3.3. The cost incurred by the relay depends on its current state, on the action it takes, as well as on the state of the other relays. In the following, $g(x_i, a, \mathbf{x}_{-i})$ denotes the cost incurred by relay $i$ if it takes action $a$ when the system is in state $\mathbf{x}$. The possible values of the costs are shown in the last column of Table 3.1.

We define a control law (or policy) as a function $\pi : \mathbb{R} \times \mathcal{S} \to \mathcal{A}$ such that $\pi(t, x) \in \mathcal{A}(x)$ for all $x \in \mathcal{S}$. Given the policies

$$\boldsymbol{\pi}_{-i} = (\pi_1, \dots, \pi_{i-1}, \pi_{i+1}, \dots, \pi_N)$$

of the other relays, the goal of relay $i$ is to minimize the expected cost

$$J_i(\tau, 0; \pi, \boldsymbol{\pi}_{-i}) = \mathbb{E}\left\{ \int_\tau^\infty g(x_i(t), \pi(s, x_i(t), \mathbf{x}_{-i}(t))\, dt \right\}, \tag{3.2}$$

over all policies $\pi$. In the above equation, $J_i(s, x; \pi, \boldsymbol{\pi}_{-i})$ represents the expected cost-to-go for relay $i$ under policy $\pi$ if it is in state $x$ at time $s$, and $\tau$ is the first time relay $i$ enters state 0. The cost for relay $i$ depends upon the states and the policies of the other relays only through $p_k^i$ (see (3.1)).

Let

$$J_i^*(t, x; \boldsymbol{\pi}_{-i}) = \lim_{M\to\infty} \inf_{\pi} J_i(t, x; \pi, \boldsymbol{\pi}_{-i}),$$

be the optimal cost-to-go for the tagged relay if it is in state $x \in \mathcal{S}$ at time $t$ when $M \to \infty$.

**Proposition 3.4.1.** *When $M \to \infty$, the optimality equations read as follows*

$$J_i^*(t,1;\boldsymbol{\pi}_{-i}) = \min\left(0, \inf_{s\geq t} G_k^i(t,s)\right), \tag{3.3}$$

$$J_i^*(t,m_s;\boldsymbol{\pi}_{-i}) = \min\left(0, C_r + \inf_{s\geq t} G_k^i(t,s)\right), \tag{3.4}$$

*where $G_k^i(t,s)$ is defined in* (3.1).

Before going to the proof, we first have some observation and present two lemmas which will be used in the proof of the Proposition. We consider a given relay (say relay $i$) and establish the optimality equations of problem (3.2) for this relay in the limiting regime $M \to \infty$. The proof proceeds in two steps: (a) assuming $M$ is large but fixed, we first use the well-know uniformization technique [55] to establish the optimality equations for an equivalent discrete-time MDP, and (b) we then establish the limits of these optimality equations when $M \to \infty$.

To simplify notations, let $q = \frac{\lambda}{M}$, $p = \frac{\mu}{M}$, $\bar{p} = 1-p$ and $\bar{q} = 1-q$. Denoting by $\mathbf{Q}$ the infinitesimal generator of the controlled CTMC shown in Figure 3.3, the equivalent discrete-time MDP has transition matrix $\mathbf{P}(a) = \mathbf{I} + \frac{1}{M}\mathbf{Q}(a)$ under action $a$, that is,

$$\mathbf{P}(a) = \begin{array}{c} \\ 0 \\ m_s \\ 1 \\ m_d \\ 2 \end{array}\begin{array}{c} \begin{array}{ccccc} 0 & m_s & 1 & m_d & 2 \end{array} \\ \begin{pmatrix} \bar{q} & q & 0 & 0 & 0 \\ 0 & 0 & \mathbb{1}_{\{a=accept\}} & 0 & \mathbb{1}_{\{a=reject\}} \\ 0 & 0 & \bar{p}\,\mathbb{1}_{\{a=keep\}} & p\,\mathbb{1}_{\{a=keep\}} & \mathbb{1}_{\{a=drop\}} \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array},$$

and costs-per-stage

$$\tilde{g}(x,a,\mathbf{x}_{-i}) = \frac{1}{M} g(x,a,\mathbf{x}_{-i}), \quad \forall a \in \mathcal{A}(x), \forall x \in \mathcal{S}.$$

Let $V_n(x)$ be the optimal cost-to-go of relay $i$ starting in state $x \in \mathcal{S}$ at time $n$, and let $q(n)$ denotes the probability that the relay be the first one to deliver the message to the destination at that time. Note that the latter probability depends on the policies $\boldsymbol{\pi}_{-i}$ of the other relays, although we do not make explicit this dependence. Lemma 3.4.1 establishes the optimality equations for the states $x = m_s$ and $x = 1$.

**Lemma 3.4.1.** *Provided that $R \geq C_r + C_d + \frac{C_s}{\mu}$, the optimal costs-to-go are given by*

$$
\begin{aligned}
V_n(m_s) &= \min\left(0, C_r + V_{n+1}(1)\right), && (3.5)\\
V_n(1) &= \min\left(0, U_{n,1}, U_{n,2}, \ldots\right), && (3.6)
\end{aligned}
$$

*where*

$$
U_{n,m} = \frac{\mu}{M} \sum_{k=0}^{m-1} (\bar{p})^k \left\{ \frac{C_s}{\mu} - \overline{R}\, q(n+k+1) \right\}. \tag{3.7}
$$

*Proof.* Since $V_n(m_d) = (C_d - R)\, q(n)$, the dynamic programming equation

$$
V_n(x) = \min_{a \in \mathcal{A}(x)} \left\{ \tilde{g}(x_n, a, \mathbf{x}_{-n}) + \sum_{y \in \mathcal{S}} p_{x,y}(a) V_{n+1}(y) \right\} \tag{3.8}
$$

yields

$$
\begin{aligned}
V_n(1) &= \min\left(0, \frac{C_s}{M} - p\overline{R}q(n+1) + \bar{p}V_{n+1}(1)\right)\\
&= \min\Bigg(0, \frac{C_s}{M} - p\overline{R}q(n+1), \frac{C_s}{M}(1+\bar{p}) -\\
&\qquad p\overline{R}\sum_{k=0}^{1}(\bar{p})^k q(n+k+1) + (\bar{p})^2 V_{n+2}(1)\Bigg),
\end{aligned}
$$

which can be developed recursively to obtain

$$
V_n(1) = \min\left(0, U_{n,1}, U_{n,2}, \ldots\right).
$$

The optimal cost-to-go in state $m_s$ is obtained directly from the dynamic programming equation (3.8). □

We note that the term $U_{n,m}$ in Lemma 3.4.1 corresponds to the cost obtained if the action "keep" is played $m$ consecutive times starting from the current decision epoch $n$, until the relay meets the destination or decides to drop the message. The optimal policy at instant $n$ is to retain the message if either of the $U_{n,m}$ is negative. Otherwise it is optimal to drop the message.

We now turn to the second part of the proof, which is based on Lemma 3.4.2.

**Lemma 3.4.2.** *Let $s, t \in \mathbb{R}$, $s > t \geq 0$. We have*

$$\lim_{M\to\infty} U_{\lfloor Mt\rfloor,\lfloor M(s-t)\rfloor} = G_k^i(t,s). \tag{3.9}$$

*Proof.* To simplify notation, let $n = \lfloor Mt \rfloor$ and $m = \lfloor M(s-t)\rfloor$. The term $U_{n,m}$ can be rewritten as follows

$$\begin{aligned} U_{n,m} &= \frac{C_s}{\mu}\left(1-\bar{p}^m\right) - \frac{\mu}{M}\overline{R}\sum_{k=0}^{m-1}(\bar{p})^k q(n+k+1), \\ &= \frac{C_s}{\mu}\left(1-\left(1-\frac{\mu}{M}\right)^m\right) - \frac{\mu}{M}\overline{R}\sum_{k=0}^{m-1}\left(1-\frac{\mu}{M}\right)^k q(n+k+1), \end{aligned} \tag{3.10}$$

Since $m = \lfloor M(s-t)\rfloor$, for the first term on the LHS, we have

$$\lim_{M\to\infty}\frac{C_s}{\mu}\left(1-\left(1-\frac{\mu}{M}\right)^m\right) = \frac{C_s}{\mu}\left(1-e^{-\mu(t-s)}\right).$$

Besides, since the discrete-time Markov chain corresponds to the original continuous-time Markov chain observed at random times according to a Poisson process with intensity $M\,t$, we can identify $q(n+k+1)$ with $p_k^i(t+\frac{k+1}{M})$, so that the second term on the LHS of (3.10) can be rewritten as follows

$$\frac{\mu}{M}\overline{R}\sum_{k=0}^{m-1}\left(1-\frac{\mu}{M}\right)^k p_k^i(t+\frac{k+1}{M}).$$

Approximating $\left(1-\frac{\mu}{M}\right)^k$ by $e^{-\frac{k\mu}{M}}$, it yields

$$e^{\frac{\mu}{M}}\sum_{k=1}^{m}\mu\overline{R}e^{-\frac{k\mu}{M}}p_k^i(t+\frac{k}{M})\frac{1}{M},$$

which can be rewritten as

$$e^{\frac{\mu}{M}}\sum_{k=1}^{m} f\left(t+\frac{k}{M}\right)(x_k - x_{k-1}),$$

where $f(x) = \mu\overline{R}e^{-\mu(x-t)}p_k^i(x)$ and $x_k = t + \frac{k}{M}$ for $k = 0,\ldots,m$. When $M\to\infty$, the term $e^{\frac{\mu}{M}} \to 1$, whereas the Riemann sum

$$\sum_{k=1}^{m} f\left(t+\frac{k}{M}\right)(x_k - x_{k-1}) \xrightarrow[M\to\infty]{} \int_t^s \mu\overline{R}e^{-\mu(x-t)}p_k^i(x)dx.$$

In view of (3.1), summing the limits of the first and second terms on the LHS of (3.10) concludes the proof. □

We are now in position to prove Proposition 3.4.1.

*Proof of Proposition 3.4.1.* The proof directly follows from Lemmata 3.4.1 and 3.4.2 since

$$\begin{aligned} J_i^*(t,1;\boldsymbol{\pi}_{-i}) &= \lim_{M\to\infty} V_{\lfloor Mt\rfloor}(1) \\ &= \min\left(0, \min_{k\geq 1} \lim_{M\to\infty} U_{\lfloor Mt\rfloor,k}\right) \\ &= \min\left(0, \inf_s G_k^i(t,s)\right), \end{aligned}$$

and the result on $J_i^*(t,m_s;\boldsymbol{\pi}_{-i})$ is obtained similarly. □

### 3.4.1 Best-Response Policy

From now on, we shall only consider the limiting regime $M\to\infty$. In words, Proposition 3.4.1 says that if relay $i$ has the message at time $t$, its best-response is to keep it if and only if there exists $s\geq t$ such that the expected cost $G_k^i(t,s)$ of keeping the message in the interval $(t,s)$ is negative. Similarly, if relay $i$ meets the source at time $t$, its best-response is to accept the message if and only if there exists $s\geq t$ such that the expected reward $-G_k^i(t,s)$ offsets the cost of receiving the message from the source. Using the optimality equations stated in Proposition 3.4.1, Theorem 3.4.1 establishes the structure of the best-response policy of a relay.

**Theorem 3.4.1.** *Given the strategies of the other relays, the best-response policy $\pi_i^*(t;x)$ of relay $i$ is a threshold-type policy: there exists a $\theta_k^i$ and $\gamma_k^i>\theta_k^i$ such that $\pi_i^*(t;m_s)=$ accept if and only if $t\leq\theta_k^i$, and $\pi_k^*(t;1)=$ drop if and only if $t>\gamma_k^i$. Moreover, $(\theta_k^i,\gamma_k^i)$ is the solution of:*

$$\gamma_k^i = \sup\{t : p_k^i(t) > \frac{C_s}{\mu(R-C_d)}\}, \tag{3.11}$$

$$\theta_k^i = \sup\{t : C_r + G_k^i(t,\gamma_k^i) < 0\}, \tag{3.12}$$

*where by convention the supremum of the empty set is* 0.

We have some observation before proving the Theorem. Define

$$g_k^i(t,s) = \mu e^{-\mu(s-t)}(\frac{C_s}{\mu} - \overline{R}\,p_k^i(s)).$$

Note that

$$G_k^i(t,s) = \int_t^s g_k^i(t,x)dx.$$

That is, $g_k^i(t,x)$ is the marginal cost of keeping the message at time $x$ given that it was accepted at time $t$. The crucial observation is that the sign of $g_k^i(t,s)$ depends only on $s$:

$$g_k^i(t,s) < 0 \iff \frac{C_s}{\mu \overline{R}} < p_k^i(s), \quad \forall t, \forall s \geq t. \tag{3.13}$$

We use this observation in Lemma 3.4.3 below.

**Lemma 3.4.3.** *Define $\phi(t) = \inf_{s \geq t} G_k^i(t,s)$ for all $t \geq 0$, and let $\gamma_k^i$ be defined as in Theorem 3.4.1. Then:*

*(a) $\phi(t) \geq 0$ for all $t \geq \gamma_k^i$ and $\phi(t) < 0$ for all $t < \gamma_k^i$,*

*(b) $\phi(t) = G_k^i(t, \gamma_k^i)$ for all $t < \gamma_k^i$, and*

*(c) $\phi(t)$ is strictly increasing in $t$ in the interval $[0, \gamma_k^i]$.*

*Proof.* (a) By definition of $\gamma_k^i$, $p_k^i(y) \leq \frac{C_s}{\mu \overline{R}}$ for all $y \geq \gamma_k^i$. According to (3.13), it yields $g_k^i(t,y) \geq 0$ for all $t$ and $y$ such that $y \geq t$ and $y \geq \gamma_k^i$. Hence

$$G_k^i(t,s) = \int_t^s g_k^i(t,y)\,dy \geq 0, \quad \forall s \geq t, \forall t \geq \gamma_k^i,$$

implying that

$$\phi(t) = \inf_{s \geq t} G_k^i(t,s) \geq 0 \quad \text{for al} \quad t \geq \gamma_k^i.$$

Similarly, for $y < \gamma_k^i$, we have $p_k^i(y) > \frac{C_s}{\mu \overline{R}}$. With (3.13), it implies that $g_k^i(t,y) < 0$ for all $t$ and $y$ such that $t \leq y < \gamma_k^i$. Hence

$$\phi(t) = \inf_{s \geq t} G_k^i(t,s) \leq G_k^i(t, \gamma_k^i) = \int_t^{\gamma_k^i} g_k^i(t,y)\,dy < 0.$$

We thus conclude that $\phi(t) \geq 0$ for all $t \geq \gamma_k^i$ and $\phi(t) = G_k^i(t, \gamma_k^i) < 0$ for all $t < \gamma_k^i$, as claimed.

(b) We know that $g_k^i(t,y) < 0$ for all $t$ and $y$ such that $t \leq y < \gamma_k^i$. This implies that for $t < \gamma_k^i$ fixed, $G_k^i(t,y)$ is a strictly decreasing function of $y$ on the interval $[t, \gamma_k^i]$, so that $G_k^i(t,y) > G_k^i(t,\gamma_k^i)$. Moreover, $g_k^i(t,y) \geq 0$ for all $t$ and $y$ such that $y \geq t$ and $y \geq \gamma_k^i$. This implies that, for $t < \gamma_k^i$ fixed, $G_k^i(t,y)$ is a non-decreasing function of $y$ on the interval $[\gamma_k^i, \infty]$, so that $G_k^i(t,y) \geq G_k^i(t,\gamma_k^i)$. As a consequence,

$$\phi(t) = \inf_{s \geq t} G_k^i(t,s) = G_k^i(t,\gamma_k^i) \quad \text{for all} \quad t < \gamma_k^i.$$

(c) We note that from assertion (b), we have $\phi(t) = G_k^i(t,\gamma_k^i)$ for all $t < \gamma_k^i$, so that $\phi'(t) = -g_k^i(t,t)$. Since $g_k^i(t,y) < 0$ for all $t$ and $y$ such that $t \leq y < \gamma_k^i$, we have $g_k^i(t,t) < 0$ for all $t < \gamma_k^i$, and thus $\phi'(t) > 0$ for all $t < \gamma_k^i$.

□

The proof of Theorem 3.4.1 now readily follows from Proposition 3.4.1 and Lemma 3.4.3.

*Proof of Theorem 3.4.1.* We know from Proposition 3.4.1 that

$$J_i^*(t,1;\boldsymbol{\pi}_{-i}) = \min(0,\phi(t))$$

As proven in Lemma 3.4.3, $\phi(t)$ is negative if and only if $t < \gamma_k^i$. Therefore, the best-response policy of the tagged relay is such that if it is in state 1 at time $t$, it should keep the message if $t < \gamma_k^i$, and drop it otherwise. Similarly, according to Proposition 3.4.1, we have

$$J_i^*(t,m_s;\boldsymbol{\pi}_{-i}) = \min(0, C_r + \phi(t))$$

Therefore, if the relay is in state $m_s$ at time $t$, it should accept the message if and only if $C_r + \phi(t) < 0$. If $t \geq \gamma_k^i$, the relay should reject the message since, as proven in Lemma 3.4.3, $\phi(t) \geq 0$. If on the contrary $t < \gamma_k^i$, we have $C_r + \phi(t)$ so the relay should accept the message if and only if $C_r + G_k^i(t,\gamma_k^i) < 0$, that is, if and only if $t \leq \theta_k^i$, as claimed. □

Note that in Theorem 3.4.1, nothing precludes that $\gamma_k^i = \infty$, or even that $\theta_k^i = \gamma_k^i = \infty$, as shown in the following example. Consider the case where the strategy of the other relays is to never accept the message. In that case, the success probability of relay $i$ is $p_k^i(t) = 1$ for all $t \geq 0$, and it follows from the assumption $R \geq R_{min}^i$ that $\gamma_k^i = \infty$. Moreover, since in that case $G_k^i(t,\infty) = \frac{C_s}{\mu} - \overline{R} < 0$ for all $t \geq 0$, we also have $\theta_k^i = \infty$. Hence, the best-response policy of player $i$ is to always accept the message and to keep it forever. This shows in particular that, under the assumption $R \geq R_{min}^i$, the vector of policies in which all relays always reject the message cannot be a Nash equilibrium of our game.

## 3.5 Nash Equilibrium

The Nash equilibrium of the forwarding game is defined as a vector

$$\boldsymbol{\pi}^* = (\pi_1^*, \ldots, \pi_N^*)$$

of policies from which no relay finds it beneficial to unilaterally deviate. Formally, $\boldsymbol{\pi}^*$ is a Nash equilibrium of the game if and only if

$$J_i(\tau, 0; \pi_i^*, \boldsymbol{\pi}_{-i}^*) \leq J_i(\tau, 0; \pi, \boldsymbol{\pi}_{-i}^*), \quad \forall \pi, \forall i \tag{3.14}$$

A direct consequence of Theorem 3.4.1 is the following structural result of any Nash equilibrium.

**Corolary 3.5.1.** *At a Nash equilibrium, if any, all players use a threshold-type policy, that is, there exist vectors $\boldsymbol{\theta}_k$ and $\boldsymbol{\gamma}_k$ such that relay $i$ uses a threshold-type strategy with parameters $(\theta_k^i, \gamma_k^i)$.*

A Nash equilibrium can be asymmetric or symmetric. An asymmetric equilibrium can be of the form: relay 1 always accepts and keeps the message until it meets the destination and relay 2 never accepts. For an example of this type of asymmetric equilibrium, assume that $\lambda = \mu = 1$, and let $k = 1$. Under the given policy of relay 1, the probability of success of relay 2 at time $t$ will be

$$p_1^2(t) = e^{-t}(1 + t). \tag{3.15}$$

From the above equation and (3.11), it follows that $\gamma^2$ will be finite. Suppose that relay 2 meets the source at time 0. This is the most favorable scenario for relay 2. If it is not profitable to accept the message at time 0, then it will never be so later on. From (3.1), if the relay meets the source at time 0, then its total cost to go if it accepts the message and keeps it until time $\gamma^2$ will be

$$\begin{aligned} C_r + G_1^2(0, \gamma^2) &> C_r - \bar{R} \int_0^{\gamma^2} (1+t)e^{-2t} dt \\ &> C_r - \bar{R} \int_0^{\infty} (1+t)e^{-2t} dt = C_r - \frac{3\bar{R}}{4} \end{aligned}$$

That is, if $\bar{R} < \frac{4}{3}C_r$, then relay 2 will always have a positive cost of accepting and its best response will be never to accept. Of course, if $\bar{R} > C_r + C_s$ then relay 1 will always accept if it knows that relay 2 will never accept because this reward is greater than the average cost incurred by one relay. Thus, we have the claimed asymmetric equilibrium.

We shall study the existence and uniqueness of only the symmetric Nash equilibria, that is, equilibria in which all relays use the same thresholds $\theta_k$ and $\gamma_k$ for message $k$. Using the fact that best-response policies are of threshold type, we obtain an explicit expression of the success probability $p_k^i(t)$. Assuming that up to message $k-1$ only symmetric equilibria have been played, we use this simple expression of the success probability in the following to establish the conditions under which there exists a unique symmetric Nash equilibrium.

## Symmetric Nash Equilibrium

Assume that all relays have played symmetric equilibria[2] for messages $1, 2, \ldots, k-1$, that is, $\theta_j^i = \theta_j$ and $\gamma_j^i = \gamma_j$ for $j = 1, 2, \ldots, k-1$. A direct consequence of Corollary 3.5.1 is that if all relays play their Nash equilibrium strategies, the success probability $p_k(t)$ of a player has a very simple structure. For $y \geq x \geq 0$, let

$$v_{x,y}(s) = e^{-\lambda(x-s)} + \frac{\lambda}{\mu-\lambda} e^{-\mu y} e^{\lambda s} \left( e^{(\mu-\lambda)x} - e^{(\mu-\lambda)s} \right), \tag{3.16}$$

for $s < x$, and $v_{x,y}(s) = 1$ otherwise. Note that if $x = \min(\theta_k, t)$ and $y = \min(\gamma_k, t)$, $v_{x,y}(s)$ represents the probability that relay $j$ be not able to deliver the message $k$ by time $t$ given that it comes into play at time $s$. Then, introducing

$$V_k(x,y) = \int_{\theta_{k-1}}^{\infty} \phi_k(s) v_{x,y}(s) ds, \tag{3.17}$$

$$= 1 - \int_{\theta_{k-1}}^{x} \phi_k(s)(1 - v_{x,y}(s)) ds, \tag{3.18}$$

the quantity

$$f_k(t) = V_k\left(\min(\theta_k, t), \min(\gamma_k, t)\right), \tag{3.19}$$

represents the probability that a relay fails to deliver the message to the destination by time $t$, either because it does not meet the source by time

[2] In the sequel, since we are treating the symmetric case, we shall not use superscripts to distinguish relays or player. For example, we shall use $p_k(t)$ instead of $p_k^i(t)$ for the probability of success.

$\min(\theta_k, t)$, or because it meets it but does not meet the destination before $\min(\gamma_k, t)$. It then follows that the probability of success of a given relay is

$$p_k(t) = f_k^{N-1}(t) = V_k\left(\min(\theta_k, t), \min(\gamma_k, t)\right)^{N-1}. \tag{3.20}$$

Note that $p_k(t)$ is constant after $\gamma_k$, and that $p_k(t) = V_k(\theta_k, t)^{N-1}$ for all $t \in [\theta_k, \gamma_k]$. As a consequence, defining

$$\omega = \left(\frac{C_s}{\mu \overline{R}}\right)^{1/(N-1)}, \tag{3.21}$$

the second threshold $\gamma_k$ is the greatest value of $t$ such that $V_k(\theta_k, t) \geq \omega$. For a given $\theta \geq 0$, let

$$\gamma(\theta) = \sup\{x : V_k(\theta, x) \geq \omega\}. \tag{3.22}$$

Note that the function $\gamma(\theta)$ takes its values in $[0, \infty]$. We establish in Lemma 3.5.1 that there exist $\theta_{min}$ and $\theta_{max}$ such that $\gamma(\theta) = \infty$ for $\theta \leq \theta_{min}$, whereas $\gamma(\theta)$ takes a uniquely defined finite value for $\theta \in (\theta_{min}, \theta_{max}]$.

**Lemma 3.5.1.** *Let $\theta_{min}$ be the solution of*

$$1 + \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s)\left(e^{-\lambda(\theta_{min}-s)} - 1\right) ds = \omega, \tag{3.23}$$

*and $\theta_{max}$ be such that*

$$V_k(\theta_{max}, \theta_{max}) = \omega. \tag{3.24}$$

*Then, for $\theta$ fixed, the equation $V_k(\theta, \gamma)$ has a unique finite solution $\gamma \geq \theta$ if and only if $\theta \in (\theta_{min}, \theta_{max}]$. Moreover, $\gamma(\theta)$ is a strictly decreasing function of $\theta \in (\theta_{min}, \theta_{max}]$.*

We shall first establish some properties of the function $V_k(x, y)$ in Lemma 3.5.2, before proving Lemma 3.5.1.

**Lemma 3.5.2.** *For $y$ fixed, the function $V_k(x, y)$ is strictly decreasing in $x$ in the interval $[0, y)$, and for $x > 0$ fixed, it is strictly decreasing in $y$ in the interval $[0, \infty]$.*

*Proof.* The proof directly follows from

$$\begin{aligned}
\frac{\partial V_k}{\partial x}(x, y) &= \int_{\theta_{k-1}}^{x} \phi_k(s) \frac{\partial v_{x,y}}{\partial x}(s)\, ds, \\
&= \lambda\, e^{-\lambda x}\left(e^{-\mu(y-x)} - 1\right) \int_{\theta_{k-1}}^{x} \phi_k(s) e^{\lambda s}\, ds,
\end{aligned}$$

which is negative for all $y > x$, and

$$\begin{aligned}\frac{\partial V_k}{\partial y}(x,y) &= \int_{\theta_{k-1}}^{x} \phi_k(s)\frac{\partial v_{x,y}}{\partial y}(s)\,ds,\\ &= -\frac{\lambda\mu e^{-\mu y}}{\mu-\lambda}\int_{\theta_{k-1}}^{x} \phi_k(s)e^{\lambda s}\left(e^{(\mu-\lambda)x} - e^{(\mu-\lambda)s}\right)ds,\end{aligned}$$

which is also negative for all $y, x \geq 0$. □

These properties of the function $V_k(x, y)$ are now used to prove Lemma 3.5.1.

*Proof of Lemma 3.5.1.* Since according to Lemma 3.5.2 the continuous function $V_k(x, y)$ is strictly decreasing in $y$ for $x$ fixed, the equation $V_k(\theta, \gamma) = \omega$ has a solution $\gamma \geq \theta$ if and only if

$$\lim_{y\to\infty} V_k(\theta, y) < \omega \leq V_k(\theta, \theta).$$

With (3.16) and (3.17), the LHS inequality directly leads to $\theta > \theta_{min}$, whereas the RHS one yields $\theta \leq \theta_{max}$. Hence, for $\theta$ fixed, the equation $V_k(\theta, \gamma) = \omega$ has a solution $\theta \leq \gamma < \infty$ if and only if $\theta \in (\theta_{min}, \theta_{max}]$.

To show that $\gamma(\theta)$ is decreasing on $(\theta_{min}, \theta_{max}]$, note that

$$\frac{d}{dx}V_k(x,\gamma(x)) = \frac{\partial}{\partial x}V_k(x,\gamma(x)) + \gamma'(x)\frac{\partial}{\partial y}V_k(x,\gamma(x)).$$

On the interval $(\theta_{min}, \theta_{max}]$, the function $V_k\left(\theta, \gamma(\theta)\right)$ is a constant, and its derivative is thus 0. From Lemma 3.5.2, both the partial derivatives of $V_k$ are negative, from which we conclude that derivative of $\gamma(\theta)$ is strictly negative. □

We use Lemma 3.5.1 to establish in Theorem 3.5.1 below the conditions under which there exists a unique symmetric Nash equilibrium.

**Theorem 3.5.1.** *There exists a symmetric Nash equilibrium with $\theta > 0$ if and only if*

$$\overline{R} \geq C_r + \frac{C_s}{\mu}. \tag{3.25}$$

*Under this condition, the symmetric Nash equilibrium is unique. Moreover, the parameters of the equilibrium are finite, i.e., $\theta_k > 0$ and $\theta_k \leq \gamma_k < \infty$ if and only if*

$$1 + \mu\frac{C_r}{C_s} < \frac{(1+b)^N - 1}{N\,b} \tag{3.26}$$

*where*

$$b = \frac{1}{\sigma\omega} \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left( e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)} \right) ds, \tag{3.27}$$

*and* $\sigma = (\mu - \lambda)/\lambda$.

Define the function

$$\hat{G}_k(\theta) = G_k(\theta, \gamma(\theta)). \tag{3.28}$$

The value of $\theta$ at an equilibrium is determined by a solution of $\hat{G}_k(\theta) = -C_r$. Thus, the number of equilibria will depend upon the number of roots of the equation $\hat{G}_k + C_r = 0$ on the positive real line.

The next result gives some properties of $\hat{G}_k$ that are then sufficient to conclude the unicity of the symmetric equilibrium.

**Lemma 3.5.3.** *On the interval* $[0, \theta_{max}]$, *the function* $\hat{G}_k$ *is*

*(a) continuous;*

*(b) strictly increasing; with*

*(c)*

$$\hat{G}_k(0) = \frac{C_s}{\mu} - \overline{R}, \tag{3.29}$$

$$\hat{G}_k(\theta_{max}) = 0. \tag{3.30}$$

*Proof.* (a) The continuity of $\hat{G}_k$ on the open interval $[0, \theta_{min}) \cup (\theta_{min}, \theta_{max}]$ follows from the definition of $G_k$. In order to show the continuity of $\hat{G}_k$ it is thus sufficient to show that

$$\lim_{\theta \to \theta_{min}^-} \hat{G}_k(\theta) = \lim_{\theta \to \theta_{min}^+} \hat{G}_k(\theta).$$

In order to prove this, observe that we can write

$$\hat{G}_k(\theta) = \int_{\theta}^{\gamma(\theta)} g_k(\theta, t) dt, \tag{3.31}$$

where

$$g_k(\theta, t) = \mu e^{-\mu(t-\theta)} \left(\frac{C_s}{\mu} - \overline{R}\, V_k(\theta, t)^{N-1}\right)$$

is the marginal cost of keeping a message at time $t$ given that it was accepted at time $\theta$. Since $V_k(\theta, t) \in [0, 1]$, we have

$$|g_k(\theta, t)| \le m_1 \mu e^{-\mu(t-\theta)}$$

for all $\theta$ and $t$, where

$$m_1 = \max\left(\overline{R} - \frac{C_s}{\mu}, \frac{C_s}{\mu}\right)$$

. It yields

$$\begin{aligned}
\left|\int_{\theta_{min}}^{\infty} g_k(\theta, t)dt - \hat{G}_k(\theta)\right| &\le \int_{\theta_{min}}^{\theta} |g_k(\theta, t)|\, dt + \int_{\gamma(\theta)}^{\infty} |g_k(\theta, t)| dt \\
&\le m_1 \left\{e^{-\mu(\theta_{min}-\theta)} - 1 + e^{-\mu(\gamma(\theta)-\theta)}\right\},
\end{aligned}$$

from which we conclude that

$$\begin{aligned}
\lim_{\theta \to \theta_{min}^+} \hat{G}_k(\theta) &= \lim_{\theta \to \theta_{min}^+} \int_{\theta_{min}}^{\infty} g_k(\theta, t)dt, \\
&= \int_{\theta_{min}}^{\infty} g_k(\theta_{min}, t)dt.
\end{aligned}$$

where the last equality is obtained using the dominated convergence theorem. Similar arguments can be used to establish that $\hat{G}_k(\theta)$ converges to the same limit when $\theta \to \theta_{min}^-$.

(b) We have

$$\begin{aligned}
\frac{dG_k}{d\theta}(\theta, \gamma(\theta)) &= g_k(\theta, \gamma(\theta))\gamma'(\theta) - g_k(\theta, \theta) \\
&\quad + \int_{\theta}^{\gamma(\theta)} \frac{\partial g_k}{\partial \theta}(\theta, x)dx \qquad (3.32) \\
&= g_k(\theta, \gamma(\theta)) - g_k(\theta, \theta) + \int_{\theta}^{\gamma(\theta)} \frac{\partial g_k}{\partial \theta}(\theta, x)dx (3.33) \\
&= \int_{\theta}^{\gamma(\theta)} \left(\frac{\partial g_k}{\partial x}(\theta, x) + \frac{\partial g_k}{\partial \theta}(\theta, x)\right) dx,
\end{aligned}$$

where (3.33) is obtained from (3.32) by observing that, for $\theta > \theta_{min}$, $v(\theta, \gamma(\theta)) = \omega$ implies that $g_k(\theta, \gamma(\theta)) = 0$, whereas for $\theta \leq \theta_{min}$, $\gamma(\theta) = \infty$ also implies $g_k(\theta, \gamma(\theta)) = 0$. Since

$$\begin{aligned}\frac{\partial g_k}{\partial x}(\theta, x) + \frac{\partial g_k}{\partial \theta}(\theta, x) &= -\overline{R}(N-1)\mu e^{-\mu(x-\theta)} V_k(\theta, x)^{N-2} \\ &\quad \times \left( \frac{\partial V_k}{\partial x}(\theta, x) + \frac{\partial V_k}{\partial \theta}(\theta, x) \right),\end{aligned}$$

we conclude from Lemma 3.5.2 that $G_k(\theta, \gamma(\theta))$ is strictly increasing in $\theta$.

(c) Equality (3.29) follows from noting that $V_k(0, x) = 1$, and using this in (3.31). Similarly, (3.30) is obtained by noting that $\gamma(\theta_{max}) = \theta_{max}$ (from Lemma 3.5.1), and using this in (3.31).

□

An immediate consequence of Lemma 3.5.3 is stated in Corolary 3.5.2.

**Corolary 3.5.2.** *There is a unique solution to $\hat{G}_k(\theta) = -C_r$ in the interval $[0, \theta_{max}]$ if and only if*

$$C_r + \frac{C_s}{\mu} \leq \overline{R}$$

We are now in position to prove Theorem 3.5.1.

*Proof of Theorem 3.5.1.* From Lemma 3.5.1 there is a unique $\gamma$ for a given $\theta > 0$ that satifies (3.11). Also, from Corollary 3.5.2 there is unique $\theta > 0$ that satisfies (3.12) if and only if $C_r + \frac{C_s}{\mu} \leq \overline{R}$. Thus, this last inequality is necessary and sufficient for the existence of a unique symmetric equilibrium.

From Lemma 3.5.3 and Lemma 3.5.1, we deduce that, for $\gamma$ to be finite the necessary and sufficient condition is

$$\hat{G}_k(\theta_{min}) < -C_r.$$

From (3.18),

$$\begin{aligned}V_k(\theta_{min}, x + \theta_{min}) &= 1 + \int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s) \left\{ \left(e^{-\lambda(\theta_{min}-s)} - 1\right) + \right. \\ &\quad \left. \frac{e^{-\mu x}}{\sigma} \left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)}\right) \right\} ds \\ &= \omega\left(1 + e^{-\mu x} b\right),\end{aligned}$$

where

$$b = \frac{1}{\sigma\omega}\int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s)\left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)}\right) ds.$$

Then,

$$\begin{aligned}\hat{G}_k(\theta_{min}) &= \frac{C_s}{\mu} - \overline{R}\int_0^{\infty} \mu e^{-\mu x} v(\theta_{min}, x+\theta_{min})^{N-1} dx \\ &= \frac{C_s}{\mu} - \overline{R}\frac{\omega^{N-1}}{Nb}\left((1+b)^N - 1\right).\end{aligned}$$

where the last equality follows from the binomial formula.

Thus, $\hat{G}_k(\theta_{min}) < -C_r$ if and only if

$$\frac{C_s}{\mu} - \overline{R}\frac{\omega^{N-1}}{Nb}\left((1+b)^N - 1\right) < -C_r,$$

which, since $\omega^{N-1} = C_s/(\mu\overline{R})$, is equivalent to

$$1 + \mu\frac{C_r}{C_s} < \frac{(1+b)^N - 1}{Nb},$$

as claimed. □

We remind the reader that $R_{min} = C_r + \frac{C_s}{\mu} + C_d$ is the minimum value that the reward $R$ should have for a single relay to attempt the delivery of a message. Theorem 3.5.1 shows that for any value of $R$ greater than this minimum value, the existence of a unique symmetric equilibrium is guaranteed. Figure 3.4 illustrates the condition (3.26) for the first message when $N = 3$, $\mu = 0.4$, $C_s = 0.5$ and $C_r = 4.0$. In that case, the minimum value of $\bar{R}$ is $C_r + \frac{C_s}{\mu} = 5.25$. We note that for the first message we have $e^{-\lambda\theta_{min}} = \omega$ and thus $b = \sigma^{-1}(1-\omega^{\sigma})$.

**Remark** The condition (3.26) of Theorem 3.5.1 can be equivalently written as

$$f(b) > 1 + \mu\frac{C_r}{C_s}$$

where

$$f(x) = \frac{(1+x)^N - 1}{Nx}.$$

Note that $b > 0$ for all $\sigma \in [-1, \infty)$ and all $\omega \in (0, 1)$. Using the binomial formula, it is easy to show that $f(x) > 1 + g(x)$ for all $x > 0$, where

$$g(x) := \frac{N-1}{2}x\left(1 + \frac{N-2}{3}x\right)$$

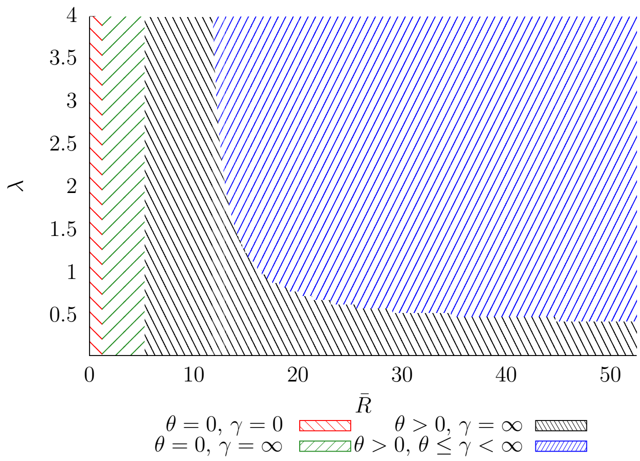



Figure 3.4: Existence of a symmetric Nash equilibrium for 3 relays.

Hence, a sufficient condition for $\theta_k$ and $\gamma_k$ to be finite is $g(b) > \mu \frac{C_r}{C_s}$. Since $g(x)$ is strictly increasing over $[0, \infty)$, this is equivalent to $b > y$, where

$$y = \frac{3}{N-2}\left(\sqrt{\frac{1}{4} + \frac{2}{3}\mu\frac{C_r}{C_s}\frac{N-2}{N-1}} - \frac{1}{2}\right),$$

is the unique solution of $g(x) = \mu \frac{C_r}{C_s}$. The latter sufficient condition can be written as

$$\overline{R} > \frac{C_s}{\mu}\left(\frac{1}{\sigma y}\int_{\theta_{k-1}}^{\theta_{min}} \phi_k(s)\left(e^{-\lambda(\theta_{min}-s)} - e^{-\mu(\theta_{min}-s)}\right) ds\right)^{-\frac{1}{N-1}}.$$

## 3.6 Transient analysis of the symmetric equilibrium

In the rest of the chapter we shall focus only on the symmetric equilibrium. First, we give an algorithm to compute the thresholds $\theta_k$ and $\gamma_k$ for message $k$ which will then be used to derive the probability of message delivery and expected message delay from these thresholds.

### 3.6.1 Recursive computation of the success probability

The thresholds $\theta_k$ and $\gamma_k$ of a symmetric equilibrium are obtained from (3.11) and (3.12), in which $p_k^i(t) = V_k(\theta_k, t)^{N-1}$ for all $t \in [\theta_k, \gamma_k]$ and all relays $i$.

The computation of the function $V_k(x,y)$ however requires the knowledge of the probability density function $\phi_k(t)$, $\int_a^b \phi_k(t)dt$ representing the probability that a given relay comes into play for the delivery of the $k^{th}$ message between time instants $a$ and $b$. In this section, we shall show how this probability density function can be recursively computed for symmetric equilibria .

To this end, let us define $I_k(x,t)$ as the probability that a relay that comes into play at time $x$ will accept the $k^{th}$ message and will not be able to deliver it to the destination by time $t \in [\theta_k, \gamma_k]$. Therefore,

$$\begin{aligned} I_k(x,t) &= \int_x^{\theta_k} \lambda e^{-\lambda(s-x)} e^{-\mu(t-s)} ds \\ &= \frac{e^{-\mu t}}{\mu - \lambda} \lambda e^{\lambda x} \left( e^{(\mu-\lambda)\theta_k} - e^{(\mu-\lambda)x} \right) \end{aligned} \tag{3.34}$$

Thus, $1 - I_k(x,t)$ is the probability that a relay will not have the $k^{th}$ message at time $t$, either because it has not met the source, or because it has already delivered the message. Similarly, $I_k(x,t_1) - I_k(x,t_2)$ represents the probability that a relay that comes into play at time $x$ will meet the source before $\theta_k$ and deliver the message to the destination in the time interval $(t_1, t_2]$. Finally, note also that

$$\frac{dI_k(x,t)}{dt} = -\mu I_k(x,t)$$

We use the definition of $I_k(x,t)$ as well as its above mentioned properties to prove Lemma 3.6.1 below, which gives a recursion to numerically compute the density $\phi_k(t)$.

**Lemma 3.6.1.** *For* $t \in [\theta_k, \gamma_k]$,

$$\phi_{k+1}(t) = h_1(\theta_k)\delta_{\theta_k}(t) + \phi_k(t) + h_2(\theta_k)\left\{\mu e^{-\mu t} + e^{-\mu\gamma_k}\delta_{\gamma_k}(t)\right\} \tag{3.35}$$

*where*

$$\begin{aligned} h_1(\theta_k) &= \int_{\theta_{k-1}}^{\theta_k} \phi_k(x)\left\{1 - I_k(x,\theta_k)\right\} dx, \\ h_2(\theta_k) &= e^{\mu\theta_k} \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) I_k(x,\theta_k) dx. \end{aligned}$$

*Proof of Lemma 3.6.1.* Let us first consider the probability that the relay be ready for competing for the delivery of the $(k+1)^{th}$ message at time $\theta_k$. This is only possible if it was ready for competing for the $k^{th}$ message at some time $x \in [\theta_{k-1}, \theta_k]$, and has not the message at time $\theta_k$. As a consequence

$$\Phi_{k+1}(\theta_k) = \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) \left[1 - I_k(x, \theta_k)\right] dx = h_1(\theta_k).$$

Consider now the probability that $T_{k+1}$ be in the interval $(\theta_k, t]$ for some $t < \gamma_k$. This can occur if the relay was ready for competing for the $k^{th}$ message at some time $x \in [\theta_{k-1}, \theta_k]$, took this message from the source at some time $s \in [x, \theta_k]$ and deliver it to the destination in $y \in (\theta_k - s, t - s]$ units of time. Another possibility is that the relay comes into play for the delivery of the $k^{th}$ message after $\theta_k$ but before $t$, in which case it will be proposed directly the $(k+1)^{th}$ message. As a consequence

$$\Phi_{k+1}(t) - \Phi_{k+1}(\theta_k) \;\; = \;\; \int_{\theta_k}^{t} \phi_k(x) dx + \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) \left(I_k(x, \theta_k) - I_k(x, t)\right) dx$$

which upon derivation with respect to $t$ yields

$$\begin{aligned} \phi_{k+1}(t) &= \phi_k(t) + \mu \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) I_k(x, t) dx \\ &= \phi_k(t) + \mu e^{-\mu t} h_2(\theta_k) \end{aligned}$$

Finally, the only possibility for the relay to come into play at time $\gamma_k$ is that it was ready for competing for the $k^{th}$ message at some time $x \in [\theta_{k-1}, \theta_k]$, took the message from the source but was not able to meet the destination by $\gamma_k$. Therefore

$$\mathbb{P}\left(T_{k+1} = \gamma_k\right) = \int_{\theta_{k-1}}^{\theta_k} \phi_k(x) I_k(x, \gamma_k) dx = e^{-\mu \gamma_k} h_2(\theta_k),$$

□

Lemma 3.6.1 can be used for the recursive numerical computation of the density $\phi_k(t)$, from which we can derive the probability of success

$$p_k(t) = \left(V_k\left(\min(\theta_k, t), \min(\gamma_k, t)\right)\right)^{N-1}$$

Figures 3.5a and 3.5b show the CDF $\Phi_k(t)$ and the success probability $p_k(t)$ for $k \in \{1, 2, 10\}$, respectively, in the case $N = 3$ relays, using the following parameters: $\lambda = 1.25$, $\mu = 0.4$, $C_s = 0.5$, $C_d = C_r = 4.0$ and $R = 30$.

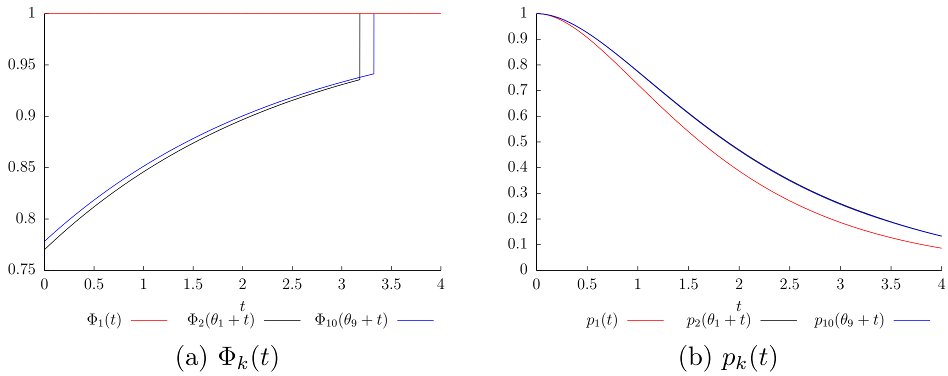



Figure 3.5: Evolution of $\Phi_k(t)$ and $p_k(t)$ with $k$.

The thresholds $\theta_k$ and $\gamma_k$ can then be obtained by solving (3.11)-(3.12) using any root finding method, such as the bisection method, as illustrated in Algorithm 1.

### 3.6.2 Performance metrics

From the point of view of the source, the main performance metrics are the probability that a message is successfully delivered and, provided that it reaches its destination, the expected time to deliver it. Our first result in this direction is on the probability of message delivery.

**Proposition 3.6.1.** *Assume that all relays play a symmetric equilibrium strategy with parameters $\theta_k$ and $\gamma_k$ for the delivery of message $k$. Let $\zeta_k$ be the probability that this message is successfully delivered, that is, the probability that at least one copy reaches the destination by time $\gamma_k$. Then*

$$\zeta_k = 1 - \left(\frac{C_s}{\mu \overline{R}}\right)^{\frac{N}{N-1}}, \tag{3.36}$$

*if $\gamma_k < \infty$, whereas*

$$\zeta_k = 1 - \left(1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s)\left(1 - e^{-\lambda(\theta_k - s)}\right) ds\right)^N, \tag{3.37}$$

*otherwise.*

**ALGORITHM 1:** Computation of successive symmetric Nash equilibria

**Require:** $\phi_1(t) = \delta_0(t)$, $\theta_0 = 0$

**for** $k = 1, 2, \ldots$ **do**
    Compute $\theta_{min}$ and $\theta_{max}$ as the solutions of (3.23) and (3.24)
    $a = 0$, $b = \theta_{max}$
    **repeat**
        $c = (a+b)/2$
        **if** $c > \theta_{min}$ **then**
            Compute $\gamma(c)$ as the solution of $V_k(c, \gamma) = \omega$
        **else**
            $\gamma(c) = \infty$
        **end if**
        $G_k = \int_c^{\gamma(c)} \mu e^{-\mu(t-c)} \left( \frac{C_s}{\mu} - \overline{R} V_k(c,t)^{N-1} \right) dt$
        **if** $G_k < -C_r$ **then**
            $a = c$
        **else**
            $b = c$
        **end if**
    **until** $|G_k + C_r| < \epsilon$.
    $\theta_k = c$, $\gamma_k = \gamma(c)$
    Compute $\phi_{k+1}(t)$ with (3.35)
**end for**

*Proof.* From (3.19), the probability that all relays fail to deliver the message to the destination by time $\gamma_k$ is $V_k(\theta_k, \gamma_k)^N$, from which we deduce that

$$\zeta_k = 1 - V_k(\theta_k, \gamma_k)^N.$$

If $\gamma_k < \infty$, it follows from (3.20) and (3.11) that

$$p_k^i(\gamma_k) = V_k(\theta_k, \gamma_k)^{N-1} = C_s/(\mu \overline{R}),$$

from which we readily obtain (3.36). If on the contrary $\gamma_k = \infty$, then (3.37) follows from (3.18) and

$$\begin{aligned}
\lim_{y\to\infty} V_k(\theta_k, y) &= 1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - \lim_{y\to\infty} v_{\theta_k,y}(s)\right) ds, \\
&= 1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - e^{-\lambda(\theta_k - s)}\right) ds.
\end{aligned}$$

□

We emphasize that Proposition 3.6.1 can be used by the source to compute the minimum reward $R$ allowing to achieve a target delivery probability. If in addition the source wishes the message to be delivered within a certain amount of time, it can use Proposition 3.6.2 below.

**Proposition 3.6.2.** *Assume that all relays play a symmetric equilibrium strategy with parameters $\theta_k$ and $\gamma_k$ for the delivery of message $k$, and let $D_k$ be the expected delivery time of this message. Provided that at least one copy reaches the destination by time $\gamma_k$, the expected delivery time is*

$$\mathbb{E}\left[D_k | D_k \le \gamma_k\right] = \frac{1}{\zeta_k} \int_{\theta_{k-1}}^{\gamma_k} \left(V_k(\min(t, \hat{\theta}_k), t)^N - V_k(\theta_k, \gamma_k)^N\right) dt \qquad (3.38)$$

*Proof.* From (3.19), we have

$$\mathbb{P}(D_k > t) = V_k(\min(\theta_k, t), t)^N$$

for all $t \le \gamma_k$. It yields

$$\mathbb{P}(D_k > t | D_k \le \gamma_k) = \frac{1}{\zeta_k} \left(V_k(\min(\theta_k, t), t)^N - V_k(\theta_k, \gamma_k)^N\right),$$

and the result directly follows from

$$\mathbb{E}\left[D_k | D_k \le \gamma_k\right] = \int_{\theta_{k-1}}^{\gamma_k} \mathbb{P}(D_k > t | D_k \le \gamma_k)\, dt.$$

□

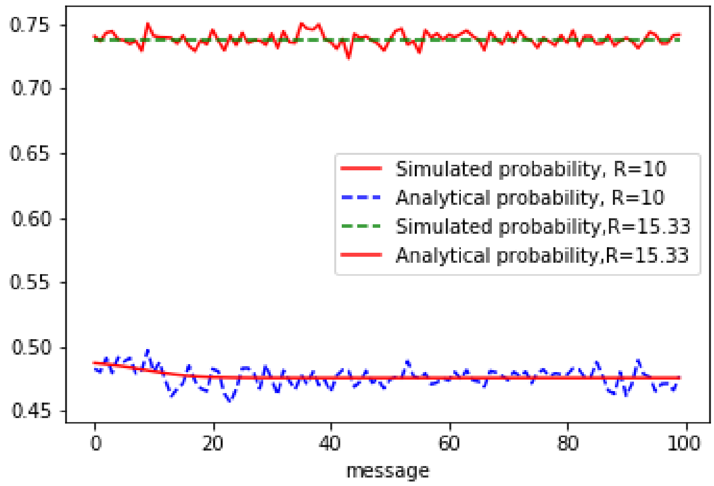



Figure 3.6: Theoretical values of the probability of success and its values obtained though simulations for $R = 10$ ($\gamma = \infty$) and for $R = 15.33$ ($\gamma < \infty$).

We do some simulations with different values of $R$ to see how expected delay and probability of success change with $R$. We take the following values for the parameters: $C_r = 10, C_s = 0.4, C_d = 4, \lambda = 0.8, \mu = 0.4, N = 15$. Figure 3.9 illustrates the convergence of the expected delay as a function of $R \in [3 \times R_{min}, 8 \times R_{min}]$ and $k$ (messages 15 and 29 have almost the same expected delay). This figure also shows that for $k$ large the messages have a greater expected delay than the first messages, whereas Figure 3.8 shows that the probability of success increases with $R$ and approaches 1 when $R \to \infty$. We do another simulation with $C_r = 2, C_s = 0.4, C_d = 2, \lambda = 0.2, \mu = 0.1, N = 10$. Figure 3.6 compares the values of the probability of success obtained with Proposition 3.6.1 against the values obtained through simulations, for $R = 10$, which yields $\gamma = \infty$, and for $R = 15.33$ which gives a finite $\gamma$. Note from (3.36) that for $R = 5.4 \times R_{min}$ the success probability is the same for all messages, whereas for $R = 3 \times R_{min}$ it decreases with $k$. Figure 3.7 shows more clearly the convergence of the expected delay with $k$ in the case $R = 3 \times R_{min}$ and $R = 5.4 \times R_{min}$.

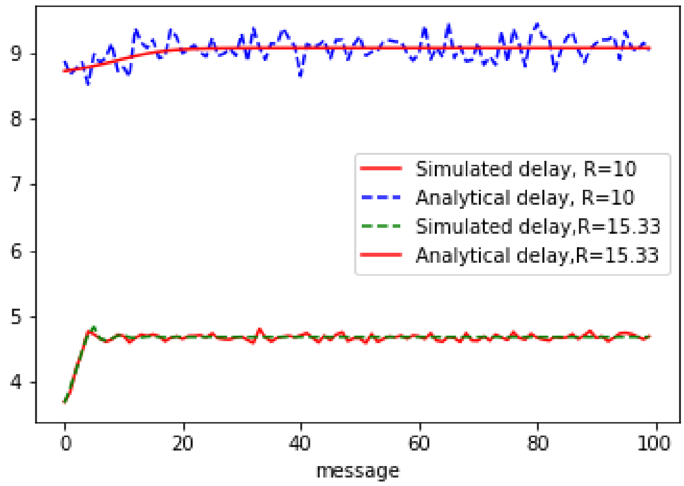



Figure 3.7: Theoretical expected delays of first 30 messages compared against values obtained through simulations for $R = 10$ and $R = 15.33$.

## 3.7 Conclusions

We considered a fixed reward incentive mechanism for a two-hop DTN with a single-destination pair and arbitrary number of competing relays. The source was assumed to be backlogged and proposes messages in a sequential way to the relays it meets. it was shown that the equilibrium policy of the relays for each message is threshold type. That is, a relay accepts the $k$th message if and only if it meets the source before a given threshold, and once it accepts the message, it keeps this message until a second threshold. A recursive formula for the computation of these thresholds was presented for symmetric equilibria.

Our results were obtained obtained under a number of crucial assumptions. One of our key assumption is that of exponentially distributed inter-contact times. Although satisfied under the Random Waypoint Mobility model, this assumption is not always met in practice and it would be natural to relax it. The problem becomes then much more complex and the decision process is no longer Markov. One can apply the theory of Semi-Markov decision processes which is more complex. This avenue is being currently explored.

Another important assumption for some of our results is that the relays have homogeneous contact processes with the source and with the destina-

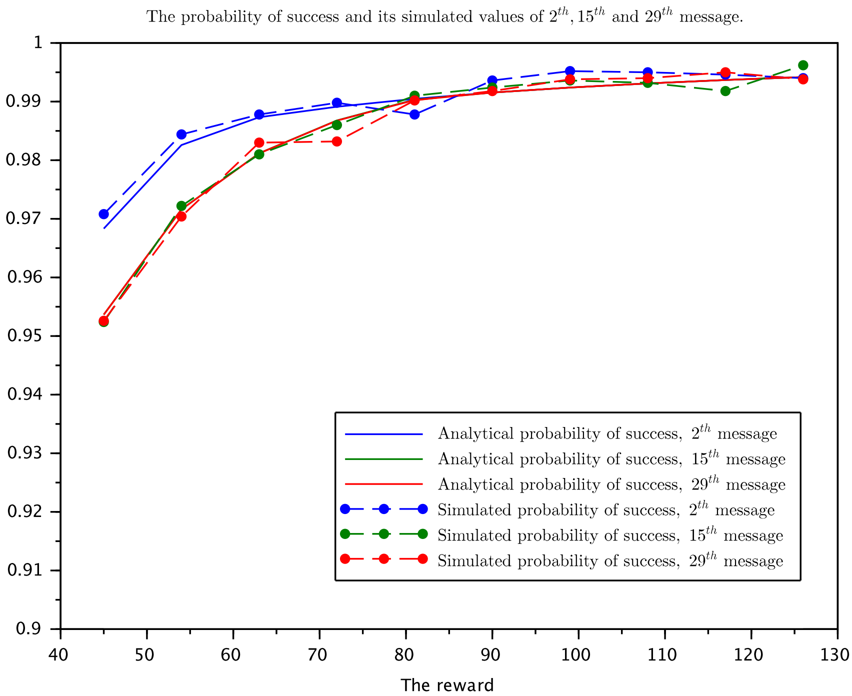



Figure 3.8: Comparison of the theoretical probabilities of success of $2^{nd}$ and $29^{th}$ messages against expected probabilities of success obtained through simulations.

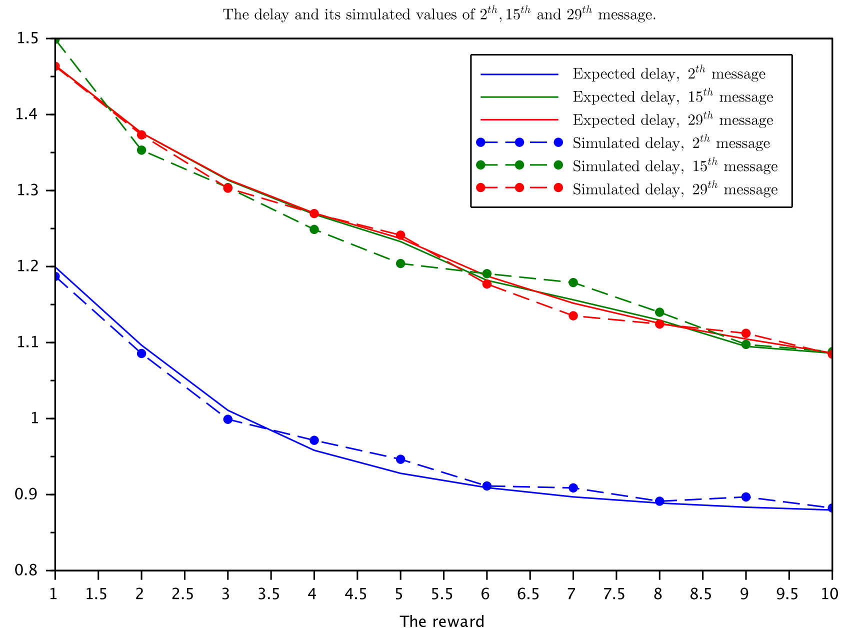



Figure 3.9: Comparison of the theoretical expected delays of $2^{nd}, 15^{th}$ and $29^{th}$ messages against expected delays obtained through simulations.

tion. In practice, it often happens that nodes are more or less heterogeneous, with diverse behaviours per each group of nodes. While it was proven that even in a heterogeneous setting all relays use a threshold strategy at a Nash equilibrium, it can be expected that in this case all Nash equilibria will be asymmetric. As discussed in Section 3.5, the characterisation of asymmetric equilibria is much more involved than that of symmetric ones.

With memory space becoming cheap for modern devices, another natural generalisation would be to assume that a relay can store more than one message. This extension however gives rise to non-trivial questions. In particular, it is not clear which message a relay should transmit when it meets the destination, assuming that it can give only one. It is not necessarily optimal to transmit the most recent message. Also, the analysis of the probability of success for a relay would be more complicated since it has to take into account which messages other relays transmit when they meet with the destination.

Another possible direction is to introduce a message arrival process at the source, for example the messages could arrive according to a Poisson process.

Extending the above models to multiple sources and destination as well as allowing the possibility for the relays to drop a message and pick another one are also part of our future plans.

# Chapter 4

# MEAN-FIELD LIMIT OF THE FIXED-REWARD INCENTIVE MECHANISM IN DELAY TOLERANT NETWORKS

We investigate the asymptotic performance of a reward incentive Delay Tolerant Network based on mean field limit. We consider a two-hop network with one source and one destination and $N$ relays. The source is backlogged and sends messages to the destination by forwarding to the relays it meets. For each message, there is a promised reward for the first one who successfully transmits it to the destination. It was shown in the previous chapter, the optimal policy for the relays is of thresholds type (a relay will accept a message until certain time and drop it after a second threshold). When the second threshold in infinite, we give the mean-field ODE and show that all the messages have the same probability of success. When the second threshold is finite we only give an ODE approximation since the dynamics are not Markovian.

## 4.1 Introduction

In this chapter, we pursue the analysis of the fixed-reward mechanism further by investigating the mean-field limit of the system dynamics when the number of relays becomes large. The backlogged source proposes a message until no relay will accept it any more after which it proposes the next message. It will be shown that the time-scale for the duration of a message being proposed by the source, that is, the first threshold, is $O(1/N)$ where $N$ is the number of relays. The second threshold will be shown to be $O(1)$. We shall focus the analysis on the case of the second threshold being infinite. This leads to a Markovian description of the system, and to the mean-field limit. We shall give the necessary and sufficient that the reward must satisfy in order for the second threshold to be infinite. We let the number of messages is proportion to the number of relays, say $k = tN$. We then let $N$ tend to infinity and study the limits. In order to do that, we consider a mean field interaction model with two states: state 1 for having a message and state 0 otherwise. We will show that:

- When the second threshold is infinite, we shall show the converge to an ODE and give its solution.

- In this case, the probability of success and the delay in limit do not depend on $t$.

- When the second threshold is finite, since the dynamics are no longer Markovian, we give an approximation for the mean field limit.

## 4.2 Related works

In literature, there has been many researches about mean-field analysis for Delay Tolerant Networks. For instance, [60] considers a network with one source and one destination. In this network, the source has only one message to send during a time $T$ (the life time of a message). The author uses multi-hop setting where a node will consider to forward a message to other node based on the available energy. He does not consider any incentive scheme but work with the trade-off between the delay and the energy which gives us a complicated optimal problem. In contradiction, mean-field leads to a simpler result with a threshold which depends on the remain energy. [26] works in the mean-field scheme of a Delay Tolerant Network in which the source wants to send an information formed by $K$ packages. The destination sends feedback to the source (via relays) about the number of packages have not received yet, based on that, the source will send again some corresponding packages. The work is based on the energy consumption and does not consider any incentive mechanism. In [53, 3], the authors study an control problem where the source can control rate of the number of copies of the message by changing the probability of forwarding the message to a relay. The authors proved the optimal strategy for the source is of threshold type policy. The mean-field limit is presented when the number of nodes is very large. The mean-field not only helps the authors reduce the complexity of the problem but also lets them analyze the network with any size, i.e. with any number of nodes.

**Organization:** The chapter is organized as follows: Section 4.3 presents our model and assumptions. The mean field model and mean field limits as well as the performance metrics are computed in Section 4.4 for the case when the second threshold in infinite. An estimation to compute $a(t) := \theta(t)N$ for finite $\gamma$ is given in Section 4.5.

## 4.3 Model description

We consider a two-hop network with $N + 2$ nodes with one source and one destination who are fixed and $N$ other nodes who play the role of relays. The source has many messages to send to its destination. $N$ relays move randomly in the network and may meet the source or the destination some time following an exponential distribution with rate $\lambda$, $\mu$ respectively. We assume that the mobility pattern of relays and the meeting times between the source and relay, between the destination and relay are i. i. d. When the source meets a relay, it will propose a message with a promised reward, $R$, for the first relay who successfully delivers the message to its destination. For a message, the fixed reward means the source proposes the same reward for all relays. A relay incurs a cost of receiving and transmitting a message of $C_r$ and $C_d$ respectively. Keeping a message costs $C_s$ per unit of time. For convenience, we denote $\bar{R} = R - C_d$. At anytime, a relay can accept or reject a message (when meeting the source), drop or keep when it has a message. There is no cost of dropping and rejecting a message. We assume that a relay only rejects or drops a message if the expected cost is positive. The two-hop network does not allow a relay to forward messages to other relays except to the destination. We also assume that a relay can store only one message at a time and it does not seek a new message while it is having one. That is, it only can accept a new message if it had rejected or transmitted or dropped the previous message. The source proposes messages to relays sequentially. The source and the relays have no feedback from the destination that the message has been transmitted or not. The source does not give any information about how many relays have the message. In our previous chapter, Chapter 3 and our previous work [48], we proved that the optimal strategy of relays is of threshold-type: it accepts until the first threshold and it drop after the second threshold. We proved the uniqueness of the symmetric solution that is, all relays will play the same thresholds. The condition to have solution and the condition to have finite solution were also given. Based on that, we provided the formula to find the expected delay and the probability of success of the source for each message.

## 4.4 Asymptotic analysis

In this section, we study the asymptotic performance of the network when the number of relays, $N$, is large. Let $\mathbf{Y}^N(\tau) \equiv (Y_1^N(\tau), Y_2^N(\tau), ..., Y_N^N(\tau))$ be a continuous-time stochastic process where $Y_i^N(\tau) \in \{0,1\}$ indicates whether relay $i$ has message at time $\tau$ or not.

Consider the discrete-time embedding of $\mathbf{Y}$ and $\theta_k$ defined by $\mathbf{X}^N(k) = \mathbf{Y}(\theta_k)$. The process $\mathbf{X}^N$ lives on $\mathcal{S}^N = \{0,1\}^N$. Here $X_n^N(k)$ is the state of relay $n$ at $\theta_{k-1}$. If $X_n^N(k) = 1$, then relay $n$ has a message at the release time of message $k$, otherwise it is free to accept message $k$ from the source. With this definition, the duration of time-slot $k$ is of length $\hat{\theta}_k$.

The process $\mathbf{X}^N$ is a discrete-time Markov chain only if $\hat{\gamma}_k$ is infinite for all $k$. Otherwise, we also need to keep track of the identity of the message held by a relay as well as the $\hat{\gamma}$ of that message to be able to define the dynamics of $\mathbf{X}^N$.

We now give the condition to have infinite $\hat{\gamma}_k$ when $N$ is large. Interestingly, the condition does not depend on $k$. For this, we first need the following results which shall be invoked later as well.

**Lemma 4.4.1.**

$$\lim_{N\to\infty} N\hat{\theta}_k = c_k. \tag{4.1}$$

*Proof.* From (3.1), we observe that $p_k(\tau)$ should be strictly positive in the limit $N \to \infty$ for a relay to accept message $k$. From (3.20) and (3.18), this is equivalent to

$$\int_{\theta_{k-1}}^{\theta_k} \phi_k(s)(1 - v_{\theta_k,\tau}(s))ds = O(1/N).$$

which is equivalent to

$$\int_0^{\hat{\theta}_k} \phi_k(s+\theta_{k-1})(1 - v_{\theta_k,\tau}(s+\theta_{k-1}))ds = O(1/N). \tag{4.2}$$

From the definition of $v$ (see (3.18)), for $\hat{\theta}_k \to 0$,

$$v_{\theta_k,\tau}(s+\theta_{k-1}) = 1 - \lambda(\hat{\theta}_k - s)(1 - e^{-\mu(\tau-\theta_k)}) + o(\hat{\theta}_k).$$

Using the definition of $\phi_k$ is (3.35), we get the following asymptotic for the LHS of (4.2)

$$\int_0^{\hat{\theta}_k} \phi_k(s+\theta_{k-1})(1 - v_{\theta_k,\tau}(s+\theta_{k-1}))ds = h_1(\theta_{k-1})\lambda\hat{\theta}_k \cdot (1 - e^{-\mu(\tau-\theta_k)}) + o(\hat{\theta}_k).$$

Next, we shall argue that $h_1(\theta_k)$ is $O(1)$ which will then imply that $\hat{\theta}_k$ has to be $O(1/N)$ for (4.2) to hold. Consider the mean-field limit of the continuous time process $\mathbf{Y}$. When $\hat{\gamma}_k = \infty$, it can be seen that the scaled process $y_1(s) = \frac{1}{N}\sum_{i=1}^{N} Y_i(s)$ converges to the ODE

$$\dot{y}_1(s) = \lambda y_0(s) - \mu y_1(s).$$

Here $y_1(t)$ is the fraction of relays that have a message at time $s$. This ODE has a unique solution for which $y_1(s) \in (0,1), \forall s$ if $y_1(0) \in (0,1)$. Thus, for all time $s$ the fraction of nodes available to compete for a message is strictly positive. The probability that a relay becomes available to take message $k$ is $\int_{\theta_{k-1}}^{\theta_k} \phi_k(s)ds$ which, from (3.35), tends to $h_1(\theta_{k-1})$ when $\hat{\theta}_k \to 0$. Thus, $h_1$ is $O(1)$ and $\hat{\theta}_k$ is $O(1/N)$. □

The above result states that the duration for which the source proposes message $k$, that is $\hat{\theta}_k$ is $O(1/N)$ as $N \to \infty$. The intuition behind this result is the following. When the number of relays is large, in order to observe a change in the occupancy measure of any state, we need to look at messages that have sequence numbers of $O(N)$. The intuitive reasoning is that a message is profitable to accept only if there are a finite number of relays that are competing to deliver this message. Otherwise, the probability of success of a relay will be zero, and it will not accept the message. Since, on an average, there will be at least $\lambda\hat{\theta}_k N M_0^N(k)$ relays that will pick message $k$, $\hat{\theta}_k$ should be $0(1/N)$ in order for the average number of competing relays to be finite for each message. Thus, we need to look at messages $k = tN$ to observe changes in the occupancy measure.

**Proposition 4.4.1.** *When $N$ is large, there exists a symmetric Nash equilibrium with $\hat{\theta}_k > 0$ if and only if $\bar{R} \geq C_r + \frac{C_s}{\mu}$. This solution is finite if and only if*

$$1 + \mu\frac{C_r}{C_s} < \frac{\frac{\mu\bar{R}}{C_s} - 1}{\ln\left(\frac{\mu\bar{R}}{C_s}\right)}. \tag{4.3}$$

*Proof.* Let us consider message of order $tN$, and let $\theta_{min}(t)$ be $\theta_{min}$ of message $k = tN$. We assume that when $N$ is large, the $\theta(t) - \theta_{min} \approx \frac{a_{min}(t)}{N}$. Therefore the Eq. (3.23) can be approximated as $1 - h_1(t)\lambda\hat{\theta}_{min}(t) = \omega$, hence

$$\left(1 - \frac{h_1(t)\lambda a_{min}(t)}{N}\right)^{N-1} = \frac{C_s}{\mu\bar{R}}.$$

When $N$ is large, the LHS of that equation tends to $e^{-\lambda a_{min}(t)h_1(t)}$. Therefore, when $N$ is large, $a_{min}(t)$ will be

$$a_{min}(t) = \frac{1}{h_1(t)\lambda}\ln\left(\frac{\mu\bar{R}}{C_s}\right). \tag{4.4}$$

Similarly, we have the approximation for $b$. First, we observe that $\lim\limits_{N\to\infty}\omega = 1$. From Eq. (3.27), $b$ can be approximated as

$$b \approx \frac{\lambda}{\mu-\lambda}(\mu-\lambda)h_1(t)\theta_{min}(t) = h_1(t)\lambda\theta_{min}(t).$$

Hence, we can get the limitation of $Nb$ and $(1+b)^N$ when $N$ is large,

$$\lim_{N\to\infty}\frac{(1+b)^N - 1}{Nb} = \frac{e^{h_1(t)\lambda a_{min}(t)} - 1}{h_1(t)\lambda a_{min}(t)}. \tag{4.5}$$

Plug the value of $a_{min}(t)$ into the Eq. (3.26) we get

$$1 + \mu\frac{C_r}{C_s} < \frac{\frac{\mu\bar{R}}{C_s} - 1}{\ln\left(\frac{\mu\bar{R}}{C_s}\right)}. \tag{4.6}$$

□

The RHS of the condition above increases in $\bar{R} = R - C_d$. That means for a small enough reward $R$, we will get a infinite $\hat{\gamma}_k$.

In the rest of this section, we shall assume that the condition (4.3) is satisfied so that $\mathbf{X}^N$ is a discrete-time Markov chain.

We observe that the transitions of each $X_n^N$ are independent from that of the others. The transition probabilities between the states are

$$\begin{aligned} p_{0,1} &= (1 - e^{-\lambda\hat{\theta}_k}) \cdot e^{-\mu\hat{\theta}_k} \\ &= \lambda\hat{\theta}_k + o(\hat{\theta}_k), \end{aligned} \tag{4.7}$$

$$\begin{aligned} p_{1,0} &= (1 - e^{-\mu\hat{\theta}_k}) \cdot (e^{-\lambda\hat{\theta}_k} \\ &\quad + (1 - e^{-\lambda\hat{\theta}_k}) \cdot (1 - e^{-\mu\hat{\theta}_k})) \\ &= \mu\hat{\theta}_k + o(\hat{\theta}_k) \end{aligned} \tag{4.8}$$

The expression for $p_{1,0}$ has the following explanation. The transition from state 1 to state 0 happens if the relay meets the destination for the first time to deliver the message that it has. Then, to remain in state 0 it should either not meet the source until the end of the interval $\hat{\theta}_k$ or if it meets the source before the end of this interval then it should again meet the destination.

### 4.4.1 Mean-field limit

In [6], a general framework for showing the convergence of discrete-time Markov chains to mean-field dynamics is given. Following their steps, let $M^N(k)$ be the occupancy measure which is the vector of frequencies of state $s \in \mathcal{S}$ at time $t$,

$$M_s^N(k) = \frac{1}{N} \sum_{n=1}^{N} \mathbb{1}_{\{X_n^N(k)=s\}}. \tag{4.9}$$

We have $M^N(k) \in \Delta := \{\mathbf{m} \in \mathbb{R}^2, m_1 + m_2 = 1, m_1, m_2 \geq 0\}$.

Define a time rescaled process $\bar{M}^N(t)$ as

$$\begin{cases} \bar{M}_s^N(t) = M_s^N(t/N) \text{ for all } t, \\ \bar{M}_s^N(\tau) \text{ is affine on } \tau \in [t, t+1/N)], \end{cases} \tag{4.10}$$

Let $\hat{a}(t) = N\hat{\theta}_{tN}$ be the rescaled interval during which message $k = tN$ is proposed . Let $\hat{a}_0$ be the rescaled interval for the first message when all the relays are available to compete for this message (see [49] for its computation). The following proposition gives the mean-field limit of the rescaled occupancy measure.

**Proposition 4.4.2.** *Let $m_0(t)$ be the solution of the differential equation*

$$\dot{m}_0(t) = -\lambda m_0(t)\hat{a}(t) + \mu(1 - m_0(t))\hat{a}(t), \tag{4.11}$$

*with $\hat{a}(t) = \frac{\hat{a}_0}{m_0(t)}$ and $m_0(0) = m$. Assume $M_s^N(0) \to m$ in probability as $N \to \infty$. Then, for all $t > 0$, as $N \to \infty$,*

$$\sup_{0 \leq \tau \leq t} \left\| \bar{M}_0^N(\tau) - m_0(\tau) \right\| \to 0, \tag{4.12}$$

*in probability.*

*Proof.* The proof is based upon the verification of conditions in [6] First, we shall show that $\hat{a}(t) = a_0/m(t)$. One computes $\theta_k$ from (3.12) and (3.1). When $N$ is large, and assuming $k = tN$ and $\hat{\gamma}_k = \infty$, one can show that

$$G_k(\theta_k, \gamma_k) = C_r + \frac{C_s}{\mu}(1 - e^{-\hat{\gamma}_k})$$
$$-\frac{(Cd - R)}{h_2(\theta_k)} \frac{1}{N} \left(p_k(\infty)^{N/(N-1)} - p_k(\theta_k)^{N/(N-1)}\right).$$

With some abuse of notation, we shall use $h_i(t)$ to denote $h_i(\theta_{tN})$. From (3.35), one has the approximation

$$h_2(t) \approx h_1(t) I_t(\theta_{tN}, \theta_{tN}) \approx h_1(t)\lambda\hat{a}(t)/N.$$

It can be shown that

$$p_k(\infty)^{N/(N-1)} \approx \left(1 - \frac{\lambda\hat{a}(t)h_1(t))}{N}\right)^N \underset{N\to\infty}{\longrightarrow} e^{-\lambda\hat{a}(t)h_1(t)}. \tag{4.13}$$

and

$$p_k(\theta_k)^{N/(N-1)} \to 1.$$

Thus, any solution of (4.13) has the form

$$\hat{a}(t)h_1(t) = c,$$

where $c$ is a constant. As argued in the proof of Lemma 4.4.1, $h_1(t) = m_0(t)$. Denoting $\hat{a}_0$ to the constant when $m_0(t) = 1$ we get the desired relationship.

Next, we check that the conditions in [6] are verified. Let $f^N(m_0)$ be the drift function which is the expected change to $M_0^N$ in one time slot,

- **Intensity vanishes at a rate** $\epsilon(N)$: We take $\epsilon(N) = \frac{1}{N}$. We need to prove that

$$\lim_{N\to\infty} \frac{f^N(m_0)}{\epsilon(N)} = f(m_0), \text{ exists for all } m_0 \in (0,1). \tag{4.14}$$

  From (4.7) and (4.8), and Lemma 4.4.1, we have

$$\frac{f^N(m_0)}{\epsilon(N)} = Nm_1\frac{\mu\hat{a}(t)}{N} - m_0\frac{\lambda\hat{a}(t)}{N}. \tag{4.15}$$

  Hence,

$$\lim_{N\to\infty} \frac{f^N(m_0)}{\epsilon(N)} = m_1\mu a(t) - m_0\lambda a(t). \tag{4.16}$$

- **Second moment of number of object transition per time slot**: There are two types of transitions from 0 to 1 and from 1 to 0. The total number of transitions of the first type has a Binomial distribution with parameters $N_i$ and $\lambda\hat{a}(t)/N_i$ where $N_i$ is the number of relays with no messages. This tends to Poisson distribution that has a finite second moment. The same argument holds for the second type of transitions.

- $f^N(m)$ **is a smooth function of** $\frac{1}{N}$ **and** $m_0$: The function

$$m_1\frac{\mu\hat{a}(t)}{N} - m_0\frac{\lambda\hat{a}(t)}{N}$$

with $\hat{a}(t) = 1/m_0$ is smooth in $1/N$ and in $m_0$.

□

The above proposition gives the fraction of relays that are available to compete for message $t$. The following result tells the duration for which this message will be proposed by the source.

**Proposition 4.4.3.** *Let* $\hat{a}(0) = \hat{a}_0/m_0(0)$. *Then,*

$$\hat{a}(t) = \frac{\hat{a}(0)}{\alpha + (1-\alpha)u}, \tag{4.17}$$

*where* $u$ *is the solution of the following equation*

$$-\beta\hat{a}(0)t = \alpha\ln(u) + (1-\alpha)u + \alpha - 1, \tag{4.18}$$

*where* $\alpha = \frac{\mu}{\mu+\lambda}, \beta = \mu + \lambda$.

*Proof.* Equation (4.11) can be rewritten in terms of $\hat{a}$ to get the following differential equation

$$\frac{\dot{\hat{a}}(t)}{\hat{a}^2(t)} = (\lambda + \mu)\hat{a}(0) + \mu\hat{a}(t). \tag{4.19}$$

The solution of this differential equation is given by

$$\hat{a}(t) = \frac{\hat{a}(0)}{\alpha + (1-\alpha)e^{-\beta\int_0^t \hat{a}(s)ds}} \tag{4.20}$$

Let $u = e^{-\beta\int_0^t \hat{a}(s)ds}$, then $\dot{u} = -\beta\hat{a}(t)u$. Plugging this substitution into the above equation and taking the integral we will gives the claimed result. □

Using this result, we can also compute the time at which message $t$ will be released by the source.

**Corolary 4.4.1.** *Let* $a(t) = \lim_{N\to\infty}\theta_{tN}$ *be the release time of message* $t$. *Then,*

$$a(t) = -\frac{1}{\beta}\ln(u),$$

*where* $u$ *is the solution of*

$$-\beta\hat{a}(0)t = \alpha\ln(u) + (1-\alpha)u + \alpha - 1.$$

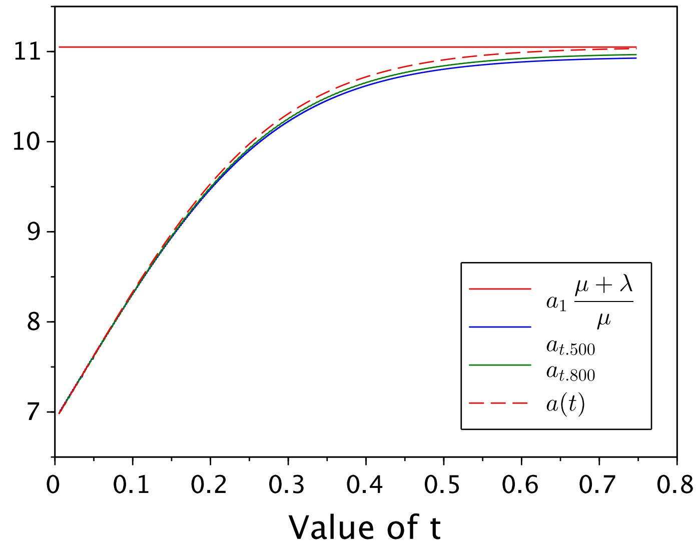



Figure 4.1: The behavior of $\hat{a}(t)$ when $\mu = 0.5$, $\lambda = 0.3$ and $N = 500; 800$.

*Proof.* The claim follows by noting that message $t$ is released at time $\int_0^t \hat{a}(s)ds$ and using the definitions in the above proposition. □

We check numerically that the mean-field ODE is gives a good approximation for $\hat{a}(t)$ for finite $N$. First, we let $\mu < \lambda$, and take $\mu = 0.5, \lambda = 0.3$ (Figure 4.1) and in Figure 4.2 the comparison is done for $\mu = \lambda = 0.4$.

### 4.4.2 Performance metrics

In this part, we will find the probability of success of message number $k = tN$ for $N$ is large as well as its expected delay. The following proposition present the probability of success and the delay when $N$ is large.

**Proposition 4.4.4.** *If $k = tN$ and $N$ is large, we have the probability of success of message $k^{th}$ (denoted by $\xi(t)$) will be*

$$\xi(t) = 1 - e^{-\lambda \hat{a}_0}, \quad \forall t. \tag{4.21}$$

*We let $D(t)$ be the delay provided that at least one copy of message has reached the destination. Then the expected value of $D(t)$ is*

$$\mathbb{E}(D(t)|D(t) < \infty) = \frac{e^{-\lambda \hat{a}_1}}{\mu(1 - e^{-\lambda \hat{a}_1})} \int_0^1 \frac{e^{\lambda \hat{a}_1 u} - 1}{u} du. \tag{4.22}$$

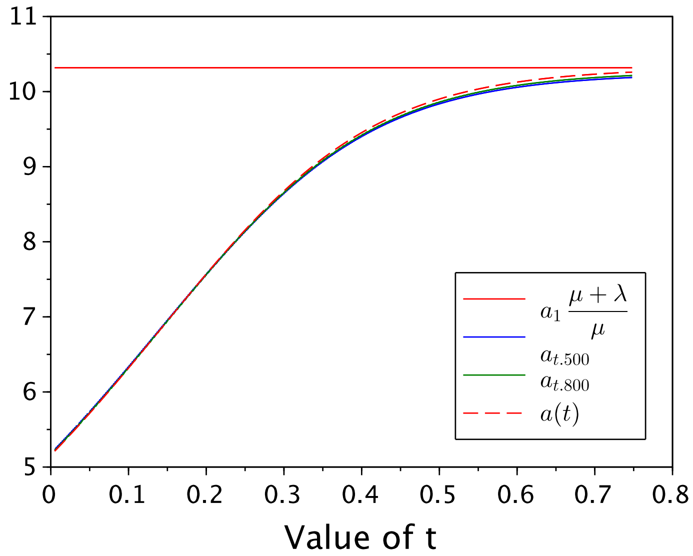



Figure 4.2: The behavior of $\hat{a}(t)$ when $\mu = \lambda = 0.4$ and $N = 500; 800$.

*Proof.* From [48], we have the probability of success when $\gamma = \infty$ is

$$\xi_k = 1 - \left(1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - e^{-\lambda(\theta_k - s)}\right) ds\right)^N. \tag{4.23}$$

We remark that for $k = tN$, we have $m_0(\theta(t)) = \frac{\hat{a}_1}{a(t)}$. Moreover, we also have

$$\begin{aligned} \xi_k &= 1 - \left(1 - \int_{\theta_{k-1}}^{\theta_k} \phi_k(s) \left(1 - e^{-\lambda(\theta_k - s)}\right) ds\right)^N, & (4.24) \\ &= 1 - \left(1 - m_0(\theta(t)) \left(1 - e^{-\lambda\hat{\theta}(t)}\right)\right)^N, \\ &= 1 - \left(1 - \lambda\frac{\hat{a}_1}{N}\right)^N. & (4.25) \end{aligned}$$

Therefore,

$$\xi(t) = \lim_{N\to\infty} 1 - \left(1 - \lambda\frac{\hat{a}_1}{N}\right)^N = 1 - e^{-\lambda\hat{a}_1}. \tag{4.26}$$

For the delay, we have, for all $k$ the expected delay given that at least one copy of message has reached the destination is

$$\frac{1}{\zeta_k}\int_{\theta_{k-1}}^{\gamma_k} \left(V_k(\min(s, \theta_k), s)^N - V_k(\theta_k, \gamma_k)^N\right) ds \tag{4.27}$$

We now estimate the $V_k(\min(s,\theta_k),s)^N$ where $k = tN$ and $N$ is large. We have, for all $s \geq \theta(t)$,

$$\begin{aligned} V_k(s,s) &= 1 - \int_{\theta(t)}^{\infty} \phi_t(y)\left(1 - e^{-\lambda(\theta(t)-y)},\right. && (4.28)\\ &\quad \left. -\frac{\lambda}{\mu+\lambda} e^{-\mu s} e^{\lambda y}\left(e^{(\mu-\lambda)\theta(t)} - e^{(\mu-\lambda)y}\right)\right) dy, \\ &= 1 - m_0(\theta(t))\lambda\hat{\theta}(t)\left(1 - e^{-\mu(s-\theta(t))}\right), && (4.29)\\ &= 1 - \frac{\hat{a}_1}{N}\lambda\left(1 - e^{-\mu(s-\theta(t))}\right). && (4.30) \end{aligned}$$

Therefore, we have

$$\lim_{N\to\infty}\left(1 - \frac{\hat{a}_1}{N}\lambda\left(1 - e^{-\mu(s-\theta(t))}\right)\right)^N = e^{-\lambda\hat{a}_1\left(1-e^{-\mu(s-\theta(t))}\right)}. \tag{4.31}$$

Hence,

$$\mathbb{E}(D(t)|D(t) < \infty) = \int_{\theta(t)}^{\infty} \frac{e^{-\lambda\hat{a}_1}}{\xi(t)}\left(e^{\lambda\hat{a}_1 e^{-\mu(s-\theta(t))}} - 1\right) ds, \tag{4.32}$$

and by changing variable we get the stated result.

□

The probability of success and the expected delay do not depend on $t$ since the average number of relays who have the message during the time between two consecutive $\theta$ are the same when $N$ is large. Figures 4.3 and 4.4 shows that the simulated probability of success and the simulated delay are close to the analytical results.

## 4.5 Estimation in the finite $\gamma$ case

When $\gamma_k$ is finite, it is more complicated since the mean field model is no longer a Markov chain. Therefore, in order to find the limit, we need to know exactly how many relays have message $k$ for all $k$ and how long they will keep that message counting from current time. That is not an easy job. In this section, we just give an estimation of $m_0(t)$.

In the following Proposition, we present an observation of $\hat{a}(t)$ and $\hat{\gamma}(t)$.

**Proposition 4.5.1.** *When $\gamma_k$ is finite, we have for all $t$,*

$$\begin{aligned} \hat{a}(t) &= \frac{\hat{a}_1}{h_1(t)}, && (4.33)\\ \hat{\gamma}(t) &= \hat{\gamma}_1. && (4.34) \end{aligned}$$

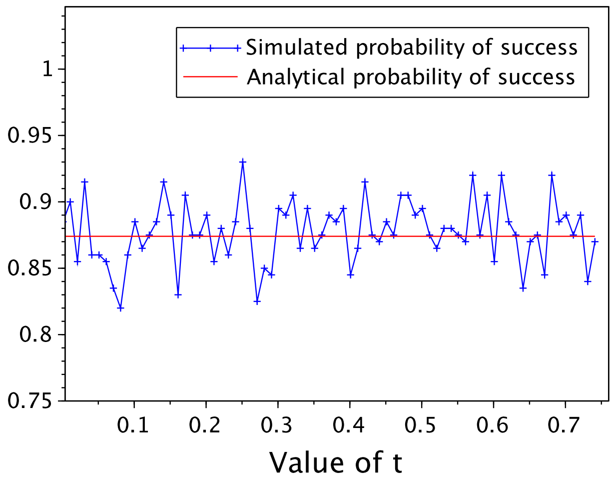



Figure 4.3: The simulated probability and its analytical value with $N = 800$, $\mu = 0.5, \lambda = 0.3$.

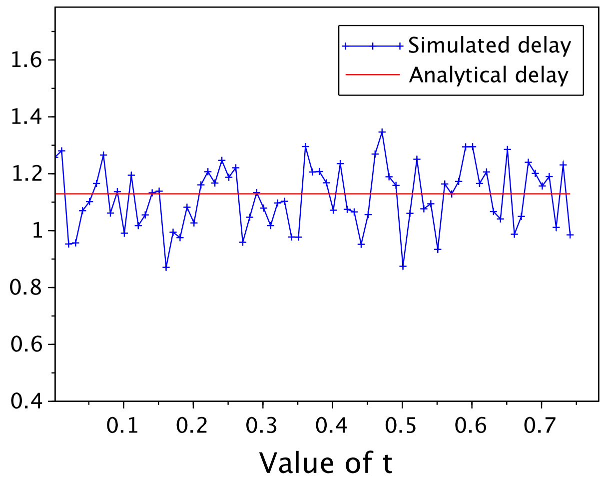



Figure 4.4: The simulated delay and its analytical value with $N = 800$, $\mu = 0.5, \lambda = 0.3$.

*Proof.* When $\gamma_k$ is finite, some following estimations still hold.

$$\begin{aligned} h_2(t) &\approx \lambda\hat{\theta}(t)h_1(t), && (4.35)\\ \int_0^{\hat{\theta}(t)} \phi_t(s)ds &\approx h_1(t), && (4.36)\\ p_t(\hat{\gamma}(t))^{\frac{N}{N-1}} &\approx e^{-\lambda\hat{a}(t)h_1(t)(1-e^{-\mu\hat{\gamma}(t)})}, && (4.37)\\ p_t(\hat{\theta}(t))^{\frac{N}{N-1}} &\approx 1. && (4.38) \end{aligned}$$

Therefore, we have the equations system to find $\hat{\gamma}(t)$ and $\hat{\theta}(t)$ as follows

$$e^{-\lambda\hat{a}(t)h_1(t)(1-e^{-\mu\hat{\gamma}(t)})} \approx \frac{C_s}{\mu\bar{R}}, \tag{4.39}$$

$$C_r - \frac{C_s}{\mu}\frac{\log(\upsilon)}{\lambda\hat{a}(t)h_1(t)} + \frac{\bar{R}}{\lambda\hat{a}(t)h_1(t)} = 0, \tag{4.40}$$

where $\upsilon = \frac{C_s}{\mu\bar{R}}$. Eq. (4.40) means when $t$ is closed to 0, we will get $\hat{a}_1$, hence for all $t$ we have

$$\hat{a}(t) = \frac{\hat{a}_1}{h_1(t)}. \tag{4.41}$$

Plugging this result into the Eq. (4.39), we also get

$$\hat{\gamma}(t) = \hat{\gamma}_1, \qquad \text{for all } t. \tag{4.42}$$

□

We also have $h_1(t) \approx m_0(a(t))$ where $a(t) = \int_0^t \hat{a}(s)ds$. Hence, we only need to estimate the function $m_0(x)$.

For convenience, we chance the time-scale and give the ODE for $m(a(t)) \equiv m_0(x)$. The change in $m_0(x)$ in a $\delta x$ will be the difference between the ones who meet the destination or drop at time $x$ and the ones who are available to accept a new message. At any time $x$, there are $\lambda m_0(x-\hat{\gamma}_1)e^{-\mu\hat{\gamma}_1}$ relays will drop their messages because the reach the second threshold. In addition there will be $m_0(x)\lambda$ who will change state to 1 and $(1-m_0(x)\mu)$ who will change state to 0. Then, we have

$$\frac{dm_0(x)}{dx} = -\lambda m_0(x) + \mu(1-m_0(x)) + \lambda m_0(x-\hat{\gamma}_1)e^{-\mu\hat{\gamma}_1}. \tag{4.43}$$

Solving (4.43) gives us $m_0(x)$, then plugging into (4.33), we will get $\hat{a}(t)$ for all $t$.

Figure 4.5 confirms that when $N$ is large, the duration of time that a relay keeps a message is the same for all messages. Figure 4.7 verifies that the estimated $\hat{a}(t)$ is close to the analytical value of $a(t)$.

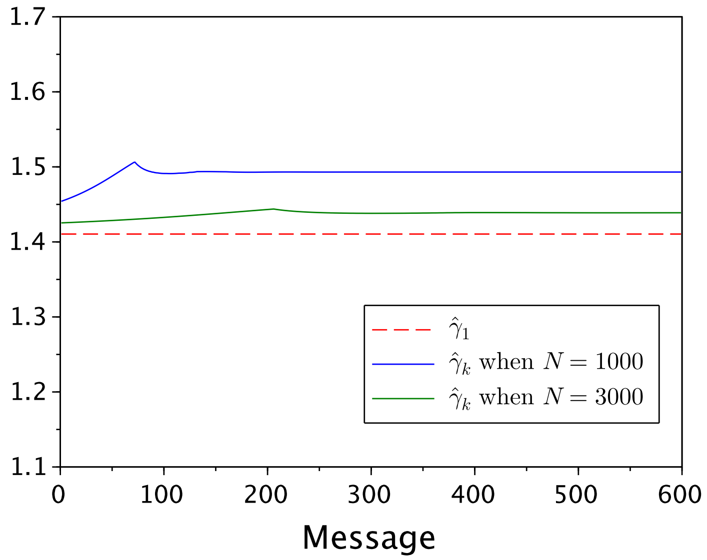



Figure 4.5: The value of $\hat{\gamma}_1$ and $\hat{\gamma}_k$ with $N = 1000; 3000$, $\mu = 0.8, \lambda = 0.4$. We see that when $N$ is large, the duration of time that a relay keeps a message is the same for all messages.

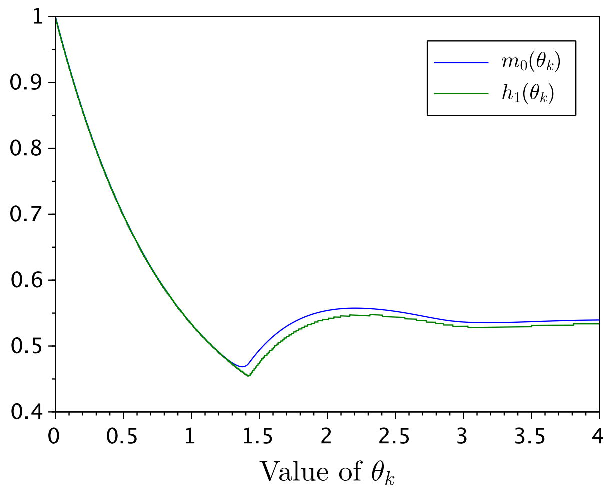



Figure 4.6: The function $m_0$ and function $h_1$ with $N = 3000$, $\mu = 0.8, \lambda = 0.4$.

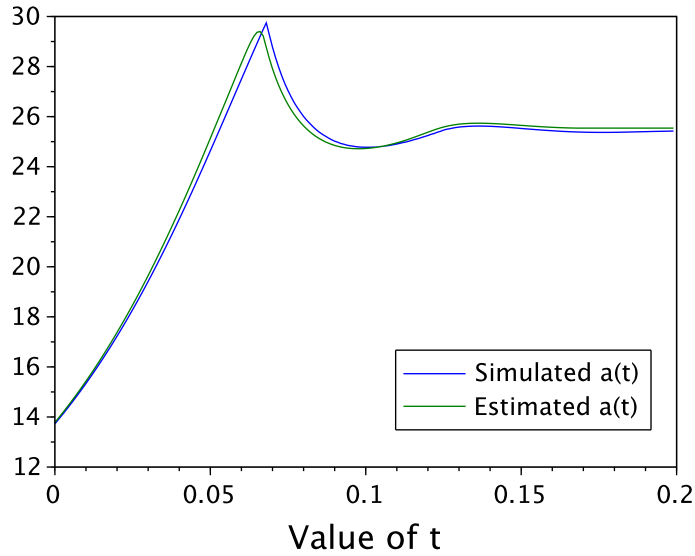



Figure 4.7: The estimated $\hat{a}(t)$ and the analytical $a(t)$ with $N = 3000$, $\mu = 0.4, \lambda = 0.8$.

For the probability of success and the delay, we do similarly to the infinite $\gamma(t)$. We have the average number of relays who have a message stays the same and equals to that number of the first message. So that we will get the same probability of success and the delay for all message.

**Proposition 4.5.2.** *When $\gamma_k$ is finite, for all $t$, the probability of success and the delay are*

$$\begin{aligned} \xi(t) &= 1 - \frac{C_s}{\mu \bar{R}}, \qquad (4.44) \\ \mathbb{E}(D(t)|D(t) < \gamma(t)) &= \frac{e^{-\lambda \hat{a}_1}}{\mu(1-\upsilon)} \int_{e^{-\mu\hat{\gamma}_1}}^{1} \frac{e^{\lambda \hat{a}_1 u} - 1}{u} du \end{aligned}$$

*where $\upsilon = \frac{C_s}{\mu \bar{R}}$.*

As in the case $\gamma(t) = \infty$, we have the same delay and probability of success for all messages as in Figure 4.8.

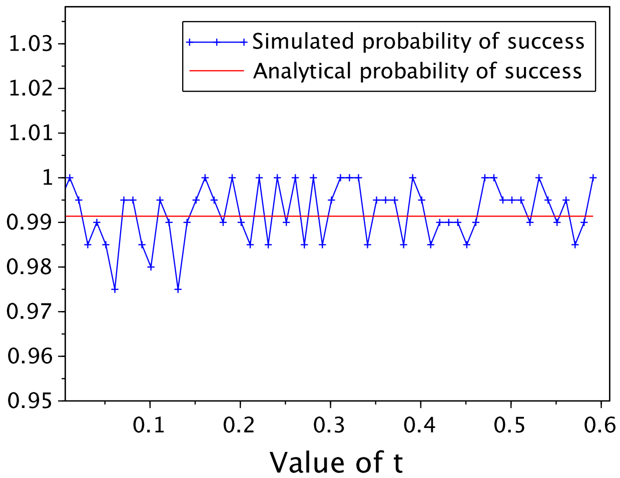



Figure 4.8: The simulated probability of success and its analytical value with $N = 1000$, $\mu = 0.8, \lambda = 0.4$.

## 4.6 Conclusion

We consider a network with $N$ relays and one pair of source-destination. The source is backlogged and proposes a fixed reward for each message. We study the mean-field limit of this game when the second threshold is infinite and show that in this limit each message is proposed for a duration of $O(1/N)$. We show that the fraction of relays without a message converges in the mean field limit to the solution of an ODE. Based on that limit, we find the formula to compute various performance metrics such as probability of success and the mean delay. It is shown that the probability of success is the same for all messages.

When the second threshold is finite, the dynamics are no longer Markovian and we propose an ODE approximation which numerically gives a good match.

# Chapter 5

# A MIXED STRATEGY FOR A COMPETITIVE GAME IN DELAY TOLERANT NETWORKS

We consider a non-cooperative game between $N$ relays in Delay Tolerant Networks with one fixed source and one fixed destination. The source has no contact with the destination, so it has to rely on the relays when it has a message to send. We assume that the source has a sequence of messages and it proposes them to relays one by one with a fixed reward for the first transmission for each message. We analyse a symmetric mixed strategy for this game. A mixed strategy means a relay decides to accept relaying the $k^{th}$ message with probability $q_k$ when it meets the source. We establish the conditions under which $q_k = 1$; $q_k = 0$ or $q_k \in (0, 1)$, and prove the existence and the uniqueness of the symmetric Nash equilibrium. We also give the formula to compute this mixed strategy as well as the probability of success and the delay of a given message. When $k$ is large, we give the limiting value of the mixed strategy $q$ and the probability of success for the messages.

## 5.1 Introduction

In Delay Tolerant Networks (DTN) [7, 27, 29, 28], the approach used by mobile nodes to communicate in the absence of a communication infrastructure is based on the so-called store-carry-forward paradigm: a source node gives a copy of its message to all mobile nodes that it meets, asking them to keep it until they can forward it to the destination. Although other routing schemes have been proposed [35, 45], in this work we shall specifically consider two-hop routing DTNs [5, 74], in which once a relay has the message, it can only transmit it to the destination.

The above approach implicitly assumes that mobile nodes accept to use their scarce energy resources for relaying messages of others out of altruism. In practice, it can be expected that some nodes will act as free-riders, that is, that they will use the network to send their own messages without offering their resources in exchange for relaying the messages of others. Clearly, if there are too many selfish nodes in a DTN, the network collapses and mobile nodes can no longer communicate with one another. A central issue in DTNs is therefore to convince mobile nodes to relay messages. Many incentive mechanisms have been proposed to avoid the free-rider problem in DTNs, including reputation-based schemes [46, 82, 33, 79, 77], barter-based schemes [67, 10, 11] and credit-based schemes [86, 14, 44, 83, 13, 62]. In contrast to most of the incentive mechanisms proposed in the literature, explicit guarantees on the probability of message delivery and on the mean time to deliver a message have been obtained for the credit-based scheme considered in [48, 50] (see also [62] for a closely-related mechanism).

In the previous two chapter, Chapter 3, Chapter 4, we consider a source which promises a fixed reward to the relay who first delivers a message to the destination. The source is backlogged and only one message at a time is proposed by the source. Inter-contact times of relays with the source and the destination are exponentially distributed. When it meets the source, a relay has the choice to either accept the message or not, and if it accepts, it can decide to drop the message at any time in the future at no additional cost. The competition between relays is modelled as a stochastic game in which each relay seeks to minimize its expected net cost, that is, the sum of its expected energy and storage costs minus its expected reward. It is proven that the optimal policy of a relay is of threshold type: it accepts a message until a first threshold $\theta$ and then keeps it until it either meets the destination or reaches a second threshold $\gamma$ (which can be infinite). Explicit formulas for computing the thresholds as well as the probability of message delivery are derived for the unique symmetric Nash equilibrium, in which all relays use the same thresholds and no player can benefit by unilaterally changing its policy.

The analysis in Chapter 3, Chapter 4 and our works in [48, 50] implicitly assumes that the source tells the relays when a message was proposed for the first time, or, in other words, when this message was generated. Our objective in this chapter is to understand whether it is profitable for the source to give this information to the relays. We thus consider the same incentive mechanism, but assuming that when it meets the source, a relay has to make its decision without knowing for how long the message is in circulation. The only information available to the relay is the value $R$ of the reward and the period of time $T$ during which the message is proposed by the source.

Since it does not know for how long a message is available, we assume that a relay decides to accept a message according to a randomized policy, that is, when relay $i$ meets the source, it accepts the $k^{th}$ message with probability $q_k^i$, and rejects it with probability $1 - q_k^i$. If it accepts the message, the relay keeps it until it reaches the destination. The value of $q_k^i$ is computed by relay $i$ so as to minimize its expected net cost, and it of course depends on $R$ and $T$, but also on the number of relays competing for the delivery of the $k^{th}$ message (some relays may be busy delivering previous messages). We note that a similar setting was considered in [2], but with a different cost structure and assuming that the source has only one message to transmit.

We establish under which condition $q_k^i > 0$ for all $i$, and show that, under this condition, there exists a unique value $q_k > 0$ such that if all relays accept the $k^{th}$ message with probability $q_k$, no relay has anything to gain by unilaterally changing its acceptance probability. In other words, the situation in which all relays accept the $k^{th}$ message with probability $q_k$ corresponds to a symmetric Nash equilibrium, and this equilibrium is unique. Explicit expressions for the probability of message delivery and the mean time to deliver a message at the symmetric Nash equilibrium are then derived. Assuming that $q_k$ converges as $k \to \infty$, we also obtain an explicit characterization of the asymptotic value of the acceptance probability $q_\infty$. Finally, we compare the performance obtained with the threshold-type strategy in the full information setting and with the randomized policy in the no information setting.

The rest of this chapter is organized as follows. Section 5.2 is devoted to model description. In Section 5.3, we establish the conditions for the existence and uniqueness of symmetric Nash equilibria and present a method for recursively computing the acceptance probabilities $q_k$. The asymptotic value of the acceptance probability is also derived in Section 5.3. Explicit expressions for the main performance metrics at the symmetric Nash equilibrium are then derived in Section 5.4. Finally, numerical results pertaining to the comparison of the full information setting and the no information setting are given in Section 5.5.

## 5.2 Assumption and Model Description

We consider a two-hop network of $N$ mobile nodes with one fixed source and one fixed destination. The source is backlogged, that is, it has an unlimited number of messages to send to the destination. Since the source and the destination are not in radio range of each other, the source cannot transmit its messages directly to the destination. Instead, it proposes a new message to the relays every $T$ units of time, promising a fixed reward $R$ to the first one to deliver the current message to the destination. We assume that the relays are moving randomly and that the inter-contact times of a given relay with the source (resp. destination) are i.i.d. and follow an exponential distribution with rate $\lambda$ (resp. $\mu$). This assumption holds (at least approximately) under the Random Waypoint Mobility model [12].

When it accepts a message, a relay incurs a one-time reception cost $C_r$ for receiving it from the source. There is then a cost $C_s$ per unit of time for keeping the message in its buffer. Finally, the relay incurs a transmission cost $C_d$ for sending the message to the destination. We however assume that the latter cost is incurred by the relay if and only if the message has not been already delivered to the destination by another relay. If on the contrary the relay is the first one to deliver the message to the destination, it incurs the cost $C_d$ but gets the reward $R$. In the following, we define $\bar{R} = R - C_d$.

When it proposes the current message (say message $k$) to relay $i$, the source informs it of the values of $R$ and $T$, but does not tell it for how long the current message is available. The relay accepts message $k$ with probability $q_k^i$, and rejects it with probability $1 - q_k^i$. If the $k^{th}$ message was rejected by relay $i$, then this relay cannot accept it later on when it meets again the source. We also assume that if the relay accepts the message, it has to keep it until it meets the destination. Finally, we assume that a relay can store only one message at a time and cannot drop a message to accept a new one.

Relay $i$ computes its acceptance probability $q_k^i$ so as to minimize its expected net cost, which depends on its probability to be the first one to deliver message $k$. Obviously, the latter probability in turn depends on the acceptance probabilities of the other relays. We say that a vector $(q_k^1, q_k^2, \ldots, q_k^N)$ is a Nash equilibrium if no relay $i$ can decrease its expected net cost by unilaterally changing its acceptance probability $q_k^i$. A symmetric Nash equilibrium is a Nash equilibrium for which $q_k^i = q_k$ for all $i$, for some value $q_k$. In the following, we shall specifically focus on symmetric Nash equilibria.

## 5.3 Acceptance Probabilities under the Symmetric Nash Equilibrium

### 5.3.1 Acceptance Probabilities

Consider a tagged relay and let us analyse the competition for the delivery of the $k^{th}$ message. Assume that all other relays accept the $k^{th}$ message with probability $q_k$. If the tagged relay accepts the message with probability $q'_k$, its net expected cost is

$$q'_k \left( C_r + \frac{C_s}{\mu} - \bar{R}P_s(q_k) \right), \tag{5.1}$$

where $P_s(q_k)$ is the probability that the tagged relay be the first one to transmit message $k$ to the destination, given the acceptance probability $q_k$ of the others. In (5.1), $C_r$ is the cost of accepting the message from the source and $C_s/\mu$ is the cost of storing the message until the relay meets the destination (recall that the inter-meeting times with the destination are exponentially distributed with mean $1/\mu$). The term $\bar{R}P_s(q_k)$ is the expected reward the relay gets the message. Thus, (5.1) gives the net expected cost for accepting the message.

For the tagged relay, the optimal value of $q'_k$ is the one which minimizes (5.1). It follows that

$$q'_k = 0 \quad \text{if} \quad C_r + \frac{C_s}{\mu} - \bar{R}P_s(q_k) > 0.$$

Hence, we conclude that if

$$\bar{R} \leq \bar{R}_{min} = C_r + \frac{C_s}{\mu},$$

no relay will accept the $k^{th}$ message. In other words, the condition $\bar{R} > \bar{R}_{min}$ is a necessary condition for the relays to have an incentive to participate in message delivery. Assuming that this condition is met, we see that

- $q'_k = 1$ is the best response of the tagged relay if $\bar{R}_{min}/P_s(q_k) < \bar{R}$.
- $q'_k = q_k$ is one of the possible best responses if $\bar{R}_{min}/P_s(q_k) = \bar{R}$.

We thus need to analyse how $P_s(q_k)$ depends on $q_k$.

To this end, let $p_k$ be the probability, as computed by the tagged relay, that an arbitrary other relay meets the source while it is proposing the $k^{th}$ message and that this relay is not already busy with a previous message.

Obviously, for the first message we have $p_1 = 1 - e^{-\lambda T}$. The derivation of $p_k$ for $k > 1$ is slightly more complex and we shall shortly explain how it can be computed by the tagged relay. From the definition of $p_k$, we obtain that $p_k\, q_k$ is the probability that an arbitrary other relay attempts the delivery of the $k^{th}$ message. Therefore, the number $A_k$ of other relays that are in competition with the tagged relay for the delivery of the $k^{th}$ message follows a binomial distribution with parameter $p_k\, q_k$, which yields

$$P_s(q_k) \;=\; \mathbb{E}\left(\frac{1}{A_k+1}\right) = \frac{1-(1-p_k\, q_k)^N}{N\, p_k\, q_k}. \tag{5.2}$$

From (5.2), we can conclude that, if $\bar{R} > \bar{R}_{min}$, there exists a unique symmetric equilibrium with $q_k > 0$, as formally stated in Theorem 5.3.1 below.

**Theorem 5.3.1.** *If $\bar{R} > \bar{R}_{min}$, there exists a unique symmetric Nash equilibrium for the $k^{th}$ message with $q_k > 0$. Moreover, we have $q_k = 1$ if*

$$\bar{R} > \frac{N p_k}{1-(1-p_k)^N}\, \bar{R}_{min}, \tag{5.3}$$

*while otherwise $q_k$ is the unique solution in $(0, 1)$ of*

$$\bar{R} = \frac{N p_k\, q_k}{1-(1-p_k\, q_k)^N}\, \bar{R}_{min}. \tag{5.4}$$

*Proof.* Before proving the lemma, we first prove that the probability $P_s(q_k)$ is decreasing in $q_k$. With $r = p_k q_k$, we have

$$\frac{\partial P_s(q_k)}{\partial q_k} \;=\; \frac{Nr(1-r)^{N-1} - 1 + (1-r)^N}{(Nr)^2} \tag{5.5}$$

The numerator is negative since it has value 0 when $r = 0$ and it is decreasing in $r$ (the derivative w.r.t $r$ is negative), and thus in $q_k$. It follows that the expected net cost $\bar{R}_{min} - \bar{R}\, P_s(q_k)$ is increasing in $q_k$ and reaches its maximum value for $q_k = 1$.

Assume $\bar{R} > \bar{R}_{min}$. If the other relays play $q_k = 1$, the best-response strategy of the tagged relay is $q'_k = 1$ if and only if $\bar{R}_{min} - \bar{R}P_s(1) < 0$, which is equivalent to (5.3). On the other hand, for $q_k \in (0, 1)$ to be a symmetric equilibrium, $\bar{R}_{min} - \bar{R}P_s(q_k) = 0$ must hold, which is equivalent to (5.4). It is easy to see from (5.4) that $\bar{R}$ is an increasing function of $q_k$ such that $\bar{R} \in [\bar{R}_{min}, \bar{R}_{max}]$, where

$$\bar{R}_{max} = \frac{N q_k p_k}{1-(1-q_k p_k)^N}\, \bar{R}_{min}.$$

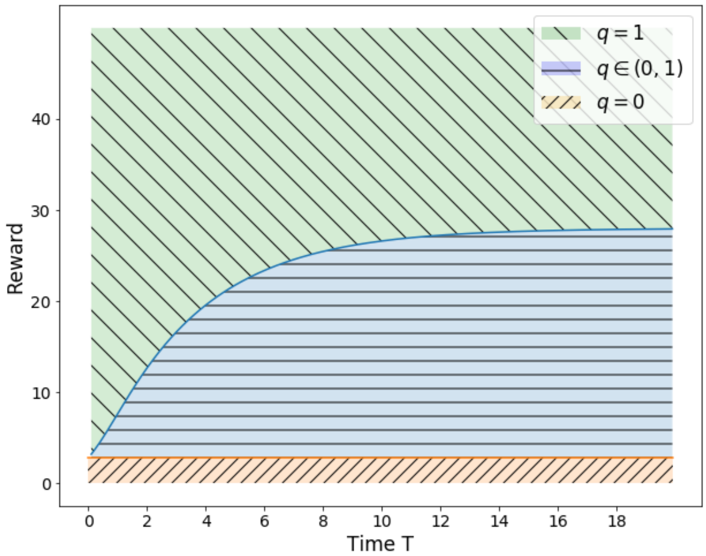



Figure 5.1: Equilibrium acceptance probability $q_1$ as a function of $R$ and $T$, when the values of the parameters are as follows: $\mu = 0.5$, $\lambda = 0.3$, $C_r = C_d = 2$ and $C_s = 0.4$.

Therefore, there is a bijective function between $\bar{R}$ and $q_k$. Hence, for any $\bar{R} \in [\bar{R}_{min}, \bar{R}_{max}]$, we always can find a value of $q_k$ such that the equation (5.4) is satisfied. □

The structure of the Nash equilibrium is illustrated in Fig. 5.1 for the first message. If $\bar{R} \leq \bar{R}_{min}$, no relay accepts the message. If

$$\bar{R} > \frac{N(1-e^{-\lambda T})}{1-e^{-\lambda NT}}\,\bar{R}_{min},$$

at the unique Nash equilibrium all relays accept the message with probability 1. Otherwise, the relays use a randomized strategy with $0 < q_1 < 1$.

### 5.3.2 Computation of the probability $p_k$

For the first message, we already know the value of $p_1$. We now explain how the value of $p_k$ can be computed by the tagged relay for subsequent messages $k > 1$. We need to consider the belief of the tagged relay regarding the number of other relays that are in competition with it for the delivery of the

$k^{th}$ message. We assume that all relays play their equilibrium strategies $q_i$, $i = 1, \ldots, k-1$ for all previous messages. Define $\Phi_k(t)$ as the probability that an arbitrary relay enters into competition for message $k$ on or before time $t$. By enter into competition on or before time $t$, we mean that there exists a time instant $t' < t$ such that the considered relay does not have any message with index smaller than $k$ in the interval $[t', t]$. We shall denote by $\phi_k(t)$ the probability density function (pdf) corresponding to $\Phi_k(t)$. If this pdf is known by the tagged relay, then it can estimate the probability $p_k$ as follows.

$$p_k = \int_{T(k-1)}^{Tk} \phi_k(x) \left(1 - e^{-\lambda(kT-x)}\right) dx.$$

Denote by $\delta_x(t)$ the Dirac delta function at point $x$. Following the same approach as in Chapter 3 and in [48], we can the following result.

**Lemma 5.3.1.** *The density $\phi_k(t)$ obeys the recursion*

$$\phi_{k+1}(t) = h_1(k)\delta_{kT}(t) + \phi_k(t) + h_2(k)\mu e^{-\mu t}. \tag{5.6}$$

Here $h_1(k)$ represents the probability that a relay is free for the $(k+1)^{th}$ message at time $kT$, and is given by

$$h_1(k) = \int_{(k-1)T}^{kT} \phi_k(x)(1 - q_k I_k(x, kT))dx, \tag{5.7}$$

and $h_2(k)e^{-\mu kT}$ is the probability that a relay be busy with the $k^{th}$ message at time $kT$, and is given by

$$h_2(k) = e^{\mu kT} \int_{(k-1)T}^{kT} q_k \phi_k(x) I_k(x, kT)dx. \tag{5.8}$$

In (5.7) and (5.8),

$$I_k(x,t) \quad = \quad \frac{\lambda}{\mu - \lambda} e^{-\mu t} e^{\lambda x} \left(e^{(\mu-\lambda)\min(t,kT)} - e^{(\mu-\lambda)x}\right),$$

represents the probability that a relay that comes into play at time $x < kT$ will meet the source and will not meet the destination by time $t$.

Since $h_2(i)e^{-\mu iT}$ is the probability that a relay has message $i$ at $iT$, it can be seen that $h_2(i)e^{-\mu(k-1)T}$ is the probability that a relay has message $i$ at time $(k-1)T$. Also, $h_1(k-1)$ is the probability that the relay does not have a message time $(k-1)T$. Since a relay either has a message or does not have one, we get the following relation:

$$h_1(k-1) + e^{-\mu(k-1)T} \sum_{i=1}^{k-1} h_2(i) = 1,$$

which yields

$$\sum_{i=1}^{k-1} h_2(i) = \frac{1 - h_1(k-1)}{e^{-\mu(k-1)T}}. \tag{5.9}$$

Using (5.6)-(5.9) and induction, we can prove that $h_1(k)$ obeys the recursions given below. We omit the proof due to lack of space.

**Proposition 5.3.1.** *The terms $h_1(k)$ can be computed with the recursion:*

$$\begin{aligned} h_1(k) &= h_1(k-1)\left(1 - q_k I_k\left((k-1)T, kT\right)\right) + \left(1 - h_1(k-1)\right)\left(1 - e^{-\mu T}\right) \\ &\quad -\left(1 - h_1(k-1)\right)\frac{q_i \lambda \mu e^{-\mu T}}{\mu - \lambda}\left(\frac{e^{(\mu-\lambda)T} - 1}{\mu - \lambda} - T\right), \end{aligned}$$

*with the initial value: $h_1(1) = 1 - q_1 I(0,T)$. This leads to the following formulas for $h_2(k)$ and $p_k$:*

$$\begin{aligned} h_2(k)e^{-\mu kT} &= 1 - h_1(k) - (1 - h_1(k-1))e^{-\mu T} \\ p_k &= h_1(k-1)\left(1 - e^{-\lambda T}\right) \\ &\quad +(1 - h_1(k-1))\left(1 - e^{-\mu T} - \frac{\mu}{\mu - \lambda}\left(e^{-\lambda T} - e^{-\mu T}\right)\right). \end{aligned} \tag{5.10}$$

Equation (5.10) has the following probabilistic interpretation. The probability that a relay can meet the source for message $k$ can be conditioned on two events at time $(k-1)T$ (i.e., at the release time of message $k$): either the relay did not have a message or had one of the previous $k-1$ messages. The two terms in (5.10) correspond to each of the two events. In the case of the first event, the probability of picking up message $k$ is just the probability of meeting the source in the interval $((k-1)T, KT]$. Since $h_1(k-1)$ is the probability of not having a message at time $(k-1)T$, the term $h_1(k-1)(1-e^{-\lambda T})$ is the probability related to the first event. Next, we look at the second event. Suppose the relay has a message at time $(k-1)T$. It can take message $k$ only if it meets the destination and then the source in an interval of length $T$ starting from $(k-1)T$. This probability is the one inside the parenthesis of the second term in (5.10). Since $(1 - h_1(k-1))$, is the probability that the relay has a message at $(k-1)T$, the second term in (5.10) corresponds to the second event.

### 5.3.3 Asymptotic Analysis when $k \to \infty$

In this section, we shall do the analysis when $k$ is large, that is, when the system is in steady-state or stationary regime. In this regime, the function $\phi_k$ will reach its limiting value so that each message will have statistically the same performance measures. This regime reflects the long-run characteristics which are obtained after a large number of messages have been transmitted. From numerical experiments, it will be seen that, for our model, after as few as 10 to 15 messages, the system reaches the steady-state.

Let $h_2'(k) = h_2(k)e^{-\mu k T}$. From Proposition 5.3.1 we get the following expressions for the limiting values of $p_k, h_1$, and $h_2'$. The proof is omitted.

**Proposition 5.3.2.** *When $k$ is large, we have*

$$\begin{aligned} h_1(\infty) := h_1 &= \frac{C(T)}{q_\infty I_\infty + C(T)}, & (5.11)\\ h_2'(\infty) := h_2' &= (1-h_1)(1-e^{-\mu T}), & (5.12)\\ p_\infty &= h_1(1-e^{-\lambda T}) + (1-h_1)D(T), & (5.13)\end{aligned}$$

*where*

$$\begin{aligned} C(T) &= 1 - e^{-\mu T} - \frac{q_\infty \mu\lambda}{\mu-\lambda}\left(\frac{e^{-\lambda T}-e^{-\mu T}}{\mu-\lambda} - Te^{-\mu T}\right), \\ D(T) &= 1 - e^{-\mu T} - \frac{\mu}{\mu-\lambda}(e^{-\lambda T}-e^{-\mu T}), & (5.14)\\ I_\infty &= \frac{\lambda}{\mu-\lambda}(e^{-\lambda T}-e^{-\mu T}). & (5.15)\end{aligned}$$

From Proposition 5.3.2, we can write the relation between $q_\infty$ and $p_\infty$ as

$$p_\infty(q_\infty) = \frac{C(T)(1-e^{-\lambda T)}}{q_\infty I_\infty + C(T)} + \frac{q_\infty I_\infty D(T)}{q_\infty I_\infty + C(T)} \tag{5.16}$$

Now, we can establish the conditions when $q_\infty = 1$ and when $q_\infty < 1$.

**Lemma 5.3.2.** *If the following condition is satisfied, then $q_\infty = 1, p_\infty = p_\infty(1)$:*

$$\bar{R} - \frac{Np_\infty(1)(C_r + C_s/\mu)}{1-(1-p_\infty(1))^N} > 0 \tag{5.17}$$

*Otherwise, $p_\infty$ and $q_\infty$ are the unique solution of the following system of equations:*

$$\begin{aligned} \bar{R} - \frac{Np_\infty q_\infty(C_r + C_s/\mu)}{1-(1-p_\infty q_\infty)^N} &= 0 & (5.18)\\ \frac{C(T)(1-e^{-\lambda T)}}{q_\infty I_\infty + C(T)} + \frac{q_\infty I_\infty D(T)}{q_\infty I_\infty + C(T)} &= p_\infty & (5.19)\end{aligned}$$

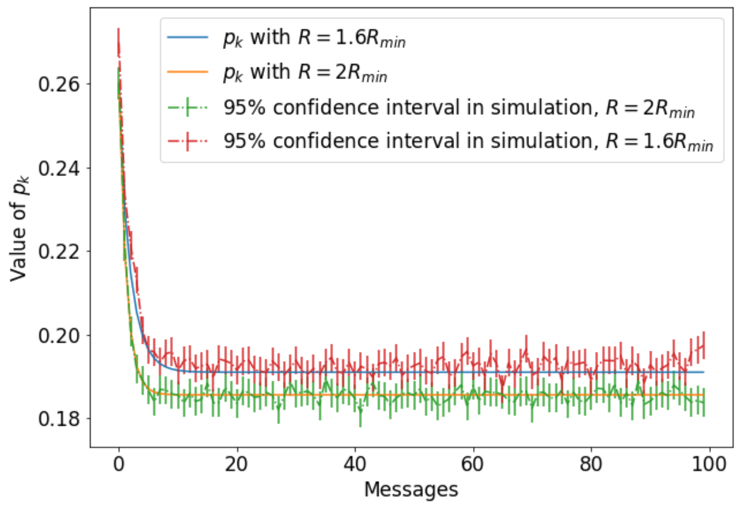



Figure 5.2: Value of $p_k$ in two cases with $R = 1.6R_{min}$ and $R = 2R_{min}$ as well as their 95% confidence interval.

The proof follows directly from Theorem 5.3.1. Notice that in case of $q_\infty < 1$, there is unique solution since the left hand side of (5.18) is decreasing in $q_\infty$.

Fig. 5.2 presents the probability $p_k$ that an individual relay, which is not busy with any previous message, meets the source while it is proposing the $k^{th}$ message. This probability is computed from analytical expressions as well as from simulations for different values of $R$, $T = 1.00357$ and $N = 10$ (the other parameters have the same value as in Fig. 5.1). In fact, the value of $T$ is the value of $\hat{\theta}_\infty = \lim_{k\to\infty} \theta_{k+1} - \theta_k$ and the value of $R$ is expressed as a multiple of $R_{min} = \bar{R}_{min} + C_d = C_r + \frac{C_s}{\mu} + C_d$. The simulations consist of generating meeting times of relays with the source and the destination, then each relay deciding whether to accept or not the message when it meets the source, and then determining which relay wins the reward. The value of $p_k$ was then averaged over $2,000$ sample paths. For the same parameter values, Fig. 5.3 presents the acceptance probabilities $q_k$ as well as their limiting value $q_\infty$. From these figures, it can be seen that the steady-state is reached quite quickly (after 10 messages).

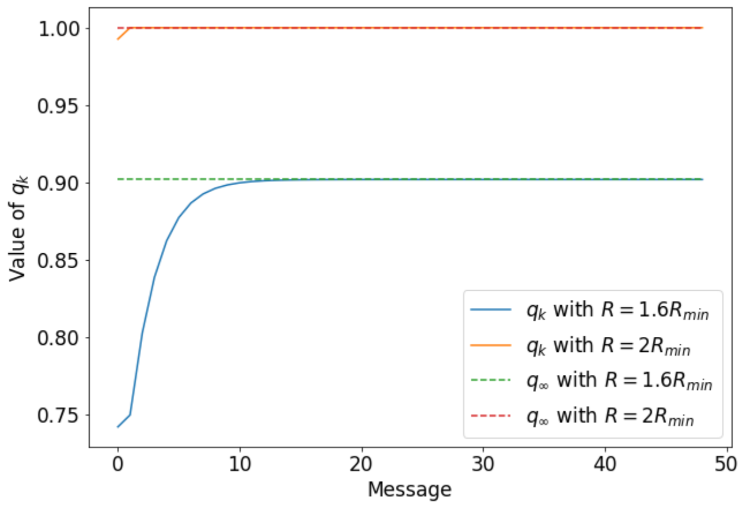



Figure 5.3: Value of $q_k$ and its limiting value.

## 5.4 Performance Metrics

In this section, we use the results obtained in Section 5.3 to establish explicit expressions for the probability of message delivery and the mean time to deliver a message at the symmetric Nash equilibrium. Together with Theorem 5.3.1 and (5.10), our first result, formally stated in Proposition 5.4.1, allows to compute the probability of message delivery of each message.

**Proposition 5.4.1.** *The probability of successful delivery of the $k^{th}$ message is $\xi_k = 1 - (1 - q_k\, p_k)^N$.*

*Proof.* Each individual relay participates to the delivery of the $k^{th}$ message with probability $q_k p_k$, from which the result follows. □

Fig. 5.4 shows the probability of message delivery for different values of $R$, and the following parameter values: $T = 1.00357$ and $N = 10$. The other parameters are the same as in Fig. 5.1. The probabilities obtained with event-driven simulations are also shown in Fig. 5.4. In the simulation, we generate the inter-contact times between the source, the destination and relays. We then let the relays follow the mixed strategy with $q_k$ computed from previous sections. We run the simulation 5000 times and take the average.

**Proposition 5.4.2.** *Let $D_k$ denote the delay of the $k^{th}$ message. It holds*

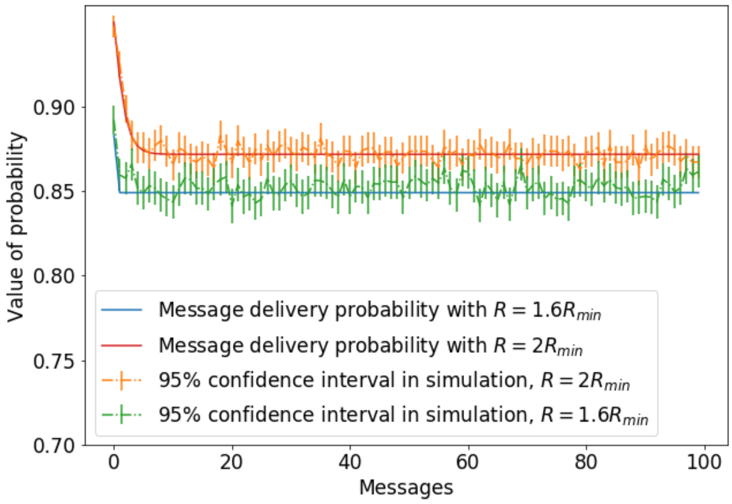



Figure 5.4: Analytical probability of message delivery and simulated probability for different values of $\bar{R}$.

*that*

$$\mathbb{E}(D_k|D_k < \infty) = \frac{1}{\xi_k}\int_{(k-1)T}^{\infty} (1 - Q(t))^N - (1 - q_k p_k)^N dt$$

*where, with the notation* $m = \min(t, kT)$, $Q(t)$ *is defined as*

$$Q(t) = q_k \int_{(k-1)T}^{m} \phi_k(x) \left[1 - e^{-\lambda(m-x)} - I_k(x,t)\right] dx, \tag{5.20}$$

*and represents the probability that an individual relay will deliver the* $k^{th}$ *message by time* $t$.

*Proof.* The probability that an individual relay that comes into play at time $x$ will meet the source by time $m \geq x$ and the destination by time $t \geq m$ is

$$\int_x^m \lambda e^{-\lambda(s-x)} \left(1 - e^{-\mu(t-s)}\right) ds = 1 - e^{-\lambda(m-x)} - I_k(x,t).$$

With $m = \min(t, kT)$, it follows that the probability that an individual relay will deliver the $k^{th}$ message by time $t$ is

$$Q(t) = q_k \int_{(k-1)T}^{m} \phi_k(x) \left[1 - e^{-\lambda(m-x)} - I_k(x,t)\right] dx,$$

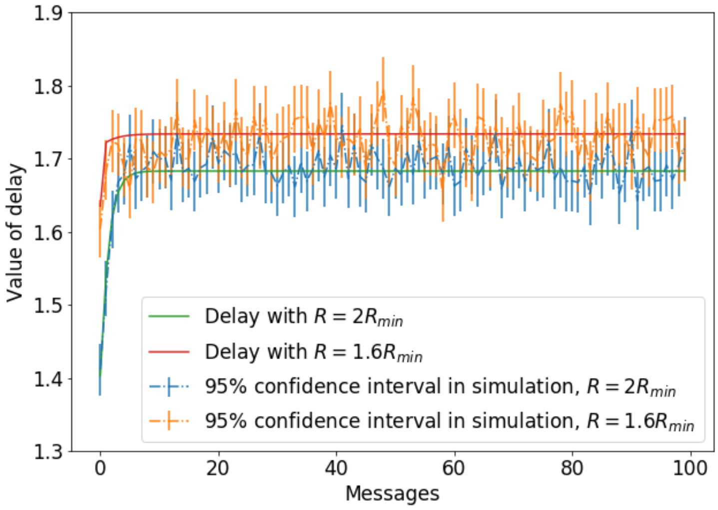



Figure 5.5: Analytical delay and simulated delay.

and hence the probability that the message is not delivered by time $t$ is $\mathbb{P}(D_k > t) = (1 - Q(t))^N$. The proof now follows from

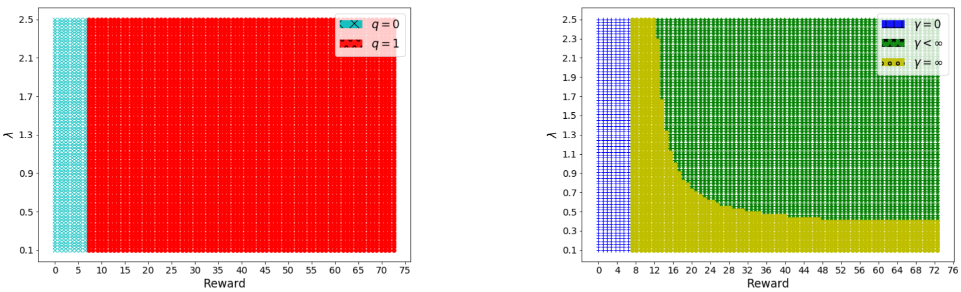
$$\begin{aligned}
\mathbb{E}(D_k|D_k < \infty) &= \int_0^\infty \mathbb{P}(D_k > t \mid D_k < \infty)\, dt, \\
&= \frac{1}{\xi_k} \int_0^\infty \mathbb{P}(D_k < \infty) - \mathbb{P}(D_k \leq t)\, dt, \\
&= \frac{1}{\xi_k} \int_0^\infty \mathbb{P}(D_k > t) - (1 - q_k p_k)^N dt.
\end{aligned}$$

□

Fig. 5.5 shows the mean message delivery time for different values of $R$. The delays obtained with event-driven simulations are also shown on the figure. The parameter values are identical to those used in Fig. 5.4.

## 5.5 Comparison between the Threshold-type Strategy and the Randomized Policy

In this section, we compare the performance obtained with the threshold-type strategy in the full information setting and with the randomized policy in the no information setting. We first consider the case where the source proposes each message for the same amount of time in both settings, that is, $T = \theta_k$ for the $k^{th}$ message ($\theta_k$ and $\gamma_k$ are the first and second thresholds, respectively, for the $k^{th}$ message). Fig. 5.6 shows the structure of the Nash equilibrium strategies for the first message in both settings. It turns out that the randomized policy is either to reject the message ($q = 0$) or to accept it ($q = 1$) depending on the value of $R$, but independently of the value of $\lambda$. In contrast, the value of $\gamma$ in the threshold-type policy depends on the value of $\lambda$. We emphasize that when $q = 1$ and $\gamma = \infty$, the two policies coincide: all relays accept the message as long as it is proposed by the source and keep it until they meet the destination (this is not the case when $\gamma < \infty$ since relays can drop the message before meeting the destination). Therefore, in this situation, the source does not need to provide the birth-time of its messages. Moreover, the relays do not need to take care of time, they just decide to accept a message or not, and then keep the message until meeting the destination. Fig. 5.7 compares the message




(a) Randomized Policy (b) Threshold-type Policy

Figure 5.6: Randomized and threshold-type policies as functions of $R$ and $\lambda$ for the first message when $T = \theta_1$. The values of the parameters are $\mu = 0.4$, $C_d = 2$, $C_r = 4$, $C_s = 0.5$ and $N = 3$.

delivery probabilities in both settings as $T$ varies. In this case, we consider the steady-state message delivery probabilities, which are obtained as $k \to \infty$, for two different values of $R$. The figure also shows the asymptotic value of the acceptance probability $q_\infty$ in the no information setting. For $R = 2R_{min} = 10$, we have $\theta = 0.65$ and $\gamma = \infty$ for the threshold policy. We observe that the message delivery probability in the no information setting increases as $T$ grows: for $T \leq \theta$, the acceptance probability $q_\infty = 1$ and the message delivery probability is lower than in the full information setting.

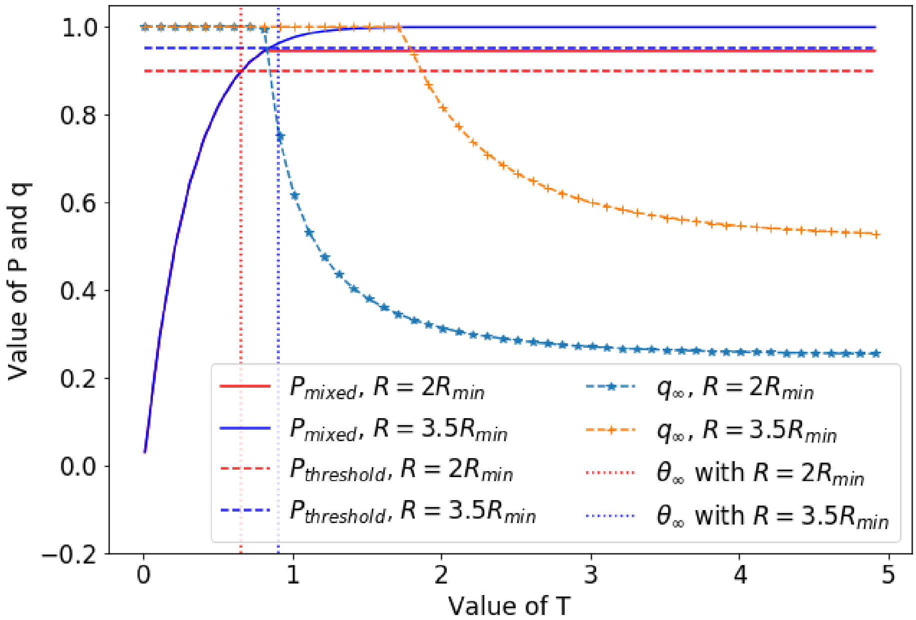



Figure 5.7: The message delivery probability in mixed strategy and threshold strategy, with $\mu = 0.4$, $C_d = C_r = 2$, $C_s = 0.4$, $N = 10$ and $\lambda = 1.5$.

Both settings coincide when $T = \theta$, as expected. For $T > \theta$, the acceptance probability $q_\infty < 1$, but the message delivery probability keeps increasing until it reaches its limiting value, which is higher than in the full information setting. For $R = 3.5R_{min} = 17.5$, we have $\theta = 0.91$ and $\gamma = 3.07$. We observe a similar behavior of the message delivery probability in the no information setting, despite the fact that in this case $\gamma < \infty$. These results suggest that by using a value of $T$ slightly larger than $\theta$, and for the same reward value $R$, the source can increase its message delivery probability if it does not tell the relays when a message was generated.

## 5.6 Conclusions

We analyzed a competitive DTN game between $N$ relays in which the source does not give information on the message generation times to the relays. The equilibrium obtained is a mixed one in which a relay accepts a message with a certain probability. This contrasts with the threshold-based equilibrium in

Chapter 3 in which the source gave message generation information to the relays. Simulations suggest giving no information on the message generation times can be advantageous to the source compared to giving information. By taking the duration for which a message is proposed to be slightly longer than the equilibrium threshold in Chapter 3, the source can improve the limiting value of its message delivery probability.

# Chapter 6

# Discussion

We have proposed and analyzed an reward incentive mechanism for DTNs with one fixed source and one destination. We assumed that nodes move randomly in the network and the inter-contact time with the source, the destination follows an exponential distribution. If a node can store unlimited messages, then they will treat all the messages as the first one. So we assume nodes can only store one message at a time. With these assumptions, we proved the optimal policy and gave the formulas and the algorithm to compute the symmetric equilibrium. We then discussed the mean-field limit when the number of players and the number of messages are large. In other setting, we assumed that nodes do not know any information and they follow a symmetric randomized policy. In this case, we proposed the condition to have all nodes participate in the delivery game with probability 1. Otherwise, they will play with a probability $p$ which is the unique solution of a given equation.

In the future work, we may consider every node as a source and as a destination as well as a relay. Each node has its own message to send to a destination, the destination can be different from message to message. In this case, the probability that a relay transmits a message before given time $t$ is needed to consider very carefully since we have to consider all the possible cases such as who is the source of that message, at what time it has the message, how long it may keep, etc.

Next step, to make the problem more general, we can relax the exponential distribution of the inter-contact time between the source, the destination and relays. With this assumption, it may complicated to find the probability of being the first relay to transmit the message. However, we still can prove the threshold policy if we have the Hazard rate of the inter-contact time is increasing. Another possible direction is letting a relay can store multiple but limited number of message at a time. Together with this assumption, we can let the source have messages follows a distribution.

In some general cases, to compute the exact formula is not simple, we may develop an learning algorithm to find the best response. One example is using reinforcement learning. Notice that, in this problem, the action space is quite large since it depends on the time. One possible solution is using function approximation in reinforcement learning or using Deep Q-Network where people use neural network to approximate the Q-value function.

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